Optics and Lasers in Engineering ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

Optics and Lasers in Engineering

journal homepage: www.elsevier.com/locate/optlaseng

A modified phase-coding method for absolute phase retrieval

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ARTICLE INFO

Article history:
Received 9 November 2015
Received in revised form
27 February 2016
Accepted 15 March 2016

Keywords:
Fringe projection
Phase-coding
Phase-shifting
System non-linearity
Phase error compensation

ABSTRACT

Fringe projection technique is one of the most robust tools for three dimensional (3D) shape measurement. Various fringe projection methods have been proposed for addressing different issues in profilometry and phase-coding is one such technique employed to determine fringe orders for absolute phase retrieval. However this method is prone to fringe order error, while dealing with high-frequency fringes. This paper studies phase error introduced by system non-linearity in phase-coding and provides a mathematical model to obtain the maximum number of achievable codewords in a given scheme. In addition, a modified phase-coding method is also proposed for phase error compensation. Experimental study validates the theoretical analysis on the maximum number of achievable codewords and the performance of the modified phase-coding method is also illustrated.

1. Introduction

Three dimensional (3D) shape measurement using optical methods is an active research area due to its widespread application in diverse fields, such as industrial metrology, manufacturing monitoring and 3D imaging [1]. Among the numerous optical methods employed [2], digital fringe projection profilometry (FPP) has been broadly used [3]. It has the advantages of being automatic, high-speed and high-resolution. However, retrieving the absolute phase for an object with an abrupt change in the surface profile with high accuracy remains a challenge.

The work on digital FPP focuses on reliable absolute phase retrieval. To extract a wrapped phase from multiple images, phase-shifting algorithm [4,5] is applied. Compared with other techniques (such as Fourier transform [6,7], Windowed Fourier transform [8], Wavelet transform [9]), phase-shifting has the advantage of pixel-by-pixel measurement resolution. Since the phases obtained contain 2π discontinuities which are introduced by an arctangent function, phase unwrapping is required to remove the discontinuities. Though spatial phase unwrapping [10,11] was proposed to obtain a continuous phase distribution, it is not suitable for an object with an abrupt change in surface profile. For such an object, other phase retrieval techniques, including temporal phase unwrapping [12], gray-coded and phase-shifting (GCPS) [13], composite phase-shifting algorithm [14] and phase-coding method [15] were proposed. However, a large number of fringe patterns are needed for the temporal unwrapping process. For composite phase-shifting algorithm only a small number of codewords can be embedded into the available range of gray values. A recent phase-coding method proposed by Wang and Zhang [15] shows a better overall performance in terms of measurement speed and reliability. The method embeds codewords into a phase domain rather than an intensity domain as the phase domain is less sensitive to surface contrast, ambient light and camera noises [15]. It is suitable for an object with an abrupt change in the surface profile. Nevertheless, due to the influence of system non-linearity, the method is limited by the number of unique codewords that can be employed. To address this problem, a two-phase phase-coding strategy was proposed [16]. If the number of unique codewords that can be employed in a conventional phase-coding method [15] is N, that would be increased to N^2 for a two-phase phase-coding method [16]. However, such a multi-phase phase-coding strategy would require a large number of fringe patterns.

In this paper, improvement is made to the conventional phase-coding method by considering the system non-linearity. This is achieved by combining a phase-coding with phase error compensation. More unique codewords are employed for reliable absolute phase retrieval without increasing the number of fringe patterns.

2. Principle of method

2.1. Principle of conventional phase-coding method

In the conventional phase-coding method [15], two sets of n-step phase-shifted fringe patterns are employed (n is the number of phase-shifting steps). The first set includes n sinusoidal fringe

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http://dx.doi.org/10.1016/j.optlaseng.2016.03.018
0143-8166/

patterns with a phase shift of $2\pi/n$ to extract the wrapped phase; while the second set includes n coded fringe patterns, into which a stair phase is embedded. If the sinusoidal fringe pattern consists of N fringe periods, phase values ranging from $-\pi$ to π would be quantified into N discrete levels. The resulting phase is called a stair phase with a constant stair height of $2\pi/N$. Each stair level with a unique phase value is treated as one unique codeword. Consequently, the number of unique codewords that are created equals to N. A unique codeword is assigned to one fringe period. The stair phase ϕ^s for the kth codeword is expressed as

$$\phi^s = -\pi + k \times \frac{2\pi}{N}, k = 0, 1, 2, \ldots, N-1, \tag{1}$$

where, k is the index of the codeword as well as the fringe order.

Using an actual stair phase value ϕ^s, the fringe order k is determined from Eq. (1) to be:

$$k = \frac{N(\phi^s + \pi)}{2\pi}. \tag{2}$$

Based on the fringe order $k(x, y)$ determined and the extracted wrapped phase $\phi'(x, y)$, an absolute phase is obtained as:

$$\Phi(x, y) = \phi'(x, y) + k(x, y) \times 2\pi. \tag{3}$$

2.2. Limitations of conventional phase-coding method

The stair phase extracted from a coded fringe pattern (hereafter called measured stair phase $\phi^{s'}$) is normally not equal to the actual stair phase ϕ^s due to system non-linearity. It is the sum of the actual stair phase ϕ^s and phase error $\Delta\phi^s$ [17]:

$$\phi^{s'} = \phi^s + \Delta\phi^s, \tag{4}$$

$$\Delta\phi^s = -c \times \sin(n\phi^s), \tag{5}$$

where c is an amplitude of the phase error, which can be estimated from a reference plane.

In the conventional phase-coding method, fringe order k' is estimated from $\phi^{s'}$:

$$k' = \text{Round}\left[\frac{N\left(\phi^{s'} + \pi\right)}{2\pi}\right], \tag{6}$$

where Round[a] is an integer closest to a.

Hence, the resulting fringe order k' can be estimated by combining Eqs. (2) and (4):

$$k' = k + \text{Round}\left[\frac{N\Delta\phi^s}{2\pi}\right]. \tag{7}$$

For $-1/2 \le N\Delta\phi^s/2\pi < 1/2$, the resulting fringe order would be the correct fringe order ($k' = k$). Hence, the condition for absolute phase retrieval by the conventional phase-coding method [15,16] is

$$\left|\frac{N\Delta\phi^s}{2\pi}\right| < \frac{1}{2}. \tag{8}$$

Substituting Eqs. (1) and (5) into Eq. (8), the condition becomes

$$N\left|\sin\left(\frac{2nk\pi}{N}\right)\right| < \frac{\pi}{|c|}, k = 0, 1, 2, \ldots, N-1. \tag{9}$$

From Eq. (9), for a n-step phase-shift, the number of unique codewords (N) is limited. Since $|\sin(2nk\pi/N)| \le 1$, only when $N < \pi/|c|$ is the condition always satisfied. For example, if c is estimated as 0.2 rad and for a three-step phase-shift (i.e., $n = 3$), $N = 16$ would exceed the limiting value and hence Eq. (9) is not satisfied. In other words, only up to 15 unique codewords can be employed for the fringe order estimation.

2.3. Proposed phase-coding method

In the proposed method, the stair phase embedded into a coded fringe pattern is modified by compensation of phase error for system non-linearity. It is more efficient to use an unevenly-spaced stair phase for fringe order estimation which is not employed in the conventional phase-coding method.

With phase error compensation, the measured stair phase $\phi^{s'}$ is expressed as Eq. (1) (i.e.,$\phi^{s'} = -\pi + k \times \frac{2\pi}{N}, k = 0, 1, 2, \ldots, N-1$). As shown in Eqs. (1) and (4), the stair phase embedded into a coded fringe pattern in the conventional phase-coding method should be modified accordingly. The resulting stair phase (hereafter called modified stair phase) is referred to as ϕ^{ms}. Combining Eqs. (4) and (5), we have:

$$\phi^{ms} - c \times \sin(n\phi^{ms}) = \phi^{s'}. \tag{10}$$

The phase error compensation is established in the premise that the modified stair phase ϕ^{ms} and the measured stair phase $\phi^{s'}$ is one-to-one mapping. To estimate ϕ^{ms} corresponding to $\phi^{s'}$ by using Newton–Raphson method [18], a function $f(x)$ is constructed:

$$f(x) = x - c \times \sin(nx) - \phi^{s'}. \tag{11}$$

As the root of $f(x)$ (i.e., $f(\phi^{ms}) = 0$), ϕ^{ms} can be estimated by the following iteration [18]:

$$\phi^{ms}_{i+1} = \phi^{ms}_i - \frac{f(\phi^{ms}_i)}{f'(\phi^{ms}_i)}, \tag{12}$$

$$f'(\phi^{ms}_i) = 1 - c \times n \times \cos(n\phi^{ms}_i), \tag{13}$$

where ϕ^{ms}_{i+1} and ϕ^{ms}_i are the respective solutions of the $(i+1)$th and ith iterations and $f'(\phi^{ms}_i)$ is the derivative of $f(x)$ at value of ϕ^{ms}_i. Since $f(x)$ is always derivable with respect to x, it is derivable at value of ϕ^{ms}_i. The initial guess of the iteration $\phi^{ms}_0 = \phi^{s'}$. The iteration is terminated when $|f(\phi^{ms}_{i+1})| < 0.001$ rad.

As ϕ^{ms} and $\phi^{s'}$ is one-to-one mapping, we have $f'(x) \ge 0$. $f'(x) = 0$ if and only if $|\cos(nx)| = 1$, which means $\phi^{ms} = \phi^{s'}$ from Eq. (11) and the Newton–Raphson method is not necessary in this case. Therefore, we have the condition $f'(x) > 0$, in terms of using the Newton–Raphson method to estimate ϕ^{ms}.

The iteration by Eq. (12) converges to ϕ^{ms} from the initial guess $\phi^{s'}$. Since $f'(x)$ is a periodic function, we take a period $x \in (-\pi/n, \pi/n)$ for explanation and two halves of the period are analyzed separately.

When $x \in (-\pi/n, 0)$, we have $f''(x) \le 0$ and hence $f(x)$ is concave. In addition, $f(-\pi/n) \times f(0) \le 0$, which means ϕ^{ms} is located within $(-\pi/n, 0)$ given the initial guess $\phi^{s'} \in (-\pi/n, 0)$. Thus, the iteration is locally convergent when $x \in (-\pi/n, 0)$ [18]. Similarly, when $x \in (0, \pi/n)$, it is shown $f(x)$ is convex and $f(0) \times f(\pi/n) \le 0$. Then, the iteration is also locally convergent given the initial guess $\phi^{s'} \in (0, \pi/n)$ [18].

3. Experiments and discussion

The experimental setup consists of a digital CCD camera (Jai CV-M9CL) and a digital-light-processing (DLP) projector (Optoma EX536) with a projection distance of 1.2–12 m. Both the camera and projector have a resolution of 1024 × 768. The camera is placed at 10 cm parallel to the projector and the object is placed perpendicular to the camera at a distance of 90 cm as shown in Fig. 1. In all the following measurements, phase-shifting is carried out using a 4-step phase-shift (i.e., $n = 4$). The error of wrapped phase due to system non-linearity is reduced [17] in the experimental results by the proposed method.

Please cite this article as: Xing Y, et al. A modified phase-coding method for absolute phase retrieval. Opt Laser Eng (2016), http://dx.doi.org/10.1016/j.optlaseng.2016.03.018