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A novel wavefront reconstruction algorithm based on interpolation coefficient matrix for radial shearing interferometry

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ABSTRACT

A novel wavefront reconstruction algorithm for radial shearing interferometer (RSI) is proposed in this paper. Based on the shearing relationship of RSI, an interpolation coefficient matrix is established by the radial shearing ratio and the number of discrete points of test wavefront. Accordingly, the expanded wavefront is characterized by the interpolation coefficient matrix and the test wavefront. Consequently the test wavefront can be calculated from the phase difference wavefront. The numerical simulation is conducted to confirm the correctness of the proposed algorithm. Compared with the previous wavefront reconstruction methods, the proposed algorithm is more accurate and stable.

1. Introduction

Radial shearing interferometry (RSI), as a high accurate interference measurement technology, is an effective and popular tool for measuring the wavefront phase distortion [1–6], reconstructing the near-field distribution from a high power laser beam [7–9], and inspecting the corneal topographic [10–12]. Compared with the traditional double-beam interference measurement technology, it is not necessary to build a reference light path with a high quality reference surface. In addition, RSI adapts a common optical path configuration. Therefore, it is not sensitive to the environment vibration.

In recent years, a variety of approaches have been proposed to retrieve the test wavefront from the phase difference wavefront in RSI. In 2007, Jeong used Zernike polynomials as the basis functions to calculate the Zernike coefficients of test wavefront [13]. A similar approach was proposed by Gu in 2011 [14]. In 2015, Kewei employed Legendre polynomials as the basis functions to calculate the Legendre coefficients of test wavefront over a square area [15]. To cope with off-axis test wavefront with general aperture shapes, Tian proposed a Gram-Schmidt orthogonalization method in 2016 [16]. In practice, the number of Zernike polynomial terms cannot be appropriately predicted to fit the wavefront in advance. However, errors caused by the mode cross-talk and aliasing cannot be eliminated when polynomials are used as basic functions to reconstruct wavefronts [15–23].

Apart from the methods described above, Li proposed an iterative method to reconstruct the test wavefront in 2002 [24]. Later, Li improved this method to treat a certain amount of lateral shear in two orthogonal directions [25]. The main idea of this method is to interpolate and iterate the phase difference wavefront to satisfy the convergent condition. Analyzing the different radial shearing ratio, the accuracy of reconstruction depends on the shape of test wavefront. When the test wavefront is irregular, the reconstruction error will increase to about 0.1–0.2 wavelength [26].

These methods mentioned above can be split into two categories: modal method [13–16, 19–23] and zonal method [27–32]. The former is based on the polynomial fitting. And the latter is depended on the assumption that the change between the two adjacent discrete points is linear. In order to reconstruct the test wavefront by our proposed algorithm, a shearing relationship of RSI is determined by the radial shearing ratio and the number of discrete points when the wavefront is sampled. Subsequently, an interpolation coefficient matrix can be obtained. So the expanded wavefront can be described by the matrix and the test wavefront. Next, the shearing relationship is changed into the relationship between the test wavefront and the phase difference wavefront. Thus, the test wavefront can be calculated. The proposed algorithm is neither based on the polynomial fitting nor the interpolation and iteration of the phase difference wavefront so the errors caused by the mode cross-talk and aliasing can be avoided and the accuracy of reconstruction can be unaffected by the high-frequency signal of test wavefront. In this paper, a numerical simulation is conducted to prove the correctness of the algorithm. The effects of the radial shearing ratio and the number of discrete points are analyzed. Compared with existing wavefront

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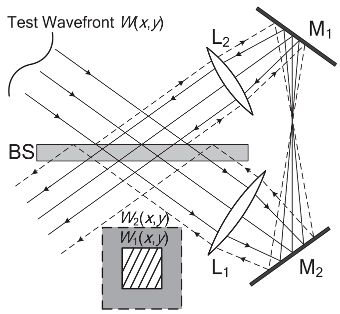

Fig. 1. Schematic of RSI with a square aperture.

reconstruction methods, the proposed algorithm is more accurate and stable.

2. Mathematical model

2.1. Principle of radial shearing interferometer

A typical radial shearing interferometer is illustrated in Fig. 1. The focal lengths of the lens L_1 and L_2 are f_1 and f_2, respectively, and the radial shearing ratio of the RSI is $s = f_2/f_1 < 1$. A collimated test beam with a square aperture enters the radial shearing interferometer and the wavefront $W(x, y)$ will be tested. The test wavefront entered through the beam splitter (BS) is contracted in the counterclockwise direction as the contracted wavefront $W_1(x, y)$ (solid line). And the test wavefront reflected off the BS is expanded in the opposite direction as the expanded wavefront $W_2(x, y)$ (dash line). The expanded wavefront and contracted wavefront interfere within their common area, and a fringe pattern is generated, as shown in the bottom of Fig. 1. The contracted wavefront can be treated as $W(x, y)$ whose domain of definition (x, y) is changed into $(x/s, y/s)$. Similarly, the expanded wavefront within the common area can be regarded as $W(x, y)$ whose domain of definition (x, y) is changed into (xs, ys). The phase difference between the two wavefronts can be defined as

$$\Delta W(x/s, y/s) = W(x/s, y/s) - W(xs, ys). \tag{1}$$

In addition, there is only the coordinate scaling transformation between the contracted wavefront and the test wavefront. As a matter of fact, the phase distribution or figure of contracted wavefront is essentially the same as that of the test wavefront. Henceforth, assuming that the test wavefront $W(x, y)$ is equal to the contracted wavefront $W(x/s, y/s)$, which means that the coordinate variables both x and y multiply s in Eq. (1). Thus, Eq. (1) can be rewritten as

$$\Delta W(x, y) = W(x, y) - W\left(xs^2, ys^2\right). \tag{2}$$

Since the expanded wavefront is the copy of a portion of $W(x, y)$, we can apply Dirac delta function to sample $W(x, y)$ to obtain the expanded wavefront. Consequently, the expanded wavefront can be expressed as

$$W\left(xs^2, ys^2\right) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta\left(x - xs^2, y - ys^2\right) W(x, y)\,dx\,dy, \tag{3}$$

where δ is the Dirac delta function within the range of $[-xs^2, xs^2]$ and $[-ys^2, ys^2]$. Therefore, Eq. (2) can be expressed as

$$\Delta W(x, y) = W(x, y) - \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \delta\left(x - xs^2, y - ys^2\right) W(x, y)\,dx\,dy. \tag{4}$$

2.2. Discrete model

In practical measurement, the interferogram is digitized by the computer. Assuming that the test wavefront $W(x, y)$ is sampled by a discrete matrix with the size of $N \times N$, which is defined within a square aperture. For RSI, according to the radial shearing ratio s, the test wavefront can be expanded to $W_2(x, y)$. As shown in Fig. 2(a), the shaded portion of test wavefront $W(x, y)$ will be naturally enlarged to an expanded wavefront denoted by $W(xs^2, ys^2)$, which has an equal area to that of $W(x, y)$ and is illustrated within the bold solid line. As is known, $W(xs^2, ys^2)$ should also be changed into the size of $N \times N$ due to the requirement of the discrete data operation, which will be described in detail in the following. For the digitized radial shearing interferograms, the phase difference wavefront $\Delta W(x, y)$ at the discrete points in the common area of the test wavefront and the expanded wavefront can be reconstructed by proper phase extraction and phase-unwrapping techniques. $\Delta W(x, y)$ is also sampled with the size of $N \times N$, as shown within the bold solid line. Namely, $\Delta W(x, y)$, $W(x, y)$ and $W(xs^2, ys^2)$ are all defined within the same area. Generally, we can denote the discrete points of the wavefronts by the notation (i, j), and the ranges of i and j are $1 \le i \le N$ and $1 \le j \le N$, respectively. The wavefront data in the ith row and jth column of $\Delta W(x, y)$, $W(x, y)$ and $W(xs^2, ys^2)$ can be accordingly represented by ΔW_{ij}, W_{ij} and $W^s{}_{ij}$, respectively. The vectors $\Delta\boldsymbol{W}$, $\boldsymbol{W}$ and $\boldsymbol{W}^s$ are indexed serially row by row of ΔW_{ij}, W_{ij} and $W^s{}_{ij}$ with the dimension of $N^2 \times 1$, respectively. Thus, the matrix form of Eq. (2) can be expressed as

$$\Delta\boldsymbol{W} = \boldsymbol{W} - \boldsymbol{W}^s, \tag{5}$$

In Fig. 2(a), the data of $W(x, y)$ over the shaded area can be described by the Kronecker delta function δ_{pq} which is the discrete version of the Dirac delta function δ. The discrete data over the shaded area is defined as Eq. (6) and denoted by the notation (m, n). Assuming Ns^2 is the integer portion of $N \times s^2$, the ranges of m and n are $1 \le m \le Ns^2$ and $1 \le n \le Ns^2$, respectively. The relationships between i and m, j and n are $i = m + (N - Ns^2)/2$ and $j = n + (N - Ns^2)/2$, respectively. The wavefront data in the mth row and nth column over the shaded area is represented by W_{mn}. The vector $\tilde{\boldsymbol{W}}$ is indexed serially row by row of W_{mn} with the dimension of $(Ns^2)^2 \times 1$.

$$\tilde{\boldsymbol{W}} = \delta_{pq} \cdot \boldsymbol{W}. \tag{6}$$

The matrix form of Kronecker delta function δ_{pq}, which is denoted by the notation (p, q) with the dimension of $(Ns^2)^2 \times N^2$, is given by

$$\delta_{pq} = \begin{cases} 1, & p = (m-1)Ns^2 + n; \\ & q = N(m-1) + (N+1)\left(N - Ns^2\right)/2 + n, \\ 0, & otherwise \end{cases} \tag{7}$$

where the ranges of p and q are $1 \le p \le (Ns^2)^2$ and $1 \le q \le N^2$, respectively.

In order to implement the discrete data operation, $W(xs^2, ys^2)$ should also be interpolated to a matrix with the size of $N \times N$ based on the data over the shaded area in $W(x, y)$. By investigating the RSI, we found that the interpolation of $W(xs^2, ys^2)$ is not only related to the radial shearing ratio s but also the number of discrete points N^2. Assuming that the number of discrete points satisfies the accuracy of measurement and the change between the two adjacent discrete points is linear [27–32].

In the one-dimensional situation, the test wavefront $W(x)$ is considered as an arbitrary line, as illustrated in Fig. 2(b), the dots "•" denote W_n, the circles "o" denote $W^s{}_j$, and the distance between two adjacent discrete points of W_n is defined as one (unit distance). The distance between two adjacent discrete points of $W^s{}_j$ can be given as

$$d = \left(Ns^2 - 1\right)/(N - 1). \tag{8}$$

According to the bilinear interpolation, W_n can be redistributed in the same domain of definition of test wavefront so that $W^s{}_j$ can be obtained as

$$W_j^s = d_{n+1} W_n + d_n W_{n+1}, \tag{9}$$

where $d_n = |d(j-1) - n|$ and $d_{n+1} = |n + 1 - d(j-1)|$ are the interpolation coefficients. Thus, Eq. (9) can be rewritten as

$$W_j^s = \left[d_{n+1} \quad d_n\right] \cdot \left[W_n \quad W_{n+1}\right]^T. \tag{10}$$

Therefore, the expanded wavefront can be calculated and its matrix form can be expressed as

$$\boldsymbol{W}_1^s = \boldsymbol{D}_1 \cdot \boldsymbol{W}_1, \tag{11}$$