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A near-wall strategy for buoyancy-affected turbulent flows using stabilized FEM with applications to indoor air flow simulation

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Abstract

We consider the numerical simulation of buoyancy-affected, incompressible turbulent flows using a stabilized finite-element method. We present an approach which combines two domain decomposition methods (DDM). Firstly, we apply a DDM with full overlap for near-wall modelling, which can be interpreted as an improved wall-function concept. Secondly, a non-overlapping DDM of iteration-by-subdomains-type for the parallel solution of the linearized problems is employed. For this scheme, we demonstrate both the accuracy for a benchmark problem and the applicability to realistic indoor-air flow problems.

Keywords: k/ϵ-model; LES; Wall function; Buoyancy; Stabilized FEM; Domain decomposition

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1. Introduction

Turbulent flows driven or significantly affected by buoyancy occur in a variety of problems including building ventilation, cooling of electrical equipment, and environmental science, cf., e.g., [15,6]. The fundamental mathematical model are the non-isothermal, incompressible Navier–Stokes equations. Their solution can become turbulent (and hence computationally infeasible), if a critical parameter, e.g., the Reynolds number or the Rayleigh number, becomes too large.

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As a remedy, so-called turbulence models are used, which are suitably modified Navier–Stokes equations whose solutions are (in some sense) close to those of the Navier–Stokes equations, but which can be computed at reasonable costs. In this paper, two different types of turbulence models are considered. Firstly, we employ the so-called k/ϵ model which is one of the most widespread turbulence models for industrial applications. It is a statistical turbulence model which was conceived to approximate the statistically averaged solution of the Navier–Stokes equations, see [4]. Secondly, we consider large-eddy simulation (LES). The aim of LES is to approximate the random motion of the large-scale flow structures (spatial averages) of the solution, see [28,7], and also [5,25] for non-isothermal LES. LES is much closer to the Navier–Stokes equations than a statistical turbulence model like k/ϵ.

Despite the fast increase in available computer power and the use of a turbulence model, the numerical solution of turbulent flow problems is still very expensive and may take several days or even weeks for large-scale 3D problems in complex geometries. One major problem is that a large number of grid points is needed to accurately resolve the solution near solid walls, where the solution often exhibits sharp gradients, called boundary layers.

Dedicated to this problem, the present paper describes a solution strategy which significantly reduces computational costs but whose results are also fairly good in accuracy for the applications in mind. Although the devised method is described for a specific application (i.e., buoyancy affected non-isothermal turbulent air flows), the key idea can be applied to a large class of wall-bounded (turbulent) flows including atmospheric and oceanographic flows, two-phase liquid gas flows and multifluid/multiphase flows with combustion, as described in the conclusion.

The *key idea* is a combination of two steps: Firstly, splitting off the boundary layer region, and secondly, applying an improved wall-function concept there. In the first step, the boundary layer region is split off using a fully overlapping domain decomposition method, following an idea devised by [30,32]: The flow problem is divided into a global (interior) problem and a problem in the near-wall region, called boundary layer problem.

The aim of [32] was to solve the same equation in both domains on different computational grids, i.e., a relatively coarse grid for computing the global solution and a fine grid for solving the boundary layer problem are used. However, for complex 3D problems, the computational costs of both subproblems are nearly of the same order of magnitude. Therefore, in the second step a much simpler computational model for the near-wall region is applied, called boundary-layer equations, which are essentially a system of coupled nonlinear ordinary differential equation. This reduces the computational costs and facilitates the implementation of the boundary layer problem significantly. From the engineering point of view, this approach can be formulated as an improved wall-function model, which accounts for effects of thermal stratification in the boundary layer, see also [24,17]. Thus, the strategy of the present paper is to couple different differential equations in both domains, cf. [16].

After splitting off the near-wall region, the boundary layer solution is needed only for providing the boundary conditions of the global (interior) flow problem. The remaining problem is then to solve the nonlinear interior problem. After semidiscretization in time and a subsequent decoupling and linearization of the arising nonlinear, highly coupled problem, the iterative process requires the fast solution of linearized Navier–Stokes problems and of advection–diffusion-reaction problems. These subproblems are discretized using stabilized FEM together with a shock-capturing technique, cf. [18], In order to parallelize the method, for the linearized problems we apply an iterative substructuring technique which couples the subdomain problems via Robin-type transmission conditions, see [26,22].

The paper is organized as follows: First we describe the mathematical model, i.e., the non-isothermal Navier–Stokes equations with a generic turbulence model (Section 2). In Section 3 we present a full-overlapping DDM for splitting off the near-wall region. We apply the k/ϵ turbulence model to the global problem and describe the simplified boundary layer problem with its simple, but nevertheless physically quite sophisticated algebraic turbulence model. In Sections 4–6 the numerical solution strategy for the global flow