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Extension of the linear matching method to geotechnical problems

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Abstract

An extension of the linear matching method for shakedown is given here for the Drucker–Prager yield condition as an approximation to the Mohr Coulomb slip condition for geotechnical materials. For this yield condition, the established criterion for the application of the method are not satisfied. We here describe adaptations of the method that yields converged solutions. The method is applied to a simple indentation problem as well as 3D rolling contact problem for both an associated and a non-associated flow rules. The developed method is appropriate for the design analysis associated with road pavements and railways foundations.

Keywords: Shakedown analysis; Finite elements; General yield conditions; Geotechnics

1. Introduction

Despite the considerable progress that has been made in applying finite element analysis methods to the field of structural engineering, there remain problems for which standard methods are extremely cumbersome and computationally intensive. This is particularly so for problems involving repeated rolling contact loading that occurs in road pavement and rail track foundation design. Design procedures remain reliant on linear elastic solutions. There is a need for simple, economical and, at the same time, efficient methods capable of describing significant design limits for structural behaviour in such cases.

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An attractive alternative to limit analysis theory for analysing the response and the stability of such geotechnical structures is to use the bound theorems of shakedown analysis, coupled with the finite element method. Such methods have two notable advantages. Firstly, it addresses the steady state directly without having to follow the plastic loading history from the start. Secondly, such methods provide, directly, the load levels at which a design significant mode of behaviour begins to occur, the onset of cyclic plastic strains in the steady state. Estimates of the critical shakedown limit load can be found by application of the two extremum shakedown principles, the Melan's [15] lower bound theorem and Koiter's [12] upper bound theorems, for elastic–perfectly plastic structures. When operating at levels below this critical level the structure will eventually "shakedown", and the ultimate response will be purely elastic, even though the response may well be plastic for a finite number of initial load applications. Above this critical level, however, the structure continues to exhibit plastic strains for however long the repeated loads are applied. This theory has been used very successfully to model various aspects of the deformation of metallic surfaces subjected to rolling and sliding loads (e.g. Ponter et al. [18], Johnson [11]). Further studies of increasing realism and complexity, related to pavement design, have been made by Collins and Cliffe [6], Collins and Boulbibane [7] and Boulbibane et al. [3] among others.

There is a wide range of approaches to the evaluation of shakedown limits in the literature. For example Casciaro and Garcea [5] have recently proposed an incremental iterative method suitable for FEM analyses for defining shakedown boundaries of frame structures, and there are many others. Here we concentrate upon a particular method, the linear matching methods [19–21], which involves the matching of the non-linear material behaviour to a linear material. This forms the basis for a powerful upper bound programming method that may be applied to significant classes of problems. As well as shakedown limits, the method has been applied successfully with considerable rigor to the evaluation of the ratchet limit of cracked bodies (Ponter et al. [21]) and the evaluation of creep design limits for a body subjected to cyclic thermal loading for finite cycle times (Boulbibane and Ponter [2]). In all this work the material deformation behaviour was assumed to be pressure insensitive. It is well known that the strength of granular materials depends on the effective confining pressure, and the load bearing capacity therefore decreases with decreasing confining pressure. Reduction in effective confining pressure may also be caused by increasing pore pressure and will result in failure. An extension of the linear matching method to a general yield condition that includes both the effective von Mises stress and the hydrostatic pressure has been given by Ponter et al. [19], Ponter and Engelhardt [20] and Parrinello and Ponter [17]. For this case, both shear and bulk moduli of the linear material are adjust so that, for the current strain rate distribution, the linear material is matched to the yield condition. Essentially, the surface of complementary energy density for the linear material, which touches the yield surface at the position of the current plastic strain rate, may lie either outside or inside the yield surface in stress space. A sufficient condition for convergence is then given by the condition that the surface of constant complementary energy lies entirely outside the yield surface (Ponter et al. [19]). At the present time it is not clear whether the condition is also necessary for convergence.

A first attempt to understand the necessity of the convergence condition has been given by the authors [4] by studying a class of elliptic yield conditions (in effective stress–pressure space). In this way, it is possible to compute solutions where the sufficient condition is both satisfied and not satisfied by varying the effective Poisson's ratios of the linear material. From this study we showed that convergence occurred even if the sufficient condition was not satisfied; it was very difficult to find conditions where convergence did not occur. This paves the way for the study in this paper, where the method is applied to the Drucker–Prager yield condition, which is often used as an approximation to the classic Mohr–Coulomb condition. Both associated and a non-associated flow rules are considered. In this case the sufficient condition is never satisfied and there are difficulties over the matching condition. We show that a reinterpretation of the matching condition provides a convergent process. The method is applied to indentation problems and the problem of a 3D homogeneous half-space subjected to rolling sliding contact. The quality of the solution is estimated by a comparison with the semi-analytic solution given by Ponter et al. [18] and the Prandtl indentation solution.