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New method for determination of diffraction light pattern of the arbitrary surface

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ABSTRACT

Diffraction phenomena have a special importance in optics. Due to their complex nature, diffraction problems cannot be solved exactly using an analytical approach for the general case. Problems for which an exact analytical solution can be found are reduced to simple ones, with a great deal of symmetry of obstacles and slits where the diffraction occurs. On the other hand, numerical methods can be very useful in solving particular problems where parameters of obstacles or slits are known. These methods can be applied in cases when the screen is at a short distance (Fresnel diffraction) as well as at a large distance (Fraunhofer diffraction).

In this paper, the methodology for finding a solution in case of diffraction problem on arbitrary objects which are at an arbitrary distance from the screen is presented. The method is based on numerical solving of the Fresnel-Kirchhoff integral by means of discretization of an obstacle and the screen on which the diffraction pattern is observed. This method can be applied for arbitrary shapes of slits for which the equation of the surface is known as well as for an arbitrary positioned screen, located even very close to the object. The developed method is employed to determine the diffraction pattern for obstacles for which the pattern is already known from the theory. Good agreement was found.

1. Introduction

Diffraction of light is related to phenomena which can be observed when the wave encounters an obstacle or a slit. There are two types of diffractions: Fresnel diffraction is diffraction where light intensity distribution is observed on the screen which is at a short distance behind an object. Fraunhofer diffraction is diffraction where the distance between the obstacle (slit) and the screen is large [1,2]. There are plenty of papers devoted to diffraction of light passing through arbitrary objects [3–9]. In referenced papers, Fraunhofer and Fresnel diffraction for rectangular, spherical and elliptical surfaces were considered.

The Fast Fourier Transformation (FFT) can be also used for determination of diffraction pattern and it gives correct results for both, Fresnel and Fraunhofer diffraction [10]. FFT reduces calculation cost, but can only be applied to planar surfaces in parallel. In the paper [10] the authors developed model for calculation Fresnel diffraction pattern for arbitrary shape source surface, and numerical FFT was applied.

According to our knowledge, there are few papers devoted to diffraction of light on 3D (3 dimension) objects. Fraunhofer diffraction has been studied for slits in the shape of a right cone [11] and a half sphere [12] which are located in a planparalel plate. In both cases, the distribution of the intensity of light is obtained via Bessel functions.

Diffraction pattern from conical objects is of importance in analysis of shapes and dimensions of etched tracks originating from the heavy charged particles in nuclear solid state detectors (SSNTD) [13–16]. These tracks are approximately conical. Larger fraction of detected particles leaves tracks which are orientated at some angle with respect to the normal of the detector surface. These tracks can be represented as rotated cones. In order to analyze these tracks, it is necessary to understand the diffraction pattern of the light passing through the rotated cones. The shapes of these tracks are not symmetrical and their projections on the plane are not circles [17]. The method for determination of diffraction pattern from rotated cones has been previously developed by authors [18]. It was shown that characteristics of the diffraction pattern depend on cone parameters. The idea for further research is to study in detail the correspondence between the diffraction pattern and cone parameters in order to enable a reverse process - to determine cone parameters by knowing the diffraction pattern. It could open a whole field of alpha particles and neutrons spectroscopy by using SSNTD.

Considering the importance of light diffraction on 2D surfaces and 3D objects, this paper presents an effort to propose a method for

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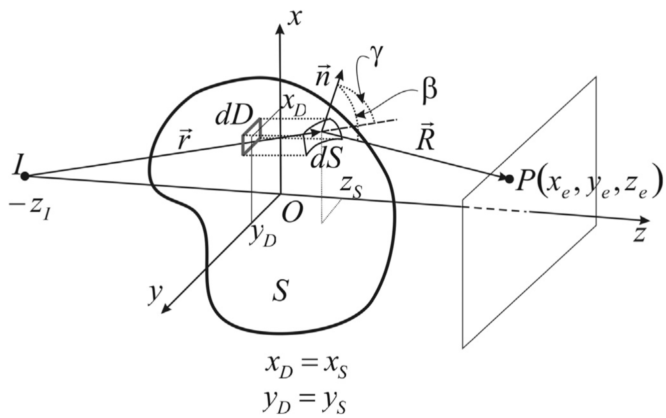

Fig. 1. The geometry of diffraction on an arbitrary surface with presented point source - I, aperture of surface - S and the screen.

determination of diffraction pattern from arbitrary objects on the screen which can be placed at an arbitrary distance. The methodology and results are presented in the text below.

2. Methodology

Let us consider light of wavelength λ, emitted from a point source I, which is diffracted by an arbitrary aperture of a surface S. The diffraction pattern is observed on the screen behind. The geometry of the considered problem with the reference coordinate system $Oxyz$ is presented in Fig. 1. The source is positioned on the z-axis, at a distance z_I from the aperture and has coordinates $(0,0,-z_I)$. The screen is a plane normal to the z-axis and at a distance z_e from the xOy plane. The segment of the aperture surface dS has coordinates (x_S,y_S,z_S) and is defined by normal vector $\vec{n}$.

With respect to the source I, the surface segment dS is defined by vector $\vec{r}$. The point P on the screen where the intensity of light was considered has coordinates $P(x_e,y_e,z_e)$ and it is defined by vector $\vec{R}$ with respect to the segment dS.

The wavefront coming from the source I reaches the surface segment $dS$and travels along towards the screen. The amplitude of the electric field oscillation of the light at point P is proportional to the segment of the surface dS and inversely proportional to the distances r and R [1,2]:

$$E_P = \iint_S -\frac{i}{\lambda}\frac{1}{rR}a(\beta,\gamma)E\, e^{i(\omega t-\vec{k}\vec{r}-\vec{k}\vec{R})}dS. \tag{1}$$

The integral shown in Eq. (1) is called the Fresnel-Kirchhoff integral. In the integrand, E is the electric field strength of the light wave of the source I, ω is the angular frequency, λ is the wavelength of light in the vacuum and k is the wave number.

The electric field strength of the wavefront, E_P at point P depends on the orientation of the segment surface, dS, which is defined by angles β and γ. The angle β is the angle between the vectors $\vec{n}$ and $\vec{R}$, while γ is the angle between the vectors $\vec{n}$ and $\vec{r}$. The factor $a(\beta,\gamma) = 0.5(\cos\gamma+\cos\beta)$ is called the inclination factor. If the distance from the aperture to the source is large, then γ is equal to 0. In that case, the plane wavefront is coming to dS. If the point P is at a large distance from the segment dS, then β=0 (Fraunhofer diffraction). When the incident wavefront is the plane wave and the type of difraction is Fraunhofer diffraction, the inclination factor is approximated by unity.

The aperture of the surface S can be represented as some function on coordinates, $F(x,y,z)=0$. Each segment dS of the surface with coordinates (x_S,y_S,z_S) must satisfy the given equation. The normal vector $\vec{n}$ can be determined as $\vec{n} = \frac{\partial F}{\partial x}\vec{e}_x + \frac{\partial F}{\partial y}\vec{e}_y + \frac{\partial F}{\partial z}\vec{e}_z$, where the condition $\frac{\partial F}{\partial z} > 0$ must be satisfied because $\vec{n}$ vector is oriented toward the z-axis.

Electric field value of the wavefront can be found at any spot on the screen according to Eq. (1). The problem that occurs when trying to solve the integral is that analytical solutions cannot be found for the general case. In that case, the integral in Eq. (1) must be solved in a non-analytical way. A variety of methods were developed for particular cases of diffraction in order to solve the integral in Eq. (1). Many of these methods are limited to specific apertures where the screen must be positioned at a large enough distance in order to consider Fraunhofer diffraction, or short enough for Fresnel diffraction [3–9]. In this paper the simple approach was derived, which leads to the solution of Eq. (1) by means of computational modeling. This method is not limited to any surface on which the diffraction occurs, as long as the equation that describes the surface, $F(x,y,z)=0$, is known. The second characteristic of the proposed method is that the screen can be positioned at an arbitrary distance from the surface, even very close to it. This method can be described in the following way.

The integral in Eq. (1) is by definition the sum of integrand over the whole surface area, when the surface element is infinitely small, $\iint_S f(x,y,z)dS \to \lim_{dS\to 0}\sum_S f(x,y,z)dS$. The idea is to substitute differential with difference calculus, where infinitively small variables are replaced with the finite ones, but very small, and the integral is replaced with the sum. The surface S needs to be divided into surface segments, dS, which are finite but small enough so that the condition $dS \to 0$ is fulfilled.

The electric field at point P on the screen, from the segment dS, which is hit by the wavefront from source I, according to Eq. (1) should bethe following:

$$dE_p = -\frac{i}{\lambda}\frac{1}{rR}a(\beta,\gamma)E\, e^{i\delta}dS \tag{2}$$

where δ is the phase displacement of the wave from the source I to the point P on the screen. The total electric field at point P, from the surface Scan be obtained by summing the fields of all surface segments dS. The intensity of light at point P on the screen is:

$$I_P = \left|\sum_S dE_p\right|^2. \tag{3}$$

Since S can be an arbitrary surface, its segmentation can vary for different shapes of the surface. It is convenient to determine the domain D of the surface$S$, which presents the projection of S on the xOy plane. The segmentation of the domain in the plane is considerably easier to carry out. Depending on the shape of the domain, Cartesian or polar coordinates can be employed in order to simplify the method. The surface segment dS can be easily obtained from domain the segmentdD. This will be explained in detail in the text below.

The methodology for determination of the intensity distribution of light on the screen can be described in the following steps.

Step 1. Define the surface equation $F(x,y,z)=0$ and distances from the screen, z_e and from the point of source, z_I.

Step 2. Define a point P on the screen with coordinates $P(x_e, y_e, z_e)$ in which the intensity of light is observed.

Step 3. Determine the surface domain D (projection of the surface S on the xOy plane), and divide it into finite, but very small segments dD.

For example, if the domain surface is rectangular then its boundaries are from x_{min} to x_{max} on the x-axis, and from y_{min} to y_{max} on the y-axis. The domain can be divided into segments of surfaces $dD=dxdy$, where $dx=(1/N_x)\cdot(x_{min}-x_{max})$ and $dy = (1/N_y)\cdot(y_{min}-y_{max})$ are segments of the domain D on the x- and y-axis. N_x and N_y are the segment numbers of the domain D on the x- and y-axis, respectively. N_x and N_y must be chosen to be large enough in order to fulfill the condition $dx \to 0$ and $dy \to 0$. The coordinates of the segment, dD in the domain D are marked with (x_D, y_D).

If the domain D is a circle of radius R_D, then $dD=\rho d\rho d\phi$, where ρ is the distance of the domain segment from the center of the circle, ϕ is