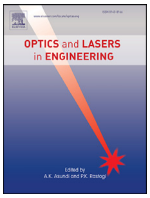
Optics and Lasers in Engineering 100 (2018) 155–160

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng



Measurement of small wavelength shift using diffraction grating and high-angular-sensitivity total-internal-reflection heterodyne interferometer

Jiun-You Lin[a,*], Jing-Hsiang Jhuang[a], Meng-Chang Hsieh[b], Chia-Ou Chang[b,c]

[a] Department of Mechatronics Engineering, National Changhua University of Education, Changhua 500, Taiwan, ROC
[b] Institute of Applied Mechanics, National Taiwan University, Taipei 10617, Taiwan, ROC
[c] College of Mechanical Engineering, Guangxi University, Nanning 530004, China

ARTICLE INFO

Keywords:
Diffraction grating
Heterodyne interferometry
Total internal reflection

ABSTRACT

This study presents a method for detecting small wavelength shifts using grating diffraction effect and high-angular-sensitivity total-internal-reflection (TIR) heterodyne interferometry. In the proposed interferometer, a half-wave plate and a quarter-wave plate that exhibit specific optic-axis azimuths are combined to form a phase shifter. When an isosceles right-angle prism is placed between the phase shifter and an analyzer that exhibits suitable transmission-axis azimuth, it shifts and enhances the phase difference of the *s*- and *p*- polarization states of the first-order diffraction beam at one TIR. The enhanced phase difference depends on the diffraction angle, which is a function of the beam wavelength; thus the wavelength shift can be easily and accurately measured by estimating the phase-difference variation. The feasibility of our method was demonstrated with a measurement resolution of approximately 0.007 nm and a sensitivity of 4.3°/nm in a measurement range of 5 nm. This method has the merits of both common-path interferometry and heterodyne interferometry.

1. Introduction

Accurate wavelength-shift measurement is often required in many research fields, such as wind speed determination, temperature monitoring, strain sensing, pressure detection, small displacement measurement, and refractive index analysis [1–8]. Several measurement methods have been proposed, including optical spectrum analysis [1–6], image interferometer [7,8], and heterodyne inteferometric methods [9–12]. Among these methods, the heterodyne interferometic method has attracted much attention of many researchers for a long time, since it applies dual-polarization frequency-modulated and common-path interferometric technique, thus providing the merits of accurate phase measurement, and excellent immunity to environmental disturbance. Based on the heterodyne interferometric principle, the transmission-type and the reflection-type measurement methods were developed. Regarding the transmission-type methods such as the uniaxial crystal [9] and the dual-path Mach-Zehnder heterodyne interferometric methods [10], the wavelength shift is inferred from the phase variation of the light beam transmitted through a quartz crystal or a glass plate in the heterodyne interferometer. Accordingly, precision in the thickness and the parallelism of two opposite sides of the crystal or the plate are strongly required to satisfy measurement conditions, thus making measurement processes tedious. In addition, the applicability of the methods is limited due to their narrow measurement range lower than 0.2 nm. The reflection-type measurement methods mainly include two kinds: surface plasmon resonance (SPR) [11] and multiple total-internal-reflection (MTIR) methods [12]. In SPR technique, the phase shift of the light beam reflected from a SPR apparatus is a function of beam wavelength due to the dispersion effect of the metal thin film coated on the apparatus. The wavelength shift can hence be determined by estimating the significant phase variation of the SPR reflected light beam. However, if the sensitivity and resolution are desired to further be improved, another coating process is needed to change the thickness or the material of the metal film, increasing the inconvenience in measurement. The MTIR estimates the wavelength shift by evaluating the phase variation of the first-order diffraction beam from a grating totally reflected inside an elongated parallelogram prism. In the prism, the light beam underwent MTIRs, attaining the improved measurement sensitivity and resolution; however, the measurement system is bulky. The method also has a drawback of a nonlinear response to the wavelength shift.

In view of these situations of the two types of methods, this study presents a method for measuring small wavelength shifts. The proposed method is based on the principle of the reflection-type heterodyne interferometric technique, but replaces the above-mentioned reflection apparatus (the elongated parallelogram prism or SPR apparatus) with a high-angular-sensitivity (TIR) apparatus. The first-order heterodyne diffraction beam from a grating propagates through a phase shifter, consisting of a half-wave and a quarter-wave plate, subsequently penetrating an isosceles right-angle prism at an angle larger than the critical angle. The light in the prism undergoes one TIR, finally traveling through an analyzer to extract the interference signal of the p- and s-polarized states.

---

[*] Corresponding author.
E-mail address: jylin@cc.ncue.edu.tw (J.-Y. Lin).

http://dx.doi.org/10.1016/j.optlaseng.2017.07.017
Received 10 February 2017; Received in revised form 9 July 2017; Accepted 31 July 2017
0143-8166/© 2017 Elsevier Ltd. All rights reserved.

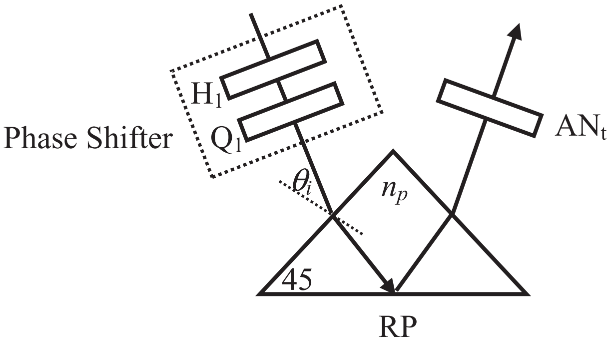


Fig. 1. The high-angular-sensitivity TIR apparatus.

When the azimuth angles of the phase shifter wave plates and the analyzer are properly selected, the final phase difference exhibited by the prism incident angle of the interference signal is substantially enhanced, thereby resulting in a high angular sensitivity. As the incident wavelength is slightly shifted, the diffracted, first-order beam is deflected at a slightly different angle. The deflection of the diffraction beam creates a small variation in the prism incident angle, finally inducing an obvious phase-difference variation of the interference signal. The phase-difference variation can be precisely measured with heterodyne interferometry. The desired wavelength shift is hence obtainable by substituting the measured results into the derived equations. Experimentally, the method displayed a linear response to the wavelength shift, and yielded measurement sensitivity and resolution levels of 4.3°/nm and 0.007 nm in a measurement range of 5 nm. The proposed method can avoid the tedious measurement process of the transmission-type methods, and provide a larger measurement range than these methods; in addition, compared with the reflection-type methods, it can also easily regulate and improve the measurement sensitivity and resolution without a thin-film coating process or a bulky MTIR prism.

2. Principle

2.1. Phase difference resulting from high-angular-sensitivity TIR apparatus

Fig. 1 demonstrates the optical configuration of the high-angular-sensitivity TIR apparatus. For convenience, the $+z$ axis is set in the direction of the propagation of light and the x axis is perpendicular to the plane of the paper. The polarization plane of a linearly polarized light is properly set at an angle θ_p from the x axis. The light is guided to pass through a phase shifter, comprising a half-wave plate H_1 (fast axis at a $\gamma/2$ angle to the x-axis) and a quarter-wave plate Q_1 (fast axis at 45° with respect to x-axis), and is subsequently incident at θ_i on one side of an isosceles right-angle prism RP with refractive index n_p. The prism is mounted on a rotation stage. The light beam enters the prism, and is totally reflected inside the prism. The light output from the prism travels through an analyzer AN_t (the transmission axis is at β to the x-axis) for interference of s- and p- polarized light. The final electric field E_t can be expressed as follows [13]:

$$E_t = \begin{pmatrix} \cos^2\beta & \sin\beta\cos\beta \\ \sin\beta\cos\beta & \sin^2\beta \end{pmatrix} \begin{pmatrix} t_p t'_p \exp(-i\delta/2) & 0 \\ 0 & t_s t'_s \exp(i\delta/2) \end{pmatrix} \times \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -i \\ -i & 1 \end{pmatrix} \times \begin{pmatrix} \cos\gamma & \sin\gamma \\ \sin\gamma & -\cos\gamma \end{pmatrix} \begin{pmatrix} \cos\theta_p \\ \sin\theta_p \end{pmatrix} = \left(A_t^+ \cos\theta_p \exp(i\phi) + A_t^- \sin\theta_p \exp(i\pi/2)\right) \begin{pmatrix} \cos\beta \\ \sin\beta \end{pmatrix}, \tag{1}$$

where the amplitudes A_t^+ and A_t^- can be written as follows:

$$A_t^{\pm} = \left\{ \frac{1}{2} [(t_p t'_p \cos\beta)^2 + (t_s t'_s \sin\beta)^2 \pm t_p t'_p t_s t'_s \sin 2\beta \cdot \sin(2\gamma + \delta)] \right\}^{1/2}, \tag{2}$$

and the phase difference ϕ can be expressed as follows:

$$\phi = \tan^{-1}\left[-\tan\left(45^\circ - \sigma\right) \cdot \tan\left(\gamma + \delta/2 - \pi/4\right)\right] + \tan^{-1}\left[\tan\left(45^\circ + \sigma\right) \cdot \tan\left(\gamma + \delta/2 - \pi/4\right)\right]. \tag{3}$$

In Eqs. (1)–(3), δ is the phase difference between the s- and p-polarizations of one TIR at the base of the prism; (t_p, t_s) and (t'_p, t'_s) are the transmission coefficients at the air-prism and prism-air interfaces, respectively; γ and σ are the parameters introduced using the phase shifter and the analyzer AN_t, respectively. δ and σ can be derived using Fresnel's equations and Jones matrix calculation [14]:

$$\delta = 2\tan^{-1}\left\{ \frac{\left[\sin^2\left[45^\circ + \sin^{-1}\left(\sin\theta_i/n_p\right)\right)\right] - (1/n_p)^2\right]^{1/2}}{\tan\left[45^\circ + \sin^{-1}(\sin\theta_i/n_p)\right] \cdot \sin\left[45^\circ + \sin^{-1}(\sin\theta_i/n_p)\right]} \right\}, \tag{4}$$

$$\sigma = \tan^{-1}\left(\frac{t_s t'_s}{t_{p'} t'_p} \tan\beta \right), \tag{5}$$

and the transmission coefficients are given as follows:

$$t_p = \frac{2\cos\theta_i}{n_p \cos\theta_i + \left[1 - (\sin\theta_i/n_p)^2\right]^{1/2}}, \tag{6}$$

$$t_s = \frac{2\cos\theta_i}{\cos\theta_i + n_p\left[1 - (\sin\theta_i/n_p)^2\right]^{1/2}}, \tag{7}$$

$$t'_p = \frac{2n_p\left[1 - (\sin\theta_i/n_p)^2\right]^{1/2}}{n_p \cos\theta_i + \left[1 - (\sin\theta_i/n_p)^2\right]^{1/2}}. \tag{8}$$

$$t'_s = \frac{2n_p\left[1 - (\sin\theta_i/n_p)^2\right]^{1/2}}{\cos\theta_i + n_p\left[1 - (\sin\theta_i/n_p)^2\right]^{1/2}}, \tag{9}$$

Eqs. (3)–(9) indicates that the phase difference ϕ depends on the parameters of γ, σ, n_p, and θ_i. If the values of σ, γ, and n_p are specific, the phase difference ϕ is a function of the prism incident angle θ_i. When the parameter σ (determined by the analyzer transmission-axis angle β) is set toward 45°, the value of tan(45° + σ) will become large, as shown in Eq. (3). Under the condition, proper selection of the parameter γ (being used to vary the phase level of $\delta/2$) can substantially increase the phase difference ϕ, and the linearity of the ϕ versus θ_i curve. In our method, γ is determined based on the azimuth angle of the fast axis of H_1, and can be chosen in the range of $(\pi/4 - \delta_{,max}/3) \leq \gamma < (\pi/4)$, where the value of δ_{max} denotes the maximal value of the phase difference δ and is expressed as follows [15]:

$$\delta_{\max} = 2\tan^{-1}\left(\frac{{n_p}^2 - 1}{2n_p} \right). \tag{10}$$

Fig. 2 displays the relation between the phase difference ϕ and incident angle θ_i at the parameter of $(\pi/4 - \delta_{,max}/3) \leq \gamma \leq (\pi/4 - \delta_{,max}/8)$ when the conditions of $n_p = 1.77862$, and $\beta = 45^\circ$ are substituted into Eqs. (3)–(10). For comparison, the relation of the phase difference δ versus the incident angle θ_i is marked as "•" in Fig. 2. It is founded that at $\beta = 45^\circ$ (σ is less than approximately 45°) increasing the γ value significantly improves the slope of the ϕ versus θ_i curve, thereby leading to the high angular sensitivity. Additionally, it is also found that at the angle β approaching 45°, the value of $[(t_p t'_p \cos\beta)^2 + (t_s t'_s \sin\beta)^2]$ is close to that of $t_p t'_p t_s t'_s \sin(2\beta) \cdot \sin(2\gamma + \delta)$ based on Eqs (2), (4) and (6)–(9). The calculation results show the amplitude of A_t^+ is larger than 0.9, and that of A_t^- smaller than 0.1.

2.2. Phase difference measurement using heterodyne interferometry

Fig. 3 shows a schematic diagram of the optical arrangement of the proposed method. When a light beam of wavelength λ output from a tunable laser is normally incident on a grating G with a period Λ, the