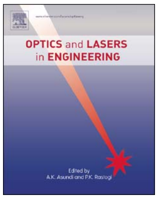
Optics and Lasers in Engineering 90 (2017) 81–90

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng



Phase extraction using multi-directional moiré fringes for multi-lateral shearing interferometry

Ke Liu, Zhenbo Ji, Chen Chen, Yanqiu Li*

Key Laboratory of Photoelectron Imaging Technique and System (Ministry of Education of China), School of Optoelectronics, Beijing Institute of Technology, Beijing 100081, China

ARTICLE INFO

Keywords:
Lateral shearing interferometry
Fringe analysis
Phase retrieval
Fast Fourier transform
Moiré technique

ABSTRACT

The multi-lateral shearing interferometers (multi-LSIs) are featured in the improved accuracy and noise resistance of wavefront reconstruction using phase differences in multiple directions. Nowadays the multi-directional phase differences are usually extracted from multi-LSIs' interferogram using the fast Fourier transform (FFT) method, whose accuracy is limited by spectral leakage effect. To improve the measurement accuracy of multi-LSIs, a phase extraction method developed from moiré technique is proposed in this paper. Using virtual gratings with properly large carrier frequencies, the desired phase information in each of the multiple directions can be modulated into low-frequency domain of the corresponding moiré pattern with larger separations between unnecessary side lobes. In this way, low-pass filters with higher cut-off frequencies can be applied in moiré technique to reduce the inaccuracy induced by spectral leakage effect. Meanwhile, phase shifting method can be applied to extract phase information from a single fringe pattern with better noise resistance by easily introducing phase shifts in computer generated virtual gratings. Simulation results show that the proposed method has higher accuracy and better anti-noise performance than the FFT method when spectral leakage effect exists. To demonstrate accuracy of the proposed method, a null test experiment of the quadriwave LSI has been conducted and experimental results show that measurement accuracy of the quadriwave LSI can be significantly improved by substituting the FFT method with the proposed method in phase extraction process.

1. Introduction

As a kind of versatile wavefront sensing device, lateral shearing interferometers (LSIs) have been widely used in the areas of X-ray optics [1], fiber optics [2], ultrafast optics [3], holography [4] and so on. Recently, a new group of LSIs, named multi-LSIs, which are based on the interference of more than one copy of test wavefront with multiple directions of shear have been proposed successively. The three-wave LSI [5,6], the cross grating LSI [7], the quadriwave LSI [8] and the spatially multiplex LSI [9] are typical multi-LSIs. Among them, the quadriwave LSI and its variations have been intensively studied in recent years and successfully applied to X-ray phase imaging [1], segmented wavefront measurement [10], piston measurement [11], infrared lenses testing [12], infrared sub-wavelength grating characterization [13], phase imaging microscopy [14] and coherent beam combination [15].

A major advantage of multi-LSIs is that more than two orthogonal phase differences are encoded in one single inteferogram. After phase extraction process with a proper method, phase differences in multi-directions can be used to reconstruct the wavefront under test, leading to a great improvement in reconstruction accuracy and noise resistance [16–19]. However, to our knowledge, the phase extraction of multi-LSIs' interferogram still use the fast Fourier transform (FFT) method [12], which has an inherent edge error due to the finite size of pupil [20] and relatively limited anti-noise performance compared with the phase shifting method [21]. Usually, the spectral leakage effect of the FFT method can be reduced by extrapolating the fringes outside the pupil boundaries [22,23], or using smoothly shaped spectrum filters [24]. However in most cases of the multi-LSIs, fringe pattern contains spatial carriers in multiple directions, making the spectral leakage effect more severe while using the FFT method. To further reduce the spectral leakage effect in phase extraction of multi-LSIs, we can adopt the approach of multiplicative moiré, which modulates the original interferogram with gratings and demodulates the moiré patterns in a more sophisticated way. In fact, multiple phase estimation using the moiré concept has already been applied in digital holographic interferometry [25] to perform simultaneous multi-dimensional deformation measurements with a single fringe pattern [26–28].

---

* Corresponding author.

http://dx.doi.org/10.1016/j.optlaseng.2016.09.014
Received 6 June 2016; Received in revised form 9 September 2016; Accepted 27 September 2016

In multiplicative moiré method, moiré patterns are formed by superimposing the interferogram to be evaluated on reference patterns, such as gratings, through multiplication operations [29]. The logic moiré[30,31], the quadrature multiplicative moiré[32] and the stair-shaped virtual grating demodulation method [33], are typical multiplicative moiré methods which use the computer generated reference patterns with easy and rapid variations. In these methods, phase shift can be introduced by virtual reference patterns in spatial domain rather than in temporal domain, as the temporal phase shifting method does. In this manner, only a single fringe pattern is required to demodulate phase information, enabling a real time measurement and removing the need of phase shifting elements. Meanwhile, the superior anti-noise performance of phase shifting methods can be retained. With the moiré patterns in hand, a filtering process is applied to moiré patterns afterwards to remove the unnecessary side lobes and keep the useful spectrums. For example, in the quadrature multiplicative moiré algorithm [28], two virtual reference patterns with phase step of $\pi/2$ multiply the fringe pattern respectively to get two moiré patterns and the desired phase information can be obtained using arctangent calculations after a low-pass filtering process. Usually, carrier frequency of the reference pattern is set very close to that of fringe pattern to improve visibility of the moiré pattern. In this way, the useful spectrums are modulated into the low-frequency domain in the spectrums of moiré pattern and can be extracted by a low-pass filter. Unfortunately in the low-pass filtering procedure, spectral leakage effect similar to the case in the FFT method would also cause obvious errors unless some remedies are applied in moiré method.

In this paper, to improve the accuracy and noise resistance of phase measurement, we extend the basic idea of multiplicative moiré method to evaluate multidirectional phase information from multi-LSIs' fringe pattern. Here the general idea is to modulate the desired phase information in each of the multiple directions into low-frequency domain of the corresponding moiré patterns using computer generated virtual gratings with different ruling directions. To mitigate spectral leakage effect in the low-pass filtering procedure, virtual gratings with properly large carrier frequencies are used to provide larger separations between unnecessary side lobes. Higher phase extraction accuracy can be achieved by careful design of the low-pass filters with larger cut-off frequencies. To demonstrate principle and for simplicity, phase extraction using moiré fringes generated by quadri-directional virtual gratings, which are especially designed for the quadriwave LSI, will be presented in detail. In Section 2, we present the principle of the proposed method. Special attentions will be paid on the design of virtual gratings and filters. In Section 3, computer simulation results of the proposed method and the FFT method will be given for comparison. The accuracy and anti-noise performance of the two methods are presented. In Section 4, experimentally obtained null test interferogram of the quadriwave LSI is analyzed by both the proposed method and the FFT method. In Section 5, we give the conclusions.

2. Principle

The quadriwave LSI simultaneously generates four tilted replicas of incident wavefront [8], which form interference pattern as shown in Fig. 1. In this way, six pairs of interference wave provide lateral shears in four different $\boldsymbol{X}_i (i = 1,2,3,4)$ directions. $\boldsymbol{X}_2$ and $\boldsymbol{X}_4$ are the unit vectors in $\boldsymbol{X}$ and $\boldsymbol{Y}$ directions, while $\boldsymbol{X}_1$ and $\boldsymbol{X}_3$ are the unit vectors in $\boldsymbol{X} - \boldsymbol{Y}$ and $\boldsymbol{X} + \boldsymbol{Y}$ directions. The goal of this paper is to extract the phase differences $\varnothing_{X_i} (i = 1,2,3,4)$ in $\boldsymbol{X}_i (i = 1,2,3,4)$ directions respectively by using moiré method..

Usually the four waves are supposed to interfere with each other with lateral shears in $\boldsymbol{X}$ and $\boldsymbol{Y}$ directions, the light intensity $I(x, y)$ of the quadriwave LSI fringe pattern with large tilt can be denoted as

$$I(x, y) = \{a_1(x, y) + b_1(x, y)\cos(2\pi fx + \varnothing_X(x, y))\} \times \{a_2(x, y) + b_2(x, y)\cos(2\pi fy + \varnothing_Y(x, y))\}, \tag{1}$$

where f is the introduced spatial carrier frequency, which is assumed to be the same value in $\boldsymbol{X}$ and $\boldsymbol{Y}$ directions in this paper; $\varnothing_X(x, y)$ and $\varnothing_Y(x, y)$ are the phase differences in $\boldsymbol{X}$ and $\boldsymbol{Y}$ directions; $a_1(x, y)$ and $a_2(x, y)$ are the background amplitudes in $\boldsymbol{X}$ and $\boldsymbol{Y}$ directions; $b_1(x, y)$ and $b_2(x, y)$ are the fringe amplitudes in $\boldsymbol{X}$ and $\boldsymbol{Y}$ directions. For simplicity, we express these terms as a_1, a_2, b_1, b_2, $\varnothing_X$ and $\varnothing_Y$ instead in the following paragraphs, due to the fact that these terms vary slowly compared with the introduced spatial carrier in most cases.

Since we consider the interference of four waves with lateral shears in four different $\boldsymbol{X}_i (i = 1,2,3,4)$ directions, Eq. (1) can be rewritten as another form using trigonometric function calculations

$$I(x, y) = a_1a_2 + \frac{1}{2}b_1b_2\cos(2\pi fx - 2\pi fy + \varnothing_{X_1}) + a_2b_1\cos(2\pi fx + \varnothing_{X_2}) + \frac{1}{2}b_1b_2\cos(2\pi fx + 2\pi fy + \varnothing_{X_3}) + a_1b_2\cos(2\pi fy + \varnothing_{X_4}), \tag{2}$$

where $\varnothing_{X_1} = \varnothing_X - \varnothing_Y$, $\varnothing_{X_2} = \varnothing_X$, $\varnothing_{X_3} = \varnothing_X + \varnothing_Y$, $\varnothing_{X_4} = \varnothing_Y$. In this expression, the desired phase differences $\varnothing_{X_i} (i=1, 2, 3, 4)$ are modulated by spatial carriers in $\boldsymbol{X}_i (i = 1,2,3,4)$ directions respectively. From the expression of Eq. (2), it seems that all the four $\varnothing_{X_i} (i=1, 2, 3, 4)$ can be deduced in ideal case by extracting the phase differences in two arbitrary $\boldsymbol{X}_i (i = 1,2,3,4)$ directions. However in the quadriwave LSI, all the four $\varnothing_{X_i} (i=1, 2, 3, 4)$ are preferred to be extracted directly from the original interferogram which contains noises, since the noise resistance of wavefront reconstruction can be improved if all the *de facto* information included in the interferogram can be taken into account [17]. Fig. 2(a) shows a typical Fourier spectrum of $I(x, y)$ with a relatively large f which separates the side lobes well enough. To extract $\varnothing_{X_i} (i=1, 2, 3, 4)$ using the FFT method, we should set four separate band-pass filters centered on the four spatial carrier frequencies in $\boldsymbol{X}_i (i=1, 2, 3, 4)$ directions respectively, shown as red square windows in Fig. 2(a) for simplicity. If f is not large enough or $\varnothing_{X_i} (i=1, 2, 3, 4)$ contain high order aberrations, spectrums around the four carriers may overlap with each other, leading to an effect known as spectral leakage, which is the main cause for the inaccuracies in the FFT method. Fig. 2(b) shows a Fourier spectrum of $I(x, y)$ when f is small. $\varnothing_{X_i} (i=1, 2, 3, 4)$ are the same as those used in Fig. 2(a), but f is half of that used in Fig. 2(a). As shown in Fig. 2(b), the spectral leakage effect is more severe in multi-LSI, owing to the fact that spatial carriers in multiple directions are used to modulate phase information. Moreover, as we can see from Eq. (2) that the fringe amplitudes in $\boldsymbol{X}_1$ and $\boldsymbol{X}_3$ directions are at least 50% smaller than those in $\boldsymbol{X}_2$ and $\boldsymbol{X}_4$ directions. As a result, the spectrum densities on carrier frequency in $\boldsymbol{X}_1$

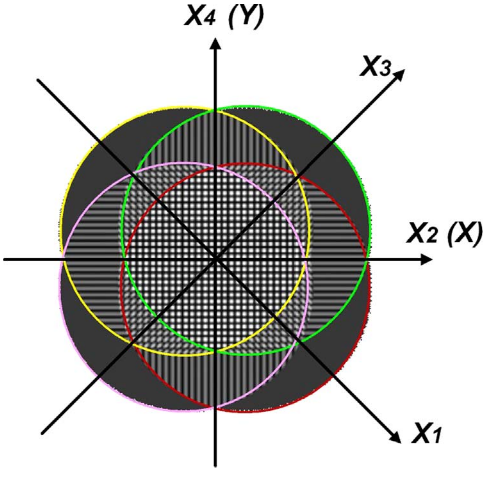


Fig. 1. Schematic interference pattern of four replicas with lateral shears and definitions of the shear directions.