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A comparative overview of frequency domain methods for nonlinear systems☆,☆☆

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ABSTRACT

The widespread acceptance of frequency domain techniques for linear and time invariant systems has been an impetus for the extension of these methodologies toward nonlinear systems. However, differences and equivalences between alternative methods have been less addressed. This paper provides a comparative overview of four classes of frequency domain methods for nonlinear systems: Volterra based models, nonlinear frequency response functions / Bode plots, describing functions and linear approximations in the presence of nonlinearities. Each method is introduced using consistent nomenclature and terminology, which allows for comparison in terms of system and signal classes for which the methods are valid as well as the type of (nonlinear) effects captured by each model. Summarizing, the paper aims to connect, and make different frequency domain methods for nonlinear systems accessible, by providing a comparative overview of such methodologies, accompanied by illustrative (experimental) examples.

1. Introduction

For linear and time invariant (LTI) systems, frequency domain techniques resulted in a widespread acceptance in the engineering community for analysis, modeling and controller design [9]. Correspondingly, the Frequency Response Function (FRF) and representations such as the Bode, Nyquist and Nichols plot have become standard engineering tools and have proven to be indispensable for the modeling and design of dynamic systems in industry. Increased performance requirements of these systems result in a different attitude toward nonlinear system behavior. Previously a certain degree of nonlinear behavior could be ignored because it did not impair system performance. In many present day dynamic systems however nonlinear behavior limits performance or is deliberately introduced to improve performance. See for example [45,89]. In both cases nonlinear effects can no longer be neglected. Hence, the widespread acceptance of frequency domain techniques for LTI systems has been an impetus for the extension of these methodologies toward nonlinear systems.

Although different approaches have been independently developed to analyze and model nonlinear systems in the frequency domain, differences and equivalences between alternative methods have been minorly addressed. In this paper an overview and comparison of the following four well established approaches is presented:

- Volterra series based approaches (Section 3)
- Frequency response function and Bode plot for nonlinear systems (Section 4)
- Describing functions (Section 5)
- Linear approximations in the presence of nonlinearities (Section 6)

Table 1 provides a comparative overview of the main characteristics of these modeling approaches and constitutes one of the

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Table 1
Overview of frequency domain modeling approaches for nonlinear systems. Legend: dark grey: effect captured / information available, light grey: effect partially captured / requires additional processing, white: effect not captured / information not available. For a description of the different frequency domain effects, see [98].

		captured nonlinear effect				information content			
		gain compression / expansion	desensitization	intermodulation	harmonics	at frequencies present in input		at frequencies not present in input	
		dependence of the gain on the input excitation level	dependence of the response at one frequency on the input at another frequency	input frequencies combine to produce new frequencies in the output	generation of frequency components in the output at multiples of the input frequency	gain	phase	gain	phase
conventional FRF						dark grey	dark grey		
generalized FRF		dark grey	dark grey	dark grey	dark grey	dark grey	dark grey	dark grey	dark grey
nonlinear FRF		dark grey			dark grey	dark grey	dark grey	dark grey	dark grey
describing funct.	**sinusoidal**	dark grey				dark grey	dark grey		
	generalized	dark grey	dark grey			dark grey	dark grey		
	HOSIDF	dark grey			dark grey	dark grey	dark grey	dark grey	dark grey
best linear appr.		dark grey	dark grey	light grey	light grey	dark grey	dark grey	light grey	

main results of this paper. This comparison is further motivated in the sequel.

First, a brief introduction to the effects of nonlinearities in the frequency domain is presented in Section 2. Next, Sections 3 – 6 introduce each model class. This includes a definition of the system and signal class for which the model types are defined. The notation and terminology used in the different sections are consistent, which allows for comparison between different approaches.

A brief overview of relevant literature is provided for each method (see Table 4, Appendix B, for a summary). Moreover, each section is concluded by either an analytical example or an experimental case study, illustrating the introduced approach. Finally, in Section 7 a comparison of the properties of the different model types is provided as well as an assessment of their applicability in practice. Finally, note that the required nomenclature and preliminaries are presented in Appendix A to increase readablity of the paper.

2. Nonlinearities in the frequency domain

A fundamental property of LTI systems is that it cannot shift energy from one frequency to the other. Hence, the response to a sinusoidal input with a particular frequency is again sinusoidal with the same frequency as the input signal. The phase shift and gain relating the input and output are characterized by the FRF at that particular frequency and as superposition holds for LTI systems, the response to more general input signals is fully captured by the FRF as well. However, for nonlinear systems the superposition principle does not hold and the response to even a simple sinusoid can be a complex, multi harmonic signal. However, the increased richness in the output allows for effective identification and reduction/utilization of nonlinear effects. See for example [23,70,72,74,85]. Hence, classical frequency domain approaches for LTI systems cannot be straightforwardly applied when nonlinearities are present and additional analysis is required to investigate if, when and how similar methodologies can be used when analyzing nonlinear systems.

In the frequency domain, nonlinear effects manifest themselves in different ways. The gain of a nonlinear system may, for example, depend on the amplitude of the input and the output may contain harmonics of the input frequency. Moreover, when a nonlinear system is subject to a multisine input, input frequencies may combine, producing new frequencies which are not present in the input signal and spectral components at a given frequency in the output may depend on other frequencies in the input. These effects are summarized in Table 1 and they are used in the following to discuss different frequency domain models for nonlinear systems. To further illustrate the effects listed in Table 1, consider the following example.

Example 1 (effects of nonlinearities in the frequency domain)**.** Consider the following harmonic signal with nonzero spectral contributions at $1\,[Hz]$ and $5\,[Hz]$.

$$u(t) = a\cos(1\cdot 2\pi t) + b\cos(5\cdot 2\pi t)$$

Now consider a static nonlinear mapping $y(t) = 4u^3(t)$, which yields:

$$\begin{aligned} y(t) = \; & \underbrace{\left(3a^3 + 6ab^2\right)\cos(1\cdot 2\pi t)}_{\text{gain comp./exp. \& desensitization}} + \underbrace{\left(a^3 + 3a^2b\right)\cos(3\cdot 2\pi t)}_{\text{harmonics \& intermodulation}} \\ & + \underbrace{\left(6a^2b + 3b^3\right)\cos(5\cdot 2\pi t)}_{\text{gain comp./exp. \& desensitization}} + \underbrace{3a^2b\cos(7\cdot 2\pi t)}_{\text{intermodulation}} \\ & + \underbrace{3ab^2\cos(9\cdot 2\pi t)}_{\text{intermodulation}} + \underbrace{3ab^2\cos(11\cdot 2\pi t)}_{\text{intermodulation}} \\ & + \underbrace{b^3\cos(15\cdot 2\pi t)}_{\text{harmonics}} \end{aligned} \tag{1}$$

The expression in (1) clearly shows the different effects indicated in Table 1. First of all, multiples of the input frequencies $1\,[Hz]$ and $5\,[Hz]$ appear at $3\,[Hz]$ and $15\,[Hz]$ (harmonics). Moreover, the $1\,[Hz]$ and $5\,[Hz]$ components in (1) depend nonlinearly on the input amplitude (gain compression / expansion) and depend on the input at other frequencies as well (desensitization). Finally, the nonlinear mapping combines input frequencies and their sum and difference frequencies to new frequencies at 3,7,9 and $11\,[Hz]$ (intermodulation).

These effects are visualized in Fig. 1 where the nonlinear deformation of the signal $u(t)$ is depicted for $a = 2$ and $b = 3$. Fig. 1a and b show the deformation of the signal in the time domain. The