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Digital image correlation method for calculating coefficients of Williams expansion in compact tension specimen

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ABSTRACT

The digital image correlation (DIC) method is used to obtain the coefficients of higher-order terms in the Williams expansion in a compact tension (CT) specimens made of polymethyl methacrylate (PMMA). The displacement field is determined by the correlation between reference image (i.e., before deformation) and deformed image. The part of displacements resulting from rigid body motion and rotation is eliminated from the displacement field. For a large number of points in the vicinity of the crack tip, an over-determined set of simultaneous linear equations is collected, and by using the fundamental concepts of the least-squares method, the coefficients of the Williams expansion are calculated for pure mode I conditions. The experimental results are then compared with the numerical results calculated by finite element method (FEM). Very good agreement is shown to exist between the DIC and FE results confirming the effectiveness of the DIC technique in obtaining the coefficients of higher order terms of Williams series expansion from the displacement field around the crack tip.

1. Introduction

Non-destructive inspections are usually performed to predict the sizes and the locations of cracks that are generated during the manufacturing processes or service life of engineering structures or components. The existence of cracks changes the load-bearing capacity of the components especially for brittle materials. The stress intensity factors (SIFs), the T-stress and the coefficients of higher-order terms in the Williams series expansions are important crack tip parameters for fracture analysis of cracked components using linear elastic fracture mechanics (LEFM) concepts.

The linear elastic stress field for a plate with a crack subjected to an arbitrary in-plane loading can be expressed by Williams series expansions [1], as presented in Eq. (1).

$$\begin{Bmatrix}\sigma_x\\ \sigma_y\\ \tau_{xy}\end{Bmatrix}=\sum_{n=1}^{\infty}\frac{n}{2}A_n r^{\left(\frac{n}{2}-1\right)}\begin{Bmatrix}(2+\frac{n}{2}+(-1)^n)\cos(\frac{n}{2}-1)\theta-(\frac{n}{2}-1)\cos(\frac{n}{2}-3)\theta\\ (2-\frac{n}{2}-(-1)^n)\cos(\frac{n}{2}-1)\theta+(\frac{n}{2}-1)\cos(\frac{n}{2}-3)\theta\\ (\frac{n}{2}-1)\sin(\frac{n}{2}-3)\theta-(\frac{n}{2}+(-1)^n)\sin(\frac{n}{2}-1)\theta\end{Bmatrix}-\sum_{n=1}^{\infty}\frac{n}{2}B_n r^{\left(\frac{n}{2}-1\right)}\begin{Bmatrix}(2+\frac{n}{2}-(-1)^n)sin(\frac{n}{2}-1)\theta-(\frac{n}{2}-1)sin(\frac{n}{2}-3)\theta\\ (2-\frac{n}{2}+(-1)^n)sin(\frac{n}{2}-1)\theta+(\frac{n}{2}-1)sin(\frac{n}{2}-3)\theta\\ -(\frac{n}{2}-1)cos(\frac{n}{2}-3)\theta+(\frac{n}{2}-(-1)^n)cos(\frac{n}{2}-1)\theta\end{Bmatrix}, \tag{1}$$

In these expansions, r and θ, are the polar coordinates as shown in Fig. 1. A_n and B_n are the constant coefficients related to the mode I and mode II parts of deformation, respectively, and n is the order of terms. Mode I and mode II stress intensity factors K_I and K_{II} are related to the coefficients of the first (or singular) terms ($n = 1$), which are shown by A_1 and B_1 in these expansions. In addition, the coefficient of the second term in the mode I expansion (A_2) is related to the constant term called T-stress. Eq. (2) expresses these relations.

$$K_I=\sqrt{2\pi}A_1,\ K_{II}=\sqrt{2\pi}B_1,\ T=4A_2. \tag{2}$$

The stress intensity factors, K_I and K_{II}, have been traditionally used for analysis of the cracks in brittle cracked components. However, the higher-order (or non-singular) terms in the Williams expansion have

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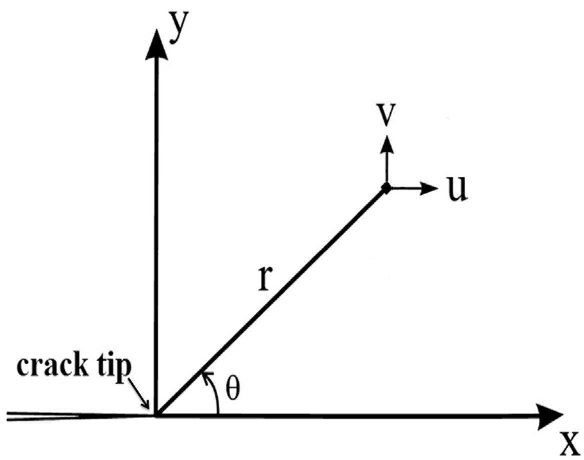

Fig. 1. The crack tip displacements in the polar coordinate system.

also an important role in analyzing the fracture processes. Previous studies have shown that the T-stress or the second term in the Williams expansion (n=2) can be of significant effects on crack propagation. For instance, Rice [2] and Betegón and Hancock [3] showed that both the sign and magnitude of the T-stress influence the shape and size of the plastic zone around the crack tip. The results presented by, Chen and Dillard [4] and Melin [5] also revealed that the path of crack growth and its directional stability are dependent on the sign and value of T-stress. Smith et al. [6] and Ayatollahi et al. [7] performed some investigations and showed that the T-stress is of significant effect on mixed mode fracture resistance of cracked components. The importance of the higher order terms in the Williams series expansion (i.e. $n \geq 3$) has also been demonstrated by a few researchers, particularly for quasi-brittle materials, see for example Chao and Liu [8] and Kardomateas et al. [9]. Ayatollahi and Akbardoost [10] have recently proposed a model which makes use of the first three terms of Williams expansion to predict size effects on mode I fracture for brittle and quasi-brittle materials. The coefficients of higher order terms in the Williams expansion can be obtained analytically only for very few cases which have simple shapes. As a result, the use of numerical approaches such as the finite element (FE) method is a good alternative for the problems which have complicated shapes. Some investigators like Rice [11], Shih and Asaro [12], Toshio and Parks [13], Ayatollahi et al. [14] and Chen et al. [15] have studied the application of FE method for the calculation of SIFs and T-stress. Karihaloo and Xiao [16] and Tong et al. [17] have used higher-order hybrid crack elements for determining the coefficients of leading terms from the elastic fields near the crack tip. More recently, Ayatollahi and Nejati [18] have used an over-deterministic method called FEOD for calculation of the coefficients of higher-order terms in cracked bodies, using the displacement field obtained from FE analysis. As an advantage, the FEOD method can be used in conjunction with the conventional finite element codes.

In addition to analytical and numerical approaches, classical experimental method, such as strain gauging, moire interferometry and optical methods have been used for calculation of the SIFs. Parnas et al. [19] used strain gauge technique to obtain the strain field near the crack tip, and then calculated SIFs from the measured strains. Because of limitation in the number of selected points, this method has some disadvantages. Optical methods such as photoelasticity and moire interferometry provide full field experimental data and can be used for determination of the SIFs in cracked components, see for example [20–22]. Digital image correlation (DIC) method has also received much attention in recent years in solid mechanics [23,24]. The output of this method is full-field in contrast to the strain gauge technique, thus there is virtually no limitation in the number of selected points. Also, different types of materials including opaque and transparent ones can be used in experiments. Because of its simplicity and accuracy DIC has been recently proposed to determine crack tip parameters like K_I and K_{II}. For example, McNeill et al. [25] and Roux and Hild [26] calculated SIF in mode I loading or Lopez-Crespo et al. [27], Réthoré et al. [28], Yoneyama et al. [29] and Zhang and He [30] measured mixed-mode SIFs by using the DIC method. In addition, Desai et al. [31] calculated SIF for a crack in a bimaterial interface using the DIC method. Three dimensional DIC method has also been used for analyzing cracked components, see for instance Roux et al. [32], Ghafoori and Motavalli [33] and Mostafavi et al. [34]. Using a comprehensive study, Yates et al. [35] measured the crack tip parameters such as K, T-stress and crack tip opening angle (CTOA) from the displacement field obtained by the DIC technique. In the recent decades, the DIC method has attracted much attention in fracture mechanics community. While most of these studies have dealt with the use of DIC method to obtain deformation in cracked components [36–39], some researchers have employed this technique to obtain fracture parameters in different materials [40,41].

In the present study, the DIC method has been used to determine the mode I coefficients of higher-order terms in the Williams series expansions for a cracked polymethyl methacrylate (PMMA) plate. First, 2D-DIC is used to obtain full field displacement while the errors due to rigid body motions and rotation are omitted. Afterwards, the obtained displacement field within the elastic zone is used to determine the coefficients of the Williams series expansions using the least-squares method. In order to verify the accuracy and efficiency of the proposed technique, the crack tip coefficients obtained by DIC method, are compared with those determined from finite element analysis.

2. DIC method for calculation of coefficients of Williams expansion

2.1. Fundamentals of digital image correlation method

Comparison between the digital images taken before and after deformation is the basis of DIC approach for calculating the displacement field. This process is generally executed in three sequential steps. The first step is the specimen preparation and the experimental setup. Since the surface of the specimens should be comparable in digits, it is required that the specimens be painted by speckle of white and black dot patterns. In the second step, digital images of the specimen surface, which is perpendicular to the optic axis of camera, are recorded before and after deformation. In the third step, using the image processing program these captured images are correlated, i.e. a small selectable region of the reference image is traced to the same region at the deformed image. This process is performed according to a number of subsets in two images. The subset is a selectable region consisting of several pixels. The size and distance between these regions depend on the magnitude of applied deformation and the degree of accuracy of results. The coordinate of a speckle in the first image is followed in the deformed image by computer program, and thus its displacements in different directions are calculated [42]. To evaluate the accuracy of the correlation process, a correlation coefficient has been suggested by researchers. The correlation coefficient is calculated by contrasting the subset in the reference with that of the deformed image [43]. This coefficient is used as an optimization criterion for pattern matching. The correlation coefficient can be written as follows:

$$C(x,\ y,\ u,\ v)=\sum_{i,j=-n/2}^{n/2}(I(x+i,\ y+j)-I^*(x+u+i,\ y+v+j))^2 \tag{3}$$

In this equation, x and y, are the coordinates of pixel for each point in the reference subset, and u and v are the displacement components for the same point. The light intensity function of images before and after loading are shown by I and I*, respectively, and $n = 0.5 \times (m - 1)$, where m is the subset size, which is always an odd number. The displacement in the selected region can be calculated