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Simultaneous estimation of unwrapped phase and phase derivative from a closed fringe pattern

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ABSTRACT

We propose a new approach for the direct estimation of the unwrapped phase from a single closed fringe pattern. The fringe analysis is performed along a given row/column at a time by approximating the phase with a weighted linear combination of linearly independent basis functions. Gaussian radial basis functions with equally distributed centers and a fixed variance are considered for the phase approximation. A state space model is defined with the weights of the basis functions as the state vector elements. Extended Kalman filter is effectively utilized for the accurate state estimation. A fringe density estimation based criteria is established to select whether the phase estimation is performed in a row by row or column by column manner. In the *seed* row/column decided based on this criteria, the optimal basis dimension is computed. The proposed method effectively renders itself in the simultaneous estimation of the phase and the phase derivative. The proposed phase modeling approach also allows us to successfully demodulate the low density fringe patterns. Simulation and experimental results validate the practical applicability of the proposed method.

1. Introduction

Advances in fringe analysis have fueled a wide spread deployment of optical interferometric techniques. In the optical setups such as electronic speckle pattern interferometry, and holographic interferometry, single cosine fringe patterns are recorded carrying the information on the interference phase. Fringe analysis basically involves an estimation of the interference phase or the phase derivative from a fringe pattern. Basically, a fringe pattern consists of either an open or a closed set of fringes. Open fringe patterns are comparatively easier to demodulate due to the presence of carrier frequency. The techniques based on Fourier transform [1], windowed Fourier transform [2], and wavelet transform [3,4] are some of the commonly used techniques to demodulate the open fringe patterns. On the other hand, in the case of a closed fringe pattern, phase sign ambiguity exists due to the absence of a carrier frequency. To overcome this phase sign ambiguity, temporal phase stepping [5] is the most commonly employed technique due to its simplicity and high accuracy. In this technique, at least three phase shifted fringe patterns are required to estimate the phase without any sign ambiguity. The experimental setup involved in the phase shifting approach needs to be isolated from external mechanical disturbances for a reliable phase estimation. This condition is not always met especially in the industrial environment. Consequently, it is often desirable to obtain the phase estimation from a single closed fringe pattern.

Number of techniques have been reported in the literature over the years for the demodulation of a single closed fringe pattern. A commonly used technique is based on regularized phase tracking (RPT) [6]. The performance of this technique is improved by using the fringe follower approach [7] which offers a fringe scanning strategy to avoid the critical points. Rivera [8] proposed a robust phase demodulation technique based on the minimization of the regularized cost function. The adaptive quadrature filter [9] based on Bayesian estimation theory and complex-valued Markov random-field prior models has also been proposed for the closed fringe demodulation. The frequency guided methods [10–12] involve an exhaustive search of the local frequencies. These techniques mainly involve phase estimation based on the local analysis of the fringe pattern. Larkin et al. proposed a different approach for fringe demodulation based on the two dimensional extension of the Hilbert transform [13]. Recently, improvements over the performance of the RPT technique have also been proposed [14,15]. A new algorithm for fringe demodulation based on the analysis of horizontal and vertical oriented fringes [16] offers improved computational efficiency.

Another way to tackle the problem of closed fringe demodulation is by analyzing the fringe pattern in a global manner. Accordingly, the techniques based on Zernike polynomial approximation of phase [17], implicit smoothing splines [18] have

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been reported. The technique proposed in [18] is shown to be able to demodulate the low density fringe patterns as well.

In applications such as deformation analysis, phase derivative estimation looks to outpace the interest in phase estimation itself due to its direct relationship with the strain developed on the object surface. The techniques mentioned above mainly provide the estimation of phase only. In this paper, we propose a partially global approach for the direct unwrapped estimation of phase and phase derivative from a single closed fringe pattern. The proposed method is based on the assumption that the phase is spatially continuous. Based on this, the phase along each row/column is approximated with the weighted linear combination of a certain number of linearly independent basis functions. This approximation of phase also allows us to compute the phase derivative along a row/column without any extra computational cost. The robust weight estimation is performed using the extended Kalman filter.

2. Theory

The intensity of a closed fringe pattern can be represented as

$$I(m,n) = I_0(m,n) + I_a(m,n)\cos(\varphi(m,n)) + \epsilon(m,n), \tag{1}$$

where $I(m,n)$ is the recorded intensity of the fringe pattern of size $M \times N$ pixels; $m \in [1,M]$ and $n \in [1,N]$ represent the pixels along the rows and the columns, respectively; $I_0(m,n)$ is the background intensity; $I_a(m,n)$ is the fringe modulation intensity and $\varphi(m,n)$ is the interference phase; $\epsilon(m,n)$ is the additive white Gaussian noise. In the proposed method, the background intensity which in general has low spatial variation is removed by high pass filtering of the fringe pattern. The fringe intensity can be normalized using one of the fringe normalization techniques proposed in [19–21]. After normalization, the fringe pattern can be represented as

$$I(m,n) = \cos(\varphi(m,n)) + \epsilon(m,n). \tag{2}$$

The proposed method is based on one dimensional signal analysis performed in a row by row or a column by column manner. The decision on whether the analysis should be performed along the rows or columns is based on the fringe density map as will be explained in the next section. Consider that it is decided to perform the analysis in a row by row manner. Accordingly, the fringe intensity in a given row m can be represented as

$$I(n) = \cos(\varphi(n)) + \epsilon(n). \tag{3}$$

Similar to the most closed fringe demodulation techniques, the proposed method is based on the assumption that the phase is continuous over the entire fringe pattern. This characteristic of spatial continuity of the phase is exploited by the proposed method by approximating the phase $\varphi(n)$ as a weighted linear combination of the pre-defined linearly independent basis functions of n. That is,

$$\hat{\varphi}(n) = \sum_{k=0}^{K-1} a_k \psi_k(n), \tag{4}$$

where K is the number of the basis functions, i.e. basis dimension, used in the phase approximation; $\psi_k(n)$ is the kth basis function with a_k as its associated weight. Using this phase approximation, the phase derivative $\frac{\partial\varphi}{\partial n}$ can be defined as

$$\frac{\partial\varphi}{\partial n} = \sum_{k=0}^{K-1} a_k \frac{d\psi_k}{dn}. \tag{5}$$

Since the basis functions are pre-defined, the derivatives of the basis functions with respect to n can be computed numerically and stored in an array. Note that the derivatives of the basis functions with respect to m can be similarly defined. For a specific set of the weights, i.e. a_ks, it is expected that $\hat{\varphi}(n)$ will provide the accurate representation of $\varphi(n)$. It is apparent that the problem of phase estimation is converted into a problem of parameter estimation where the weights are the parameters to be estimated. Note that the same estimates of weights provide the estimation of phase derivative as well. This is one of the important advantage of the proposed method. The estimation of the fringe intensity based on the phase model considered in Eq. (4) can be represented as

$$\hat{I}(n) = \cos(\hat{\varphi}(n)), \tag{6}$$

$$= \cos\left(\sum_{k=0}^{K-1} a_k \psi_k(n)\right). \tag{7}$$

It can be recognized from the above equation that the accurate estimate of the weights of the basis functions minimizes the error between the intensity sample $I(n)$ and its estimate $\hat{I}(n)$. Now, it remains to define the linearly independent basis functions. The selection of appropriate basis functions plays a crucial role in the accurate demodulation of the closed fringe pattern. It is expected that the basis functions are linearly independent and are continous functions of the spatial variables m or n. Different types of basis functions satisfying this criteria are reported in the literature. These basis functions include Fourier, Legendre, and Chebyshev and polynomial basis functions. We have studied these basis functions in the context of phase and phase derivative estimation [22,23] from a complex interferogram. In this study, we have considered *Gaussian radial basis functions* (GRBF)s. These basis functions have been widely used in many applications for the purpose of function approximation. Likewise, in the proposed method, these basis functions are used for the phase approximation. The basis functions $\psi_k(n)$ for $k \in [1, K-1]$ are defined as

$$\psi_k(n) = \exp\left(-\frac{(n-n_k)^2}{2\sigma^2}\right) \tag{8}$$

with $\psi_0(n) = 1$. Here, n_k represents the locations of the centers of the peaks of the Gaussian functions; σ is the *standard deviation* which controls the width of the Gaussian. The selections of n_k and σ depend on the signal length N. In this study, we have considered the Gaussian centers to be equally distributed with a pre-defined value of σ. For a given basis dimension, the basis functions are computed in advance and are stored in the memory location. Next, we describe the procedure of the weight estimation based on the extended Kalman filter (EKF).

2.1. EKF based parameter estimation

Noting the fact that the weights enter into Eq. (7) in a nonlinear manner, we propose to use extended Kalman filter based nonlinear parameter estimation technique [24]. For the EKF implementation, let us first define the state space model as follows:

$$\boldsymbol{\mu}_n = \boldsymbol{F}\boldsymbol{\mu}_{n-1}, \tag{9}$$

$$I(n) = \boldsymbol{g}_n(\boldsymbol{\mu}_n) + \epsilon(n). \tag{10}$$

The state vector $\boldsymbol{\mu}_n$ is defined as

$$\boldsymbol{\mu}_n = [a_0, a_1, \ldots, a_{K-1}]^T \tag{11}$$

Here, T represents the transpose operation. The weights of the GRBFs in the given row are assumed to be constants. Consequently, the state transition matrix $\boldsymbol{F}$ is simply an identity matrix of size $K \times K$. The fringe intensity at n correspond to the measurement variable. The estimate of fringe intensity is considered to be a nonlinear function $\boldsymbol{g}_n(\cdot)$ of the state vector. That is,

$$\hat{I}(n) = \boldsymbol{g}_n(\boldsymbol{\mu}_n) = \cos(\boldsymbol{\mu}_n^T \boldsymbol{\psi}) \tag{12}$$

where