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Temporal phase unwrapping using orthographic projection

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ABSTRACT

This paper proposes a novel temporal phase unwrapping method which is a generalization of the three commonly proposed approaches: hierarchical, heterodyne, and number theoretical phase unwrapping. The proposed unwrapping method is based on the orthographic projection of wrapped phases from the measurement space to the space of co-dimension 1. In the space of co-dimension 1 all unknown integer period-order numbers are computed simultaneously using a nearest-neighbor search on a fixed constellation of points. The proposed method offers new key insights about relationship between unwrapping success rate, noise limit, and maximum unwrapping range. We give example how the proposed method may be applied in the multiple frequency phase shifting profilometry.

1. Introduction

The problem of phase unwrapping arises in many applications such as structured light profilometry [1], optical interferometry [2], interferometric synthetic aperture radar (InSAR) [3], and magnetic resonance imaging [4], to name a few. The problem is characterized by the fact the phase is measurable modulo-2π only. Formally, let Φ denote the true phase value and let ϕ denote the phase measured modulo-2π; then

$$\Phi = \phi + 2\pi k, \tag{1}$$

where Φ is the *true* or *absolute* phase, ϕ is the *wrapped* or *principal* phase, and $k \in \mathbb{Z}$ is an unknown integer which models the phase ambiguity and is sometimes called *period-order* or *fringe-order* number. The task of phase unwrapping methods is to *unwrap* the *wrapped* phase ϕ and obtain the *true* phase Φ.

Various approaches to phase unwrapping proposed in the literature may be classified into two broad groups: *spatial* [5] and *temporal* [6] phase unwrapping. Spatial phase unwrapping methods use only one frequency and unwrap the phase via spatial analysis of a two-dimensional wrapped phase map. Temporal phase unwrapping methods use more than one frequency and unwrap the phase via temporal analysis of multiple wrapped phase values for each sample point separately. In essence, temporal methods assume *temporal invariance* of phase values while *spatial* methods assume *spatial continuity* of phase values. The temporal approaches to phase unwrapping provide an exact solution and are therefore the method of choice for all applications where the phase is *temporally invariant*.

Approaches to temporal phase unwrapping are usually classified into three groups [6]: hierarchical methods [7–13], heterodyne methods [14–18], and number-theoretical methods [19–23]. In this paper we propose a novel temporal phase unwrapping method that may be considered a generalization of all of the three aforementioned approaches. It unwraps the phase in the same way regardless of wavelengths selected; the proposed method therefore has minimal constraints on wavelengths. The proposed method is based on the orthographic projection of wrapped phases from the measurement space to the space of co-dimension 1. In the space of co-dimension 1 all unknown integer period-order numbers are computed simultaneously using a nearest-neighbor search over a fixed constellation of points. The nearest-neighbor search replaces the rounding operator of the existing approaches and offers optimal performance with regard to noise. The proposed method is simple to implement and is not computationally expensive as it requires one matrix multiplication and one nearest-neighbor search to determine the correct period-order numbers.

Besides describing a novel temporal phase unwrapping method we also provide several new key insights: (a) for a desired unwrapping success rate there exists a hard limit on allowed noise which is directly computable given the chosen wavelengths without the need to run numerical simulations; (b) the exponential frequency sequence with basis 2 has the largest tolerance to noise; and (c) the choice of frequencies whose ratio is rational provides better noise tolerance than using frequencies whose ratio is irrational.

This paper is structured as follows: In Section 2 we give a brief overview of both spatial and temporal phase unwrapping methods. In Section 3 we describe the proposed temporal phase unwrapping

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method. In Section 4 we give noise analysis and we compare the proposed method to each of the three classes of temporal phase unwrapping methods. In Section 5 we present computer simulations and we show how to apply the proposed temporal phase unwrapping method to the problem of multiple phase shifting surface profilometry. We conclude in Section 6.

If the reader is only interested in implementing the proposed unwrapping method then reading the introduction to Section 3 and Subsections 3.1 and 3.3 is sufficient.

2. A brief review of phase unwrapping

Approaches to phase unwrapping may be classified into two distinct groups: *spatial* and *temporal* approaches. Spatial approaches are especially suitable for problems where phase may change in time or where repeated measurements are prohibitively costly or impossible to perform. Spatial approaches are also limited in their ability to handle discontinuities in phase; due to the periodic structure of the phase observation there exist discontinuities that cannot be uniquely determined using spatial analysis of the observed phase data [24]. Temporal approaches were introduced due to their ability to handle any kind of discontinuities in phase. They are limited to problems where phase is temporally invariant and where repeated measurements are possible.

2.1. Spatial phase unwrapping

In spatial phase unwrapping the true phase is determined from one two-dimensional phase measurement. As there is only one wrapped phase value per sample point the unwrapping is an ill-posed problem; Eq. (1) does not have an unique solution unless further information is added [5]. To make the problem solvable spatial phase unwrapping methods assume the phase difference between two adjacent points varies smoothly, i.e. the absolute wrapped phase difference is less than π. This assumption is also known as *Itoh's* condition [25]. If Itoh's condition is satisfied then the true phase derivative may be estimated from the finite differences of the wrapped phase values making the true phase recoverable up to a constant by integration. Unfortunately, the Itoh's condition is easily violated if the phase measurements are noisy or if the true phase contains discontinuities thus making naive integration inapplicable in real-world applications. To overcome this the areas where phase derivatives can not be estimated are usually detected and are then avoided in the integration process.

Approaches to spatial phase unwrapping may be roughly classified as: (a) path-following methods [3,26–29]; (b) optimization methods [24,30-34]; and (c) parametric methods [35,36].

The path-following methods unwrap the wrapped phase on paths either by integrating the estimated phase differences or by adding a whole number of cycles to the unwrapped phase thus reducing the phase difference to less than half-cycle. Regardless of how the unwrapping on paths is implemented, the path-following methods significantly differ in the way the paths which achieve the path-independent phase unwrapping are chosen: branch-cuts are used in [3] and quality or reliability maps are used in [26–29].

The optimization methods do not unwrap the phase on paths but instead cast the phase unwrapping problem as an optimization of some objective function. Most often the objective function used is L^p norm of the difference between absolute phase differences and wrapped phase differences [30–34], although Bayesian approaches which find MAP phase estimate are also used [24,34]. The solution for L^2 (least-squares) unwrapping is given by Ghiglia and Romero [30] and is extended to regularized least-squares by Marroquin and Rivera [31]. Ghiglia and Romero extended their least-squares solution to cases $p < 2$ in [32]. An efficient solution for L^1 norm is given by Costantini [33].

The parametric methods constrain the phase to some predetermined parametric model, usually a low-order polynomial [35,36]. The parametric approaches yield excellent results only if the parametric model accurately represents the true phase.

2.2. Temporal phase unwrapping

In temporal phase unwrapping the true phases are determined from repeated wrapped phase measurements using different frequencies for each measurement. Under the assumption of temporal invariance one then obtains a system of equations of the form given by Eq. (1). Such system of simultaneous equations may be solved exactly under certain conditions. Most often the condition used limits the allowed range of true phase values to some predetermined interval.

Approaches to temporal phase unwrapping may be classified as: (a) hierarchical phase unwrapping [7–13]; (b) heterodyne phase unwrapping [14–18]; (c) number-theoretical phase unwrapping [19,20,22,23]; and (d) optimization approaches [37–42].

The hierarchical phase unwrapping methods use a sequence of signals with increasing frequencies whose wrapped phase is then measured. The signals are selected so the lowest-frequency signal (the coarsest) contains only one period per allowed phase range; it therefore provides the true phase measurement by definition. The remaining signals which have larger frequencies are then incrementally unwrapped by computing the period-order number from the unwrapped phase of a preceding coarser signal. The main difference between various variants described in the literature is in the selection of frequencies [6]. The selected frequencies may form linear [7], exponential [43], reverse exponential [10], etc. sequence. To be efficient one wants to use the shortest possible frequency sequence which allows reliable measurements regarding noise and application constraints.

The heterodyne phase unwrapping uses signals of similar wavelengths which may be virtually combined to produce a beat interference signal. Most often only two wavelengths λ_1 and λ_2 are used [14–16]. For two-wavelength approach if the condition $\lambda_1 < \lambda_2 < 2\lambda_1$ is fulfilled then the phase may be unwrapped by analyzing the virtual beat signal at the wavelength $\lambda_2 - \lambda_1$. The true phase of a smaller wavelength signal is recoverable in $[0, 2\pi\lambda_{eq}/\lambda_1\rangle$ range, where $\lambda_{eq} = \lambda_1\lambda_2/(\lambda_2 - \lambda_1)$. Extension to more than two wavelengths is given in [17,18].

The number-theoretical phase unwrapping methods were first proposed by Gushov and Solodkin [19]. They are based on the divisibility properties of integers and use the Chinese-remainder theorem to solve a system of congruence equations of the form

$$\mathrm{Round}\left[\frac{\lambda_i}{2\pi}\Phi_i\right] \equiv \mathrm{CircularRound}\left[\frac{\lambda_i}{2\pi}\phi_i\right] \bmod \lambda_i, \tag{2}$$

$i = 1,...,n$. The number-theoretical phase-unwrapping requires that the used wavelengths λ_i are pairwise co-prime and that the range of the true phase is equal-to or smaller-than the least common multiple of wavelengths. The method is extended to use a look-up table for speed in [20]. Improvements which address the noise-sensitivity of the method are proposed in [20,22,23].

In the optimization approaches phase unwrapping is considered as an optimization problem. There are two approaches: the method of excess fractions [37–41] and L_1-minimization [42]. The method of excess fractions defines a residual error in terms of excess fractions. The phase is unwrapped by finding integer period-order numbers which minimize the residual error in the least-squares sense. The L_1-minimization approach selects integer period-order numbers which minimize the absolute phase difference.

For more details about temporal phase unwrapping methods we refer the reader to a review by Zuo et al. [6].

3. Phase unwrapping using orthographic projection

In temporal phase unwrapping more than one wavelength (or