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Compressed-sensing wavenumber-scanning interferometry

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ABSTRACT

The Fourier transform (FT), the nonlinear least-squares algorithm (NLSA), and eigenvalue decomposition algorithm (EDA) are used to evaluate the phase field in depth-resolved wavenumber-scanning interferometry (DRWSI). However, because the wavenumber series of the laser's output is usually accompanied by nonlinearity and mode-hop, FT, NLSA, and EDA, which are only suitable for equidistant interference data, often lead to non-negligible phase errors. In this work, a compressed-sensing method for DRWSI (CS-DRWSI) is proposed to resolve this problem. By using the randomly spaced inverse Fourier matrix and solving the underdetermined equation in the wavenumber domain, CS-DRWSI determines the nonuniform sampling and spectral leakage of the interference spectrum. Furthermore, it can evaluate interference data without prior knowledge of the object. The experimental results show that CS-DRWSI improves the depth resolution and suppresses sidelobes. It can replace the FT as a standard algorithm for DRWSI.

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1. Introduction

Depth-resolved wavenumber-scanning interferometry (DRWSI) is an extension of the traditional phase-shifting technique [1]. It uses phase information of the Fourier transform in the frequency domain to measure three-dimensional profiles and displacements inside a structure with a high precision [2,3]. In general, three kinds of algorithms are employed for data evaluation in DRWSI: the Fourier transform (FT) [2], the nonlinear least-squares algorithm (NLSA) [4,5], and the eigenvalue decomposition algorithm (EDA) [6]. However, because the wavenumber series of laser's output in DRWSI is usually accompanied by nonlinearity and mode-hop [7], FT, NLSA, and EDA, which are only suitable for equidistant interference data, often lead to non-negligible phase errors.

To address this problem, two solutions have been reported: (1) the linear interpolation of the wavenumber series [8] and (2) the random-sampling Fourier transform (RSFT) in the wavenumber domain [7]. Interpolation is equivalent to low-pass filtering and thus results in the loss of the high frequencies in the spectrum. Moreover, the deviation of the interpolated data from the measured data can easily produce distortion in the spectrum. The RSFT yields less spectrum distortion than linear interpolation but has two weaknesses: (i) the spectral leakage of the RSFT is the same as that of the FT, which makes the depth resolution and the phase

accuracy lower than those of NLSA and EDA; and (ii) as the wavenumber scans with the mode-hop, the RSFT is equivalent to convolute multiple windows. Consequently, the sidelobes in the amplitude spectrum are augmented, which often suppresses the mainlobes [7,9].

In 2006, Candès et al. proposed the compressed-sensing theory (CST) for reconstructing a signal in sub-Nyquist sampling if the signal exhibits sparsity or compressibility [10]. In DRWSI, the sampled interference signal in the wavenumber domain appears as a low-dimensional non-sparse vector owing to the limited range of wavenumber scanning, whereas the interference spectrum after using DTFT (Discrete-time Fourier Transform) is a high-dimensional sparse vector. The key for evaluating DRWSI data is obtaining the interference spectrum from the sampled interference signal. Therefore, the problem to be solved in DRWSI is coincident with the topic of CST.

In this work, a compressed-sensing theory for DRWSI (CS-DRWSI) is presented to replace the FT while improving the depth resolution and suppressing the sidelobe, when the nonlinearity and mode-hop of wavenumber series occur. The experimental results show that the CS-DRWSI can replace the FT as a standard algorithm for data evaluation in DRWSI. The remaining part of this paper is organized as follows. In Section 2, an optical setup of DRWSI is introduced. In Section 3, the compressed-sensing theory for DRWSI is presented. In Section 4, an experiment, in which the wavenumber series scanned nonlinearly and mode-hop, was done

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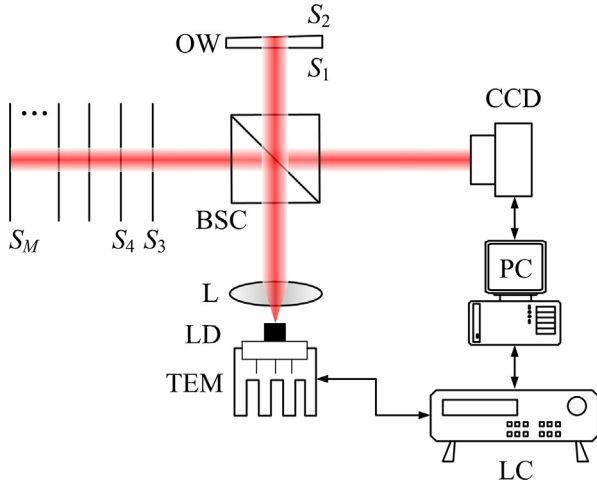


Fig. 1. Optical setup. TEM is a temperature module; LD is a laser diode; LC is a laser controller; PC is a personal computer; L is a convex lens; OW is an optical wedge; BSC is a beam-splitter cube; CCD is a CCD camera; $S_1, S_2, S_3, \dots, S_M$ are the measured surfaces in a stack.

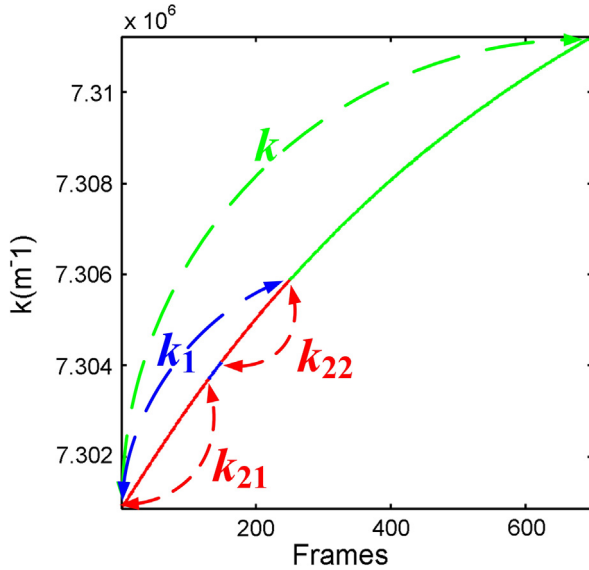


Fig. 2. Wavenumber series k, k_1 , and k_2 , where k_1 has a narrow scanning range, and k_2 scans with mode-hop.

to verify the CS-DRWSI. Finally, the conclusion and discussion are provided in Section 5.

2. Optical setup

As shown in Fig. 1, the optical setup employed for DRWSI was based on a Michelson interferometer. The light source was a distributed-feedback laser diode (LD-0860-0150-DFB-1) driven by a laser controller (ILX Lightwave Co., LDC-3724 C). A CCD camera (VDS Vosskuhler GmbH, 1280×1024) was used to acquire a wavenumber-domain interferogram. An optical wedge, whose optical path difference (OPD) was set as $\Lambda_{120} = 9.06$ mm at $x = 0$ mm and $y = 0$ mm, served two functions: providing a reference plane and monitoring the output of the laser [7].

3. Compressed-sensing theory for DRWSI

The light reflected from multiple surfaces S_1, S_2, \dots, S_M in a stack forms interferograms [7], as follows:

$$I(x, y, \mathbf{k}) = \sum_{p=1}^M \sum_{q=1}^M \sqrt{I_p(x, y) \cdot I_q(x, y)} \cdot \cos[2\pi \cdot \Lambda_{pq}(x, y) \cdot (\mathbf{k} - \mathbf{k}_1) + \phi_{pq}(x, y)]$$

$$\phi_{pq}(x, y) = 2 \cdot k_1 \cdot \Lambda_{pq}(x, y) + \phi_{pq0}(x, y), \quad (1)$$

where vector $\mathbf{k} = [k_1, k_2, \dots, k_N]^T$ is the wavenumber series of the laser's output, N is the total number of frame index acquired by a camera; I_p ($p = 1, 2, \dots, M$) is the reflected light intensity from S_p ; Λ_{pq} and ϕ_{pq0} ($p = 1, 2, \dots, M, q = p + 1$) are the optical path difference and the initial phase difference between S_p and S_q , respectively; and (x, y) are the spatial coordinates, which are omitted in the following text below.

In DRWSI, the depth-resolved phase maps $\phi_{pq}(x, y)$ are determined from the interference spectrum [7,8]. To evaluate the interference spectrum, an inverse Fourier transform of the interference signal is written in a matrix form:

$$I(\mathbf{k}) = \hat{\mathbf{F}} \cdot \tilde{I}(\mathbf{f}), \quad (2)$$

where,

$$I(\mathbf{k}) = [I(k_1) \ I(k_2) \ \dots \ I(k_N)]_{1 \times N}^T,$$

$$\tilde{I}(\mathbf{f}) = [\text{conj}(\tilde{I}(f_1)) \ \text{conj}(\tilde{I}(f_2)) \ \dots \ \text{conj}(\tilde{I}(f_L))]_{1 \times L}^H,$$

$$\hat{\mathbf{F}} = [\hat{f}_1 \ \hat{f}_2 \ \dots \ \hat{f}_L]_{N \times L}^T,$$

$$\hat{f}_l = [1 \ \exp[-j \cdot 2\pi \cdot f_l \cdot (k_2 - k_1)] \ \dots \ \exp[-j \cdot 2\pi \cdot f_l \cdot (k_N - k_1)]]^H, \\ l = 1, 2, \dots, L. \quad (3)$$

Here, the superscripts T and H represent the transposition and conjugate transpose, respectively; “conj” is the conjugate operator; $\mathbf{f} = [f_1, f_2, \dots, f_L]^T$ is the frequency vector in the Fourier space; L is the number of data points in the frequency domain; j is the imaginary unit; $\hat{\mathbf{F}}$ is the partial inverse Fourier matrix, which connects the interference signal $I(\mathbf{k})$ and the interference spectrum $\tilde{I}(\mathbf{f})$. Notably, because of the limited wavenumber scanning range Δk , the dimension of $\hat{\mathbf{F}}$ meets the condition of $N < L$.

In Eq. (2), the interference spectrum $\tilde{I}(\mathbf{f})$ via a recorded interference signal $I(\mathbf{k})$ is reconstructed using the ℓ_1 minimization as follows,

$$\begin{cases} \min \sum_{i=1}^L \{ |Re[\tilde{I}(f_i)]| + |Im[\tilde{I}(f_i)]| \} \\ \text{subject to } I(\mathbf{k}) = \hat{\mathbf{F}} \cdot \tilde{I}(\mathbf{f}) \end{cases}, \quad (4)$$

where the symbols “Re” and “Im” represent the real and imaginary parts of a complex number, respectively.

The constraint of Eq. (4) is rewritten as the real and imaginary parts:

$$I(\mathbf{k}) = \{Re(\hat{\mathbf{F}}) \cdot Re[\tilde{I}(\mathbf{f})] - Im(\hat{\mathbf{F}}) \cdot Im[\tilde{I}(\mathbf{f})]\} + j \cdot \{Re(\hat{\mathbf{F}}) \cdot Im[\tilde{I}(\mathbf{f})] + Im(\hat{\mathbf{F}}) \cdot Re[\tilde{I}(\mathbf{f})]\}. \quad (5)$$

Because the interference signal $I(\mathbf{k})$ is presented as the real numbers, the Eq. (5) is converted into the field of real number as follows,

$$\begin{cases} Re(\hat{\mathbf{F}}) \cdot Re[\tilde{I}(\mathbf{f})] - Im(\hat{\mathbf{F}}) \cdot Im[\tilde{I}(\mathbf{f})] = I(\mathbf{k}) \\ Re(\hat{\mathbf{F}}) \cdot Im[\tilde{I}(\mathbf{f})] + Im(\hat{\mathbf{F}}) \cdot Re[\tilde{I}(\mathbf{f})] = \mathbf{0} \end{cases}. \quad (6)$$

After the Eq. (6) is reformulated as a matrix equation, the Eq. (4) becomes

$$\begin{cases} \min \sum_{i=1}^L \{ |Re[\tilde{I}(f_i)]| + |Im[\tilde{I}(f_i)]| \} \\ \text{subject to } \begin{bmatrix} I(\mathbf{k}) \\ \mathbf{0} \end{bmatrix}_{2N} = \begin{bmatrix} Re(\hat{\mathbf{F}}) & -Im(\hat{\mathbf{F}}) \\ Im(\hat{\mathbf{F}}) & Re(\hat{\mathbf{F}}) \end{bmatrix}_{2N \times 2L} \cdot \begin{bmatrix} Re[\tilde{I}(\mathbf{f})] \\ Im[\tilde{I}(\mathbf{f})] \end{bmatrix}_{2L \times 1} \end{cases}. \quad (7)$$

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