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Improved FRFT-based method for estimating the physical parameters from Newton's rings

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A R T I C L E I N F O

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A B S T R A C T

Newton's rings are often encountered in interferometry, and in analyzing them, we can estimate the physical parameters, such as curvature radius and the rings' center. The fractional Fourier transform (FRFT) is capable of estimating these physical parameters from the rings despite noise and obstacles, but there is still a small deviation between the estimated coordinates of the rings' center and the actual values. The least-squares fitting method is popularly used for its accuracy but it is easily affected by the initial values. Nevertheless, with the estimated results from the FRFT, it is easy to meet the requirements of initial values. In this paper, the proposed method combines the advantages of the fractional Fourier transform (FRFT) with the least-squares fitting method in analyzing Newton's rings fringe patterns. Its performance is assessed by analyzing simulated and actual Newton's rings images. The experimental results show that the proposed method is capable of estimating the parameters in the presence of noise and obstacles. Under the same conditions, the estimation results are better than those obtained with the original FRFT-based method, especially for the rings' center. Some applications are shown to illustrate that the improved FRFT-based method is an important technique for interferometric measurements.

1. Introduction

Interferometry plays a key role in the field of optical measurements. As a nondestructive technique, it has been applied in diverse areas of science and engineering for estimating various physical parameters, such as curvature radius, wavelength, refractive index, displacement, and strain. In these applications, the fundamental step is analyzing the interference fringe patterns, which can be mathematically modeled as [1]

$$I(x, y) = a(x, y) + b(x, y)\cos[\varphi(x, y)] \tag{1}$$

where $I(x, y)$, $a(x, y)$, $b(x, y)$, and $\varphi(x, y)$ are known as the recorded intensity, background intensity, fringe amplitude, and phase distribution, respectively. In general, the information for the measured physical quantities is encoded in the phase and its derivatives of the recorded fringe patterns. Therefore, numerous methods for extracting the phase and its derivatives have been proposed. A widely used technique for phase measurement is phase-shifting [2]; this technique requires multiple fringe patterns for analysis, and therefore, it is more susceptible to external disturbances. Thus, a fringe analysis method needed to be developed that extracted the phase from a single fringe pattern [3,4]. In a similar manner, direct estimation of the phase derivative was also developed [5–8]. In some of these works, the phase was modeled as one-dimensional polynomial phase signals (PPS) for each row of the fringe patterns. By computing the polynomial phase coefficients, the phase and its derivative could be estimated, and the measured physical quantities could also be obtained.

In optical metrology, various complex fringe patterns can be represented using the following two elementary fringe patterns: straight equispaced fringe patterns and quadratic phase fringe patterns, which are also called *Newton's rings*. These elementary fringe patterns are of great importance because all other fringe patterns can be decomposed or approximated in terms of these two patterns [9,10]. Therefore, analyzing these two elementary patterns is very important in interference fringe analysis. Moreover, Newton's rings are a classical example of interference fringes, and often encountered when using interferometers, like Michelson, Twyman-type, and Fizeau-type interferometers. Newton's rings are produced when two spherical wavefronts interfere due to their different curvature; the rings are characterized by circular concentric fringes whose relative spacing becomes narrower with increasing distance from the center of the pattern. By

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analyzing Newton's rings, one can estimate the curvature radius of optical components, measure wavelength, detect any displacement and deformation, and test optical connector end faces.

Various methods for analyzing Newton's rings have been widely used to measure the physical parameters. A common step in these methods involves finding the center of the circular rings and computing the curvature radius. Some methods rely on analyzing the fringe structure in the interferogram. For instance, the method described in [11] for extracting fringe skeletons is the classical technique. It is based on a topological consideration of the interferogram to detect the positions of the strongest and weakest stripes of the fringes (centerlines of the fringes), where the value of the fringe intensity is a maximum or minimum. Then, the measured parameters are obtained through the fringe skeletons. However, this method is sensitive to noise; therefore, a variety of filtering methods such as the spin filtering method [12], wavelet methods [13], and windowed Fourier transform method [3], have been proposed. In [14], the researchers believed that the techniques based on topology used relatively less information regarding Newton's rings, and therefore, they presented a new algorithm for processing these elementary fringe patterns by considering all the pixels in an image. Because the first step of the algorithm involves finding the center of the circular fringes, the curvature-radius estimation errors are generally influenced by the ring center. The aforementioned methods are based on a geometrical approach and require a certain amount of statistical calculation. In addition to these methods, many studies have also introduced numerical processing methods because the phase difference term in the expression for intensity in fringe patterns can be described by a simple quadratic equation [15] ,

$$I(x, y) = I_0 + I_1 \cdot \cos(\alpha_x x^2 + \alpha_y y^2 + \omega_x x + \omega_y y + \varphi_0), \tag{2}$$

where I_0 is the mean intensity in the fringe pattern; I_1 is the amplitude; (α_x, α_y) is determined by the rings' density, which depends on the curvatures of the interfering waves; (ω_x, ω_y) is related to the coordinates of the rings' center; and φ_0 is the phase. Nascov et al. [16] proposed the least-squares fitting (LSF) method to analyze fringe patterns; their method is implemented with an iterative procedure. The numerical algorithm that they proposed determines the fringe parameters with a high degree of accuracy; however, the main drawback of the algorithm is in finding the initial values for the parameters $(I_0, I_1, \alpha_x, \alpha_y, \omega_x, \omega_y, \varphi_0)$. Furthermore, if these initial values are not sufficiently close to the unknown exact values, the iterative procedure associated with the method will perform erratically. To overcome this limitation, Nascov et al. [15] showed that the fringe parameters could be determined using discrete Fourier analysis. Compared with the LSF method, this method based on the discrete Fourier transform does not require initial values, but it is as precise as the one based on LSF. Similarly, in [17], Lu et al. also implemented parameter estimation of optical fringes that did not require initial values for the parameters. The difference between the two methods, the one proposed in [15,17], lies in that, in the former, one parameter is determined based on another retrieved parameter, whereas in the fractional Fourier transform (FRFT) stated in the latter, all fringe parameters describing the quadratic phase function are determined simultaneously. Moreover, when some portions of the fringe patterns are blocked out, the FRFT still helps in effectively estimating the abovementioned parameters. However, the ring center position obtained by the FRFT approach deviates from the exact position by small error margins [17].

In this paper, an improved FRFT-based method for analyzing patterns of Newton's rings is proposed. It combines the advantages of the FRFT [17] with the LSF algorithm [16] in analyzing Newton's rings. This combined method can be used to directly analyze the patterns without any filter processing, and can provide improved accuracy in calculating some parameters, especially the position of the ring center. Furthermore, because of the use of FRFT, our proposed method can be used to perform fringe analysis even when portions of the fringe patterns are masked because of the presence of obstacles.

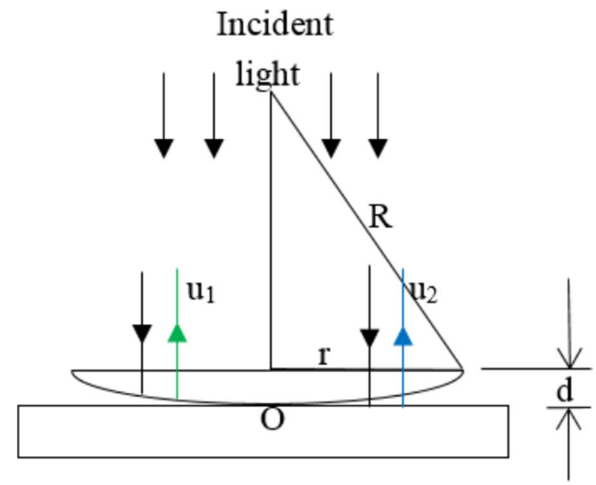

Fig. 1. Schematic diagram of Newton's rings.

The paper is organized as follows. In Section 2, a mathematical description for Newton's rings is derived. Then, in Sections 3 and 4, the performances of the FRFT method and the LSF method are presented in detail by applying them to some computer-simulated Newton's rings fringe images. Next, in Section 5, the performance of the proposed method is assessed with simulated and actual Newton's rings. Its practical applications are also described in this section. Finally, the discussion and conclusions are given in Section 6.

2. Mathematical description of Newton's rings

The basic setup for generating fringe patterns from Newton's rings is shown in Fig. 1. A plano-convex lens is placed on a flat glass, and there is an air layer of uneven thickness between the surfaces. When monochromatic light rays are incident on the setup vertically, two sets of reflected light rays are obtained: one is from the curved surface of the plano-convex lens, and the other is from the plane surface. When these two sets of reflected light rays are superposed, they generate interference fringe patterns, commonly known as Newton's rings; an example is shown in Fig. 2.

Before analyzing the fringe patterns, the intensity distribution of Newton's rings is considered, which is described by

$$\begin{aligned} I(x, y) &= I_0 + I_1 \cdot \cos(\nabla\varphi) \\ &= I_0 + I_1 \cdot \cos\left(\frac{2\pi}{\lambda_0}\delta\right) \\ &= I_0 + I_1 \cdot \cos\left(\frac{2\pi}{\lambda_0 R} r^2 + \pi\right), \end{aligned} \tag{3}$$

where I_0 and I_1 represent the mean intensity and the amplitude of the sine variation of the fringes, respectively; $\nabla\varphi$ is the phase difference and $\nabla\varphi = \varphi_1(x, y) - \varphi_2(x, y)$, where $\varphi_1(x, y)$, and $\varphi_2(x, y)$ are the phases of the two reflected wavefronts $u_1(x, y)$ and $u_2(x, y)$, respectively, which

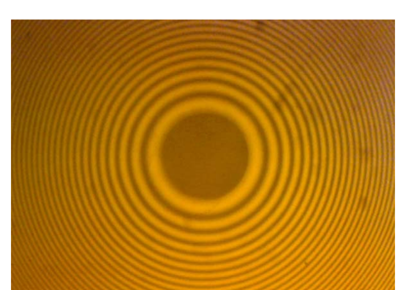

Fig. 2. Image of Newton's rings.