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Digital image correlation based on a fast convolution strategy

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ABSTRACT

In recent years, the efficiency of digital image correlation (DIC) methods has attracted increasing attention because of its increasing importance for many engineering applications. Based on the classical affine optical flow (AOF) algorithm and the well-established inverse compositional Gauss–Newton algorithm, which is essentially a natural extension of the AOF algorithm under a nonlinear iterative framework, this paper develops a set of fast convolution-based DIC algorithms for high-efficiency subpixel image registration. Using a well-developed fast convolution technique, the set of algorithms establishes a series of global data tables (GDTs) over the digital images, which allows the reduction of the computational complexity of DIC significantly. Using the pre-calculated GDTs, the subpixel registration calculations can be implemented efficiently in a look-up-table fashion. Both numerical simulation and experimental verification indicate that the set of algorithms significantly enhances the computational efficiency of DIC, especially in the case of a dense data sampling for the digital images. Because the GDTs need to be computed only once, the algorithms are also suitable for efficiently coping with image sequences that record the time-varying dynamics of specimen deformations.

1. Introduction

Subset-based digital image correlation (DIC) numerically determines a series of best-matching subsets between reference and deformation images by comparing the gray intensities of the images' speckle patterns using some continuity condition such as the well-adopted affine shape function [1–7]. So far, several classes of image registration algorithms, such as optical flow gradient-based algorithms [8–10], Newton-Raphson (NR) iterative algorithms [11–13], and weight window-based algorithms [14–16], have been developed and are employed extensively in many engineering measurement fields [17–28]. In recent years, the focus has been on improving the computational efficiency of DIC algorithms because the traditional algorithms are usually computationally complicated, which makes it difficult to meet specific speed requirements [29–30]. Some strategies for improving the efficiency reduce the computational burden related to the initial guess when searching for the integer-pixel correlation [31–33]. Several algorithms have been developed that are based on basis functions or sum-table techniques and decrease the computational complexity of the zero-normalized cross-correlation coefficient criterion that is usually employed to calculate the integer-pixel correlations [34–35]. Other algorithms enhance the computational efficiency of the sub-pixel registration calculations in DIC by avoiding redundant computations [36]. The recently introduced inverse compositional Gauss–Newton (IC-GN) algorithm speeds up the sub-pixel registration matching by not having to update the Hessian matrix in each iterative step and therefore significantly lowering the corresponding computational complexity [37–40].

With the development of parallel computing based on graphics processing units (GPUs) and the corresponding compute unified device architecture (CUDA) platform developed by NVIDIA, it becomes possible to apply certain hardware-based high-performance computing technologies to accelerate the correlation matching between digital images before and after deformation. Some researchers have employed the GPU-based parallel computing strategy to improve the efficiency of the integer-pixel correlation calculations. The computational speed of the fast Fourier transform (FFT)-based cross-correlation matching can be enhanced by a factor of ~20 [41–43]. Zhang et al. [44] developed a parallel DIC method that provides a high-efficiency correlation computation based upon NVIDIA's CUDA platform. The method performs the integer-pixel initial guess estimation using an FFT-based cross-correlation algorithm, while the sub-pixel registration calculation was implemented using the well-established IC-GN algorithm. Pan et al. [45] developed a superfast DIC algorithm by modifying the reliability-guided displacement tracking strategy using a multithread computing technique. Based

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on the integral image technique and fast interpolation method, Jiang et al. [40] proposed a high-efficiency computational strategy for the IC-GN algorithm. Likewise, Shao et al. [46] developed a three-dimensional DIC method by employing the efficient IC-GN algorithm in combination with a seed point-based parallel method to monitor the pulse of humans in a real-time way.

In this paper, we propose an alternative method for speeding up the DIC calculations by combining a set of convolution algorithms based on the well-developed FFT with the classical affine optical flow or IC-GN algorithms. Due to the efficient implementation of the FFT-based fast convolution between correlation subsets, the presented algorithm effectively improves the computational efficiency without sacrificing the measurements' accuracy and precision. Likewise, the computational complexity of the proposed algorithms appears to be less relevant to both the size of the correlation subsets and the density of the sampling points of the digital images, which makes it more suitable for large-scale global deformation measurements based upon a dense data sampling. It is anticipated that the set of algorithms can provide a new coping strategy for implementing the DIC approach with real-time function in combination with the high-performance GPU-based parallel computing technique.

2. Theoretical framework and implementation of algorithms

To begin with, we define the gray intensities of the subsets with $(2M+1)\times(2M+1)$ pixels on the reference (undeformed) and target (deformed) images as $F(X, Y)$ and $G(x, y)$, respectively, where (X, Y) and (x, y) are the location coordinates within their respective subsets. To obtain a more robust noise-proof performance and at the same time minimize the effect of the potential scaling and offset of the illumination lighting on the measurement results [11,15], we replace $F(X, Y)$ and $G(x, y)$ with the zero-mean normalized subsets $f(X, Y)$ and $g(x, y)$, respectively. This can be expressed as

$$\begin{cases} f(X,Y) = \left[F(X,Y) - F_m\right]/\Delta F \\ g(x,y) = \left[G(x,y) - G_m\right]/\Delta G \end{cases} \tag{1}$$

with

$$\begin{cases} F_m = \frac{1}{(2M+1)^2} \sum_{X=X_0-M}^{X_0+M} \sum_{Y=Y_0-M}^{Y_0+M} F(X,Y) \\ G_m = \frac{1}{(2M+1)^2} \sum_{X=X_0-M}^{X_0+M} \sum_{Y=Y_0-M}^{Y_0+M} G(x,y) \\ \Delta F = \sqrt{\sum_{X=X_0-M}^{X_0+M} \sum_{Y=Y_0-M}^{Y_0+M} \left[F(X,Y) - F_m\right]^2} \\ \Delta G = \sqrt{\sum_{X=X_0-M}^{X_0+M} \sum_{Y=Y_0-M}^{Y_0+M} \left[G(x,y) - G_m\right]^2} \end{cases} \tag{2}$$

where F_m and G_m are the mean values of the gray intensities of the reference and deformed subsets. The coordinates (x, y) and (X, Y) can be related via the following affine shape function:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} X \\ Y \end{bmatrix} + \begin{bmatrix} u & u_X & u_Y \\ v & v_X & v_Y \end{bmatrix} \begin{bmatrix} 1 \\ \Delta X \\ \Delta Y \end{bmatrix} \tag{3}$$

with $\mathbf{p} = [u, u_X, u_Y, v, v_X, v_Y]^T$, where (u, v) denotes the displacement vector of the subset center located at (X_0, Y_0) on the reference image, $u_X = \partial u/\partial X$, $u_Y = \partial u/\partial Y$, $v_X = \partial v/\partial X$, and $v_Y = \partial v/\partial Y$ are the derivatives of the displacement components with respect to the coordinates X and Y on the reference configuration, and $X - X_0 = \Delta X$ and $Y - Y_0 = \Delta Y$ are the components of the distance to the subset center (X_0, Y_0). To quantify the similarity between the reference and deformed images, we define a cost function for an arbitrary location on a digital speckle image as

$$e(x, y; X_0, Y_0; \mathbf{p}) = g(x, y) - f(X, Y) \tag{4}$$

To explore the inherent relationship of the gray intensity of a single material point over the reference and deformed images, we expand $f(X, Y)$ using Eq. (3) as a first-order Taylor's series, which is expressed as follows:

$$\begin{aligned} f(X,Y) \approx\ & f(x,y) - f_X \times \left(u + u_X \Delta X + u_Y \Delta Y\right) \\ & - f_Y \times \left(v + v_X \Delta X + v_Y \Delta Y\right) \end{aligned} \tag{5}$$

where (f_X, f_Y) stands for the derivatives of the image's gray intensity. Substituting Eq. (5) into Eq. (4) yields

$$\begin{aligned} e(x, y; X_0, Y_0; \mathbf{p}) =\ & g(x,y) - f(x,y) + f_X \times \left(u + u_X \Delta X + u_Y \Delta Y\right) \\ & + f_Y \times \left(v + v_X \Delta X + v_Y \Delta Y\right) \end{aligned} \tag{6}$$

Subsequently, we establish a vector of the cost functions corresponding to each subset of size $(2M+1)\times(2M+1)$, i.e.,

$$\mathbf{e}(x, y; X_0, Y_0; \mathbf{p}) = \begin{bmatrix} e(-M + x_0, -M + y_0; X_0, Y_0; \mathbf{p}) \\ e(-M + x_0, -M + y_0 + 1; X_0, Y_0; \mathbf{p}) \\ \vdots \\ e(M + x_0, M + y_0; X_0, Y_0; \mathbf{p}) \end{bmatrix} \tag{7}$$

where (x_0, y_0) denotes the subset center of the deformed image, which can be calculated beforehand by certain well-developed integer-pixel searching algorithms [13,34,35,41,42,43]. As a vector that includes $(2M+1)\times(2M+1)$ elements, Eq. (7) gives the deviations between $f(X, Y)$ and $g(x, y)$ on all pixel locations within an arbitrary subset.

2.1. Affine optical flow (AOF) method

The AOF method usually requires that the following total cost function holds over each particular subset of size $(2M+1)\times(2M+1)$ on the reference image [2,37,47]:

$$\begin{aligned} \mathbf{E_A}(X_0, Y_0; \mathbf{p}) &= \left[\mathbf{e}(x, y; X_0, Y_0; \mathbf{p})\right]^T \left[\mathbf{e}(x, y; X_0, Y_0; \mathbf{p})\right] \\ &= \sum_{X=X_0-M}^{X_0+M} \sum_{Y=Y_0-M}^{Y_0+M} \left[e(x, y; X_0, Y_0; \mathbf{p})\right]^2 \end{aligned} \tag{8}$$

Minimizing the total cost function requires

$$\nabla\left[\mathbf{E_A}(X_0, Y_0; \mathbf{p})\right] = 2 \sum_{X=X_0-M}^{X_0+M} \sum_{Y=Y_0-M}^{Y_0+M} \left\{\left[e(x, y; X_0, Y_0; \mathbf{p})\right] \times \partial\left[e(x, y; X_0, Y_0; \mathbf{p})\right]/\partial \mathbf{p}\right\} = \mathbf{0} \tag{9}$$

Considering that $\partial[e(x, y; X_0, Y_0; \mathbf{p})]/\partial \mathbf{p} = [f_X, f_X\Delta X, f_X\Delta Y, f_Y, f_Y\Delta X, f_Y\Delta Y]^T$ and substituting Eq. (6) into Eq. (9) gives us

$$\mathbf{C}_A\mathbf{p} = \mathbf{R} \quad \text{or} \quad \mathbf{p} = \mathbf{C}_A^{-1}\mathbf{R} \tag{10}$$

The coefficient matrix $\mathbf{C}_A$ and the residual vector $\mathbf{R}$ can be expressed as

$$\mathbf{C}_A = \begin{bmatrix} \sum\sum f_X^2 & \sum\sum f_X^2\Delta X & \sum\sum f_X^2\Delta Y & \sum\sum f_X f_Y & \sum\sum f_X f_Y\Delta X & \sum\sum f_X f_Y\Delta Y \\ \sum\sum f_X^2\Delta X & \sum\sum f_X^2\Delta X^2 & \sum\sum f_X^2\Delta X\Delta Y & \sum\sum f_X f_Y\Delta X & \sum\sum f_X f_Y\Delta X^2 & \sum\sum f_X f_Y\Delta X\Delta Y \\ \sum\sum f_X^2\Delta Y & \sum\sum f_X^2\Delta X\Delta Y & \sum\sum f_X^2\Delta Y^2 & \sum\sum f_X f_Y\Delta Y & \sum\sum f_X f_Y\Delta X\Delta Y & \sum\sum f_X f_Y\Delta Y^2 \\ \sum\sum f_X f_Y & \sum\sum f_X f_Y\Delta X & \sum\sum f_X f_Y\Delta Y & \sum\sum f_Y^2 & \sum\sum f_Y^2\Delta X & \sum\sum f_Y^2\Delta Y \\ \sum\sum f_X f_Y\Delta X & \sum\sum f_X f_Y\Delta X^2 & \sum\sum f_X f_Y\Delta X\Delta Y & \sum\sum f_Y^2\Delta X & \sum\sum f_Y^2\Delta X^2 & \sum\sum f_Y^2\Delta X\Delta Y \\ \sum\sum f_X f_Y\Delta Y & \sum\sum f_X f_Y\Delta X\Delta Y & \sum\sum f_X f_Y\Delta Y^2 & \sum\sum f_Y^2\Delta Y & \sum\sum f_Y^2\Delta X\Delta Y & \sum\sum f_Y^2\Delta Y^2 \end{bmatrix} \tag{11}$$

and