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A novel 2nd-order shape function based digital image correlation method for large deformation measurements

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ABSTRACT

Compared with the traditional forward compositional matching strategy, the inverse compositional matching strategy has almost the same accuracy, but has an obviously higher efficiency than the former in digital image correlation (DIC) algorithms. Based on the inverse compositional matching strategy and the auxiliary displacement functions, a more accurate inverse compositional Gauss-Newton (IC-GN2) algorithm with a new second-order shape operator is proposed for nonuniform and large deformation measurements. A theoretical deduction showed that the new proposed second-order shape operator is invertible and can steadily attain second-order precision. The result of the numerical simulation showed that the matching accuracy of the new IC-GN2 algorithm is the same as that of the forward compositional Gauss-Newton (FC-GN2) algorithm and is relatively better than in IC-GN2 algorithm. Finally, a rubber tension experiment with a large deformation of 27% was performed to validate the feasibility of the proposed algorithm.

1. Introduction

A highly efficient and accurate measurement of strain and deformation is very important for studying the mechanical properties and inherent deformation law of different materials and has been a hot topic in the field of mechanics and materials science research. In recent years, as a mature and effective strain measurement technique, two-dimensional digital image correlation (2D-DIC) has many measurement advantages such as high accuracy, noncontact, full-field, three-dimensional, dynamic measurement, low demands on experiment environment, and simple specimen surface treatment [1,2]. Moreover, 3D-DIC was developed by Sutton et al. [3] for out-of-plane deformation measurements. Shao et al. [4] applied 3D-DIC to real-time human pulse monitoring in medical measurements. As a core in DIC techniques, the local matching algorithm between images before and after deformation is always the focus of attention.

At present, the image local matching algorithm mainly includes the feature matching algorithm [1], the phase matching algorithm [5], and the region matching algorithm [6]. These matching algorithms all depend on the interpolation precision. Schreier et al. [7] proved that the use of higher order image interpolation function can significantly improve the searching precision of DIC. Su et al. [8–10] and Gao et al. [11] explored the influences depending on different interpolation methods and different degrees of noise, and they used mean bias error and standard deviation error to represent the systematic error and the random error. In the matching effect, the feature matching algorithm can only match the specifically required image features, and the matching result is sparse rather than wholly continuous [1]. However, the phase matching algorithm easily falls into the phase singularity and is not suitable for whole matching [5]. By calculating the correlation coefficients of the subset gray levels between the reference image and the target image, the region matching algorithm can obtain dense matching results [6]. In 2D-DIC and 3D-DIC, the region matching algorithm has been widely accepted and developed rapidly.

The region matching algorithm uses a preset correlation function as the optimization objective to find the best matching position. In the matching process, the appropriate subset shape function is selected according to the deformation characteristics of the region of interest (ROI), and the parameters of the subset shape function are used to describe the deformation characteristics of the local coordinate system [1]. Similar to the shape function in the finite element method (FEM), the higher order shape function can be used to describe more complicated mechanical deformation, but it will also require more computations. Usually, the first-order shape function is suitable for small deformation measurements, and can effectively characterize the translation, rotation, and uniform deformation. The second-order shape function can effectively reflect the complex deformation, thus making the nonuniform and large deformation measurement feasible [12]. Lu et al. [13] proved that the second-order shape function has

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better precision than the first-order one and has been successfully applied to the large deformation measurement of materials. Wang et al. [14] and Wang et al. [15] have investigated the influence of data noise on the matching accuracy, which can be reduced by the more accurate image interpolation and the selection of the appropriate correlation function. Schreier et al. [16] and Xu et al. [17] have respectively researched the systematic errors arised from the shape functions and subset sizes, and they found a larger subset size would probobally cause a more significant errors.

The region matching algorithm is a complicated nonlinear optimization problem and is generally optimally solved by the Newton-Raphson iteration method or trust-domain iteration method. The commonly used iterative updating methods are the forward additive Gauss-Newton (FA-GN) algorithm and the forward compositional Gauss-Newton (FC-GN) algorithm, both of which have been proved to be equivalent to each other by Shum and Szeliski [18]. The traditional FC-GN algorithm is described as follows. The deformation of the target subset is realized by the shape function and is searched by the maximum correlation with the reference subset in the iteration. The subpixel interpolation of the target image for each iteration is completed to obtain the partial derivatives, the Jacobian matrix and its inverse matrix calculation, which consumes most of the computational time in the subpixel matching process.

Baker and Matthews [19] found that the inverse compositional Gauss-Newton (IC-GN) algorithm can effectively reduce the calculation time because in each matching process the image partial derivative, the Jacobian matrix and its inverse matrix are calculated only once until the matching is accomplished. The Jacobian determinant is closed to 1 when the FC-GN form is transformed to its inverse form, the IC-GN algorithm; therefore the two algorithms are equivalent. On the basis of the IC-GN algorithm, Pan et al. [20,21] added the zero-normalized sum of squared differences (ZNSSD) correlation function and a seed parallel searching strategy. Shao et al. [22] proposed an anti-noised IC-GN algorithm using the SSD correlation function. Gao et al. [23] suggested that a second-order inverse compositional Gauss-Newton (IC-GN2) algorithm be applied to 3D-DIC matching and validated using simulated speckle images. In addition, the second-order shape function was applied to the IC-GN2 algorithm for out-of-plane deformation [24], 3D deformation of satellite antenna surface [25], and civil engineering materials [26,27].

However, Gao's IC-GN2 algorithm has an imperfect second-order shape operator. After a brief introduction of the FA-GN, FC-GN, and IC-GN algorithms, the IC-GN2 algorithm with a new second-order shape operator is proposed in this paper. Theoretical error analysis showed that the new IC-GN2 algorithm can achieve a stable second-order accuracy. The simulated speckle numerical comparison and the rubber tension experiment for large deformation measurement verified the validity of the IC-GN2 algorithm with the proposed second-order shape operator.

2. Fundamental principles

2.1. FA-GN algorithm

The displacement shape function plays an important role in the traditional FA-GN algorithm, since it describes the displacement mode of a point. Therefore, the deformation relationship between the target subset and the reference subset can be established. The first-order displacement shape operator can be written as

$$W(\mathbf{x}, p) = \begin{bmatrix} 1 & 0 & 0 \\ u & u_x + 1 & u_y \\ v & v_x & v_y + 1 \end{bmatrix} \begin{bmatrix} 1 \\ x \\ y \end{bmatrix} = \mathbf{A}(p)\mathbf{x}, \tag{1}$$

where $\boldsymbol{x} = [1\ x\ y]^T$ is the polar coordinate of the reference image under the local coordinate system. $p = [u\ u_x\ u_y\ v\ v_x\ v_y]^T$ is the vector concluding first-order deformation parameters of the target image and the subscripts denote the partial derivatives at the subset center. $\boldsymbol{A}(p)$ is a linear transformation matrix with size of 3×3 for the first-order deformation vector p. In the DIC process, the first-order deformation vector p is obtained through minimizing the image correlation function as

$$C(p) = \sum_x [f(\mathbf{x}) - g(W(\mathbf{x}, p))]^2, \tag{2}$$

where $f(\boldsymbol{x})$ and $g(W(\boldsymbol{x}, p))$ denote the gray matrices in the reference subset and the target subset, respectively. Every iteration in the FA-GN algorithm aims to minimize the following deviation function as

$$\sum_x [f(\mathbf{x}) - g(W(\mathbf{x}, p + \Delta p))]^2 \approx \sum_x \left[g(W(\mathbf{x}, p)) + \nabla g(W(\mathbf{x}, p)) \frac{\partial W(\mathbf{x}, p)}{\partial p} \Delta p - f(\mathbf{x}) \right]^2. \tag{3}$$

After the deformation increment Δp is obtained, the deformation parameter is updated by $p \leftarrow p + \Delta p$.

2.2. FC-GN algorithm

The FC-GN was first proposed by Shum and Szeliski [18]. The deformation parameter p is indirectly updated by the compositional form $W(W(\cdot, \Delta p), p)$ of the shape operator $W(\cdot, p)$ in the matching process, instead of being directly updated by the deformation parameter p. The increment Δp for each iteration aims to minimize the following deviation function as

$$\begin{aligned} &\sum_x [f(\mathbf{x}) - g(W(W(\mathbf{x}, \Delta p), p))]^2 \\ &\approx \sum_x \left[g(W(W(\mathbf{x}, 0), p)) + \nabla g(W(W(\mathbf{x}, 0), p)) \frac{\partial W(W(\mathbf{x}, 0), p))}{\partial W(\mathbf{x}, 0)} \frac{\partial W(\mathbf{x}, 0)}{\partial p} \Delta p - f(\mathbf{x}) \right]^2 \\ &= \sum_x \left[g(W(\mathbf{x}, p)) + \nabla g(W(\mathbf{x}, p)) \frac{\partial W(\mathbf{x}, p)}{\partial \mathbf{x}} \frac{\partial W(\mathbf{x}, 0)}{\partial p} \Delta p - f(\mathbf{x}) \right]^2. \end{aligned} \tag{4}$$

Instead of directly updating the parameter p, this strategy updates the shape operator as

$$W(\mathbf{x}, p) \leftarrow W(W(\mathbf{x}, \Delta p), p) \tag{5}$$

Comparing Eq. (5) with Eq. (3), we can see that the FC-GN algorithm needs to compute the partial derivatives of $\frac{\partial W(\mathbf{x}, 0)}{\partial \mathbf{x}}$ and $\frac{\partial W(\mathbf{x}, 0)}{\partial p}$, while the FA-GN algorithm only computes the partial derivative of $\frac{\partial W(\mathbf{x}, 0)}{\partial p}$. The above two iterations are equivalent to the following updating process as

$$\text{FA} - \text{GN updating:}\ \ W(\mathbf{x}, p + \Delta p) \approx W(\mathbf{x}, p) + \frac{\partial W}{\partial p} \Delta p \tag{6-1}$$

$$\text{FC} - \text{GN updating:}\ \ W(W(\mathbf{x}, \Delta p), p) \approx W\left(\mathbf{x} + \frac{\partial W}{\partial p} \Delta p, p\right) \approx W(\mathbf{x}, p) + \frac{\partial W}{\partial \mathbf{x}} \frac{\partial W}{\partial p} \Delta p \tag{6-2}$$

2.3. IC-GN algorithm

The IC-GN algorithm was first proposed by Baker et al. [19], which is a modified version of the FC-GN algorithm. It minimizes the following deviation function as