Measurement 95 (2017) 193–200

Contents lists available at ScienceDirect

Measurement

journal homepage: www.elsevier.com/locate/measurement

Bootstrap-based frequency estimation method

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ARTICLE INFO

Article history:
Received 18 October 2015
Received in revised form 26 September 2016
Accepted 6 October 2016
Available online 8 October 2016

Keywords:
Frequency estimation
Resampling
Bootstrap
Sample mean

ABSTRACT

In this paper, the bootstrap method was used to estimate the frequency of a signal. Here, the periodic signal was transformed into a rectangular wave and the interval of time between two consecutive positive edges of this rectangular wave was estimated. First, in order to estimate this interval of time, a counter was used to count the number of clock pulses that have occurred during every interval of this type. In this case, the clock frequency was the sampling frequency. Next, this process was repeated several times and a vector of estimated values of the period of the signal was created. After that, this vector was considered as a random sample and it was used to create thousands of resamples with the same size as the above-mentioned random sample and that were created by sampling with replacement from the original random sample. Then, for each resample, the sample mean was calculated and the inverse of the mean of these sample means was used as an estimation of the frequency of the signal. Finally, a prototype of this statistic method to frequency estimation was built. The results of the experiment were satisfactory.

1. Introduction

In this paper, a bootstrap-based method for the estimation of the frequency of a signal is presented. Here, the periodic continuous-time signal under analysis was sampled by using periodic sampling but, instead of obtaining a discrete-time representation of it, the main interest was to have a good approximation of the period of the signal by counting the clock pulses that have occurred during each cycle of it. The clock frequency was the sampling frequency and the numerical value of the reading of the counter was multiplied by the sampling period, in order to have an estimation of the period of the signal. Then, the inverse of this estimation was an estimation of its frequency, as well. However, the frequency estimation value that is obtained from this simple process should be improved, because in real life applications signals are corrupted by noise and interferences, and the electronic circuits that are used to process them do not have an ideal performance. Nevertheless, if this process is repeated several times and the readings of the counter are stored, then a vector of samples that come from a random process can be built, and this vector can be used to create new vectors by sampling with replacement from the original random sample (i.e., the original vector). Thus, this vector can be used to infer information about the population from which it was drawn, because it represents this population, and the population mean, among other parameters, can be estimated from the corresponding sample mean (i.e., the corresponding sample statistic). Therefore, the resamples obtained from this vector could be seen as being samples that would have been obtained if many samples of the population had been taken. In short, the frequency estimation method that is presented in this paper is based on this fundamental idea to estimate the mean value of the frequency of a signal by using the bootstrap method [1,2].

Other engineering applications of the bootstrap method for the estimation of parameters of a signal have been reported in the scientific literature. Some of these applications in which the estimation of the frequency of a signal is of interest are the following: In [3], the authors used a bootstrap procedure for selecting the model and the model order of a polynomial phase signal using both the method of least-squares and the polynomial-phase transform. That paper focussed on modeling polynomial phase signals, the authors considered the case where the distribution of the additive noise is unknown, and the number of observations is small. However, the distribution of the noise is not relevant, provided that it

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http://dx.doi.org/10.1016/j.measurement.2016.10.012
0263-2241/

has a finite variance. Then, the modeling involved the selection of the model and the conditional estimation of the parameters. In [3], in order to estimate the coefficient associate with the highest order of the polynomial expansion of the phase, the discrete polynomial-phase transform [4] was used to estimate the frequency of the resulting transformation, provided that the order of the phase is known. In addition, in [5] the authors presented a detection scheme that was a test that used the peak of the periodogram to estimate the signal parameters and the bootstrap to estimate the distribution of a test statistic, which was based on a consistent estimator for the amplitude of the sinusoidal signal that represented the model for the problem under analysis in that paper and a studentized version it. Also, in that paper the model was a sinusoid with constant amplitude corrupted by a wide-sense stationary, zero-mean, weakly dependent interference process of unspecified distribution. In [5], the amplitude and the phase were unknown. Furthermore, in [6] the authors presented general bootstraps procedures that were applied to an approach to detect multiple signals embedded in noisy observations from a sensor array.

In addition, in [7] the authors propose a frequency domain bootstrap method for approximating the distribution of Whittle estimators. And, in [8] a dynamic bootstrap grey method is proposed in order to process multi-sensor stress measurement results with poor information.

The bootstrap-based frequency estimation method that is presented in the present paper is not based on the application of transforms to the signal. Also, having information about the model of the signal is not relevant to estimate the frequency of it. This frequency estimation method is a time-domain method that can be used to estimate the frequency of any real-valued, periodic signal that meet the Dirichlet conditions [9]. In short, these conditions are: the periodic signal must be absolutely integrable over a period and of bounded variation over a period.

In addition, the bootstrap-based frequency estimation method presented in this paper is not a filter-based method, signal filtering is not necessary, and information about the distribution of the noise is not relevant to estimate the frequency of the signal. Moreover, in this paper it has been shown that this method performed satisfactorily in the presence of noise, it is neither a recursive method nor an adaptive one, therefore it does not have convergence problems. Also, it is not necessary to invert any matrix, which avoids problems of dealing with ill-conditioning systems [10], the implementation of the algorithm is not computationally intensive and does not require considerable computational resources, the behavior of the frequency estimation method is not characterized by sets of differential equations, the electronic circuitry that is involved here is very simple, and the implementation of this method is easy.

In the scientific literature, also there is a wide range of papers that present frequency estimation methods of signals. The signals under test in these papers, as in most of the engineering applications, meet the Dirichlet conditions as well. However, they are not bootstrap-based methods. For example, in [11] an iterative interpolation method for estimation parameters that characterize a linear combination of sinusoids is presented. In [12], a comparative evaluation between different algorithms to estimate frequency for power quality assessment is presented. In [13], a method to estimate time-varying frequency for single-phase electric power systems based on three equally spaced samples is presented. Algorithms for the frequency estimation of a signal that are based on the fast Fourier transform, the discrete Fourier transform, and the discrete cosine transform can be found in [14–18]. A fast and accurate algorithm for frequency, amplitude and phase estimation of the signals with white Gaussian noise is presented in [19]. Frequency estimation methods based on Newton-type algorithms can be found in [20,21]. In addition, a supervised Gauss-Newton algorithm for power system frequency measurement is presented in [22]. Moreover, an algorithm for parameter estimation of two channel single-tone signals that uses least squares optimization to estimate the amplitudes, phases and common frequency of the signals is presented in [23]. Other applications of the method of least squares to the estimation of the frequency of a signal can be found in [24–26]. Furthermore, a frequency measurement method based on an enhanced version of the zero-crossing technique is presented in [27]. Also, a Doppler Effect removal based on instantaneous frequency estimation and time domain re-sampling for wayside acoustic defective bearing detector system is presented in [28]. In that paper, the Short Time Fourier Transform-Viterbi algorithm based instantaneous frequency estimation is applied to obtain the fitting instantaneous frequencies, which are employed with the Morse theory to attain parameters necessary for re-sampling of the signal in the time domain. Phase-Locked Loop (PLL) and adaptive notch filters (ANFs) have also been applied to the frequency estimation problem [29–32]. In accordance with [25], the principal idea of phase locking is to actively generate a signal whose phase angle is adaptively tracking variations of that of a given signal by means of a control loop, whereas the ANF is a concept similar to the PLL in the sense that it passively extracts its phase angle output from the given signal. The dynamic behavior of ANFs is characterized by a set of differential equations, and the frequency estimation systems that are based on ANF need appropriate pre- and postfiltering stages [31]. Also, a tradeoff between good dynamics and estimation accuracy is also required while tuning the parameters of the PLL [20].

In short, the above-mentioned papers are just a small sample of the large amount of research works that have been reported in the scientific literature on the design of algorithms to carry out the estimation of the frequency of signals under test. The present paper is aimed at presenting a simple bootstrap-based frequency estimation method. The organization of this paper is as follows: Section 2 presents the frequency estimation method. Section 3 presents the experimental results. And the conclusions are given in Section 4.

2. Frequency estimation method

2.1. Basic idea of the bootstrap

The most important idea of the bootstrap method [1,2] is that, in the absence of information about a population from which we are interested in estimating unknown parameters of it (such as the mean), samples collected from this population carry with them sufficient information about the population from which they have been extracted and therefore resampling of these samples using sampling with replacement allows us to calculate the statistics for each resample. The distribution of these resample statistics is a bootstrap distribution. Thus, we can estimate unknown population parameters from the knowledge of their corresponding sample statistics. Furthermore, it is important to point out that it is assumed that there is a relationship between the population of interest and the observed samples from this population. Therefore, resamples that have been created from the original random sample represent what would have been obtained if we had taken many samples of the population.

One thing to note is that when the bootstrap method is applied we are not using the resamples as if these observations were real data. The bootstrap does not replace in any way that collect more samples of the population helps us improve the accuracy of the analysis of the population. Actually, for the problem under study, what the bootstrap does is to use the original random sample to estimate how its sample mean varies due to random sampling.