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The phase ambiguity in dispersion measurements by white light spectral interferometry

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ABSTRACT

In this work, we address the phase ambiguity in white light spectral interferometry. This ambiguity prevents one from obtaining the refractive index over a broad spectral range with high accuracy. We first determine the error when the refractive index is fitted to a linear combination of power functions. We demonstrate that the error is proportional to wavelength and independent of sample thickness. We show how to reduce the error over the entire spectral band by measuring the spectral phase at the output of the interferometer for some suitable wavelengths as a function of sample orientation.

1. Introduction

In optics, dispersion refers to the dependence of the optical properties of a material or device upon wavelength. Dispersion concerns any phenomena related to refraction and affects the propagation of pulses in matter. Accurate knowledge of dispersion is crucial in many fields of science and industry, such us optical design, optical imaging, optical communication, laser physics, low-coherence metrologies, and ultrafast optics. Additionally, dispersion can be used to obtain information about other physical properties, and for the development of theoretical and numerical physical models. From the pioneering work of Sainz and coworkers [1–3], the analysis of interference of incoherent light in the spectral domain has been shown to be a powerful tool to measure material dispersion over a broad spectral range [4–12]. In white light spectral interferometry (WLSI), mainly two different parameters have been calculated to quantify dispersion: the refractive and the group index. Group index is directly determined by the derivative of the phase in the spectral domain [10], once the thickness of the sample and the path difference between the interferometer arms are known. Alternatively, the value of the so-called "equalization wavelength" can be measured as a function of the difference of path length in the interferometer arms [11], with the same goal. The recovery of refractive index is much more complex, however, because of a 2π phase ambiguity arising from the fact that the inverse trigonometric functions are multi-valued. Several authors have tried to work around this ambiguity by assuming that the refractive index verifies a given dispersion relationship, and fitting the phase accordingly [3,4,7,8]. Specifically, in [3] and [7], authors fitted the refractive index to either a quadratic or a cubic polynomial in wave number, respectively, while the authors in [4] used a quadratic polynomial in squared wave number; on the other hand, a Sellmeier equation was considered in [8]. In each case, the similitude of the fitted and real refractive indices was checked by considering materials of known refractive index. However, in a practical case, with unknown refractive indexes, how can the accuracy of the measurements be determined? Indeed, is it possible to estimate?

In this paper, we consider the limitations of refractive index retrieval by WLSI that arise from phase ambiguity. After reviewing the origin of the phase ambiguity, we discuss the error in refractive index due to phase fitting, and arrive at an analytical expression that depends on the actual refractive index and on the functional form of the fit. Thus, it is shown theoretically and experimentally, how the phase ambiguity can be overcome by calculating separately, and with low resolution, the refractive index at given wavelengths.

2. The phase ambiguity

In white light spectral interferometry, the output of a Michelson or Mach–Zehnder interferometer illuminated with a broadband source is decomposed into its spectrum by a suitable spectrometer equipped with an array detector. For the dispersion measurement, a sample of thickness d is placed in one arm of the interferometer.

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In a well-compensated interferometer, the irradiance at the detector as a function of wave number, $\sigma = 1/\lambda$, can be written as:

$$I(\sigma) = I_0(\sigma) + V(\sigma)\cos\varphi(\sigma) \tag{1a}$$

$$\varphi(\sigma) = 4\pi\sigma(dn - d - L) \tag{1b}$$

where I_0 is the background irradiance, V is the visibility function, φ is the optical phase difference at the output of the interferometer, n is the refractive index, and L is the difference in length between the interferometer arms. For simplicity, in Eq. (1b) we have assumed that the refractive index of air is unity. In the literature, we can find different methods to extract the phase from irradiance measurements [13–15]. However, since the arccosine function is multivalued, we only obtain its principal value between $-\pi$ and $+\pi$ for each σ. After applying an unwrapping procedure, we obtain a continuous phase with a minimum in the interval $[0, 2\pi]$ (see Fig. 1). This phase, φ_p, differs from that given in Eq. (1b) by an even multiple of π, that means:

$$\varphi_p + 2k\pi = 4\pi\sigma(dn - d - L) \tag{2}$$

where $k \in \mathbb{Z}$ is the interference order. To retrieve the refractive index, we must first measure d and L, and find the value of the interference order k. Assuming that d and L are determined, we can sum $4\pi\sigma(d + L)$ on both sides of Eq. (2) and incorporate this term into a new phase variable, φ', to rewrite the equation as:

$$\varphi' = 4\pi\sigma dn - 2k\pi \tag{3}$$

In summary, because k is unknown, the phase measurement alone is not sufficient to estimate the refractive index unless some assumptions are made, as will be shown in the next section.

3. Applying a dispersion relation

As stated earlier, some authors have tried to retrieve the refractive index by assuming that n verifies some dispersion relationship, given for example by the Cauchy or Sellmeier formulas, or by simply considering a Taylor expansion of the refractive index about a suitable value $n_0 = n(\sigma_0)$. The only condition on the functional form of the refractive index is that it must not contain a term inversely proportional to wave number. In this case, the unique term in the right hand side of Eq. (3) which is independent of σ is $2k\pi$. In that case, the phase can be fitted to simultaneously obtain the refractive index and the value of k. Hereinafter, we consider that the refractive index can be written as a linear combination of power functions with m different real exponents.[1] Therefore, the phase is fitted as:

$$\varphi'(\sigma) \cong \varphi'_f(\sigma) = \sum_{j=1}^{m+1} a_j \sigma^{p_j} \tag{4}$$

In this equation, the first power p_1 is taken to be zero because of the constant term $2k\pi$ in Eq. (3), while the other m terms are correlated to the refractive index. In general, there will be some difference between the real phase, φ', and the fitted phase, φ'_f. Let us call this difference $\Delta\varphi(\sigma)$. If we substitute φ' by φ'_f in Eq. (3), we get a fitted index n_f:

$$n_f = \frac{\varphi'_f + 2k\pi}{4\pi d\sigma} = \sum_{j=2}^{m+1} \frac{a_j}{4\pi d}\sigma^{p_j - 1}. \tag{5}$$

[1] This excludes at first sight the Sellmeier equation, but this one can be well approximated to a sum of power functions by performing a Taylor expansion about a particular wave number

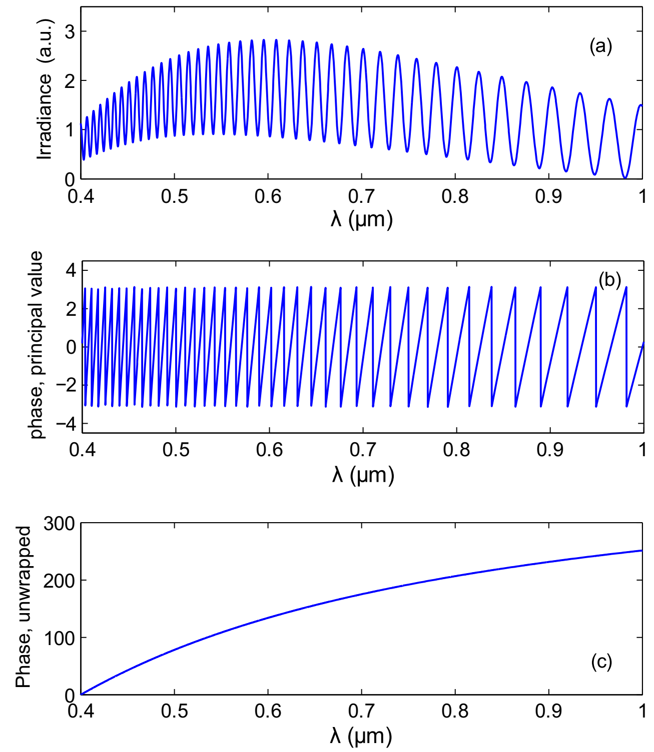

Fig. 1. Typical spectra detected in WLSI showing well-resolve fringes (a) and the phase extracted before (b) and after (c) phase unwrapping.

Furthermore, we can estimate of the interference order k as $-a_1/2\pi$. On the other hand, the real refractive index can be obtained directly as $n = (\varphi' + 2k\pi)/(4\pi\sigma d) = (\varphi'_f + \Delta\varphi + 2k\pi)/(4\pi\sigma d)$. Then, the difference between real and fitted refractive index is:

$$\Delta n = n - n_f = \left(\frac{a_1 + \Delta\varphi}{2\pi} + k\right)\frac{1}{2d\sigma} = \left(\frac{a_1 + \Delta\varphi}{2\pi} + k\right)\frac{\lambda}{2d}. \tag{6}$$

Under normal circumstances, the phase will be well-fitted by Eq. (4), so $\Delta\varphi \to 0$. Hence, hereinafter we neglect $\Delta\varphi$. In this situation, the error is zero only if the estimation of k is exact, that is, if the first coefficient of the fitted phase verifies $a_1 = -2k\pi$. However, this condition is not usually fulfilled, and so $\Delta n \neq 0$.

To estimate the value of the coefficient a_1, we consider at this point a typical fitting by applying the least-squares method. As it is shown in the Appendix, a_1 differs from $-2k\pi$ by a quantity proportional to d. Therefore, the error in refractive index is inversely proportional to the wave number, that is,

$$\Delta n = B/\sigma = B\lambda \tag{7}$$

where B is a constant. The dependence of Δn with λ or σ is related to an error in k, as can be easily shown by deriving Eq. (3) and considering no error in phase: $\Delta n = \Delta k/(2d\sigma)$. More surprisingly, B is not only independent of σ, but it is also independent of the sample thickness d [the sum $a_1/2\pi + k$ in Eq. (6) scales with d, as it is shown in the Appendix]. This is unfortunate because we could otherwise try to infer the value of k by using samples of different thicknesses.

Eq. (7) can be modified by applying the condition that k is an integer, so we can round the coefficient $a_1/2\pi$ to the nearest integer. In this case, the refractive index error is,

$$\Delta n = \mathrm{floor}(2Bd)/(2d\sigma) \tag{8}$$

The rounding function, floor, is zero for arguments less than 0.5, so this expression will give the correct refractive index for a sufficiently small sample thickness. However, as the thickness