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Regenerated phase-shifted sinusoids assisted EMD for adaptive analysis of fringe patterns

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ABSTRACT

Fringe patterns are often produced from optical metrology. It is important yet challenging to reduce noise and remove a complicated background in a fringe pattern, for which empirical mode decomposition based methods have been proven useful. However, the mode-mixing problem and the difficulty in automatic mode classification limit the application of these methods. In this paper, a newly developed method named regenerated phase-shifted sinusoids assisted empirical mode decomposition is introduced to decompose a fringe pattern, and subsequently, a new noise-signal-background classification strategy is proposed. The former avoids the mode-mixing problem appearing during the decomposition, while the latter adaptively classifies the decomposition results to remove the noise and background. The proposed method is testified by both simulation and real experiments, which shows effective and robust for fringe pattern analysis under different noise, fringe modulation, and defects.

1. Introduction

A fringe pattern from optical metrology is usually expressed as

$$I(x, y) = A(x, y) + B(x, y)\cos[\omega x + \phi(x, y)] + n(x, y), \quad (1)$$

where $A(x, y)$ and $B(x, y)$ are the background and amplitude intensity, respectively; ω denotes the spatial frequency; $\phi(x, y)$ is the phase distribution; and $n(x, y)$ is random noise. Normally the main purpose of fringe analysis is to retrieve $\phi(x, y)$. It is thus necessary to remove the irrelevant terms $A(x, y)$ and $n(x, y)$ in order to enhance the core part of the fringe pattern, $B(x, y)\cos[\omega x+\phi(x, y)]$, which is called a phase-modulated (PM) signal.

In the Fourier transform profilometry (FTP), when the phase is simple, the PM signal is band limited in the Fourier domain and can be easily separated from other parts if a high carrier frequency is provided [1,2]. However, when the phase is more complicated, a large error will appear because the spectrum of the PM signal is mixed with the spectra of other parts. In addition to this limitation, the Heisenberg's uncertainty principle also limits almost all the spectral analysis methods such as the windowed FTP [3,4], the ridge of wavelet transform (WT) [5] or the S-transform [6,7].

Empirical mode decomposition (EMD) based methods provide another approach for fringe pattern analysis. EMD is a data-driven technique that aims to decompose a non-stationary signal into a series of mono components (named as intrinsic mode functions, IMFs) [8]. It has been used to suppress noise [9] and eliminate the background to suppress the zero spectrum [10,11] in FTP. Furthermore, EMD combined with Hilbert transform (HT) is increasingly used for phase extraction in recent years, where EMD extracts the PM signal and HT constructs an analytic signal of the PM signal for phase retrieval [12–15]. A signal can be decomposed by EMD as follows,

$$I(x) = \sum_{1\le k\le K} \mathrm{IMF}_k(x) + r(x), \quad (2)$$

where k is the index of an IMF; K is the total number of IMFs; $r(x)$ is the residue. As the IMFs range from high frequency to low frequency, they are classified into three groups as follows,

$$I(x) = \sum_{k<k_1} \mathrm{IMF}_k(x) + \sum_{k_1\le k\le k_2} \mathrm{IMF}_k(x) + \left[\sum_{k_2<k\le K} \mathrm{IMF}_k(x) + r(x)\right], \quad (3)$$

where k_1 and k_2 are two critical indexes dividing all the IMFs into three groups; the first group $\sum_{k<k_1} \mathrm{IMF}_k(x)$ represents the noise, the second group $\sum_{k_1\le k\le k_2} \mathrm{IMF}_k(x)$ is the desired PM signal and the last group means the background. The classification result directly

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influences the accuracy of the phase retrieval. There are two technical challenges in getting Eq. (3). First, in EMD results, the so-called mode-mixing problem (MMP) often occurs, which gives an IMF uncertain physical meaning. Second, even without the MMP, it is difficult to correctly classify the three groups, i.e., k_1 and k_2 of Eq. (3) are not easy to be determined.

For a fringe pattern, the MMP is usually caused by uneven distribution of noise, including either high-frequency noise or low-frequency noise. Additionally, if the MMP exists in the high-frequency IMFs, it will spread to low-frequency IMFs. In [13], soft thresholding is used to eliminate the mixed PM signal existing in IMF_1. Considering the possible MMP in any IMFs, in [14,15], local parts of each bidimensional IMF (BIMF) are picked out according to local amplitudes and then combined into the final result through a weighted mean. The methods show good results regardless of solving the MMP, but they still encounter challenges for various applications if the noise or background has the similar amplitude with the PM signals. Ensemble EMD (EEMD) [16] has been thought as the mainstream technique to solve the MMP. It processes an ensemble of the white-noise added signals and takes the average of all results as the end result. EEMD has been expanded to bidimensional EEMD [17] and multivariate EEMD [18,19], and further improved into Complete EEMD (CEEMD) to reduce the residue noise or spurious modes [20,21] in these years. All ensemble-based methods suffer from long computing time to process a big ensemble of data. In order to improve the efficiency, we have conducted some studies of adding a designed “noise” to achieve the results of EEMD [23,23]. However, these methods focus on the MMP in the high-frequency IMFs caused by noise, but ignore the MMP in other scales.

As for the classification of noise and background, $k_1=2$ and $k_2=\mathrm{K}$ are often set, postulating that only IMF_1 is the noise and only the $r(x)$ is the background [12,24]. To cope with more complicated situations, in [25], the standard deviation of $\sum_{k=1}^{\alpha} \mathrm{IMF}_k(x)$ is computed as $\sigma(\alpha)$, and $k_1=\alpha-1$ is set when $\sigma(\alpha)$ becomes dramatically larger than $\sigma(\alpha-1)$, where a threshold is needed. In [17], the power of autocorrelation parameters for each BIMF is computed and then the abrupt change of the computed values is found to set k_1. As for background removal, k_2 is often chosen manually [10,26]. In [27], the marginal entropies of BIMFs are computed to obtain mutual information, in order to evaluate the correlation between IMFs, and then the mutual information is used to determine k_2 [28]. In [13], the mean of an IMF is tested whether the IMF belongs to the background through a threshold, providing that an IMF belonging to the background has a larger mean. Unfortunately, the MMP of the low-frequency IMFs always leads to unpredictable cases, thus, only the correlation between IMFs or the characteristic analysis of an IMF itself cannot work robustly to correctly determine k_2. In [29], the method realizes $k_1=k_2=\mathrm{K}=1$ cleverly and presents a good result for background reconstruction based on the condition that the fringe pattern is clean. In another work [30], the time average subtraction method is used to eliminate the background, but this is unavailable for a single frame of fringe pattern analysis. We also developed some criteria to set k_2 based on the overall changes of frequencies and amplitudes between IMFs [22,23], but more robust measures are needed in practice.

In this paper, a novel regenerated phase-shifted sinusoids assisted EMD (RPSEMD) method is introduced to solve the MMP of EMD, and then a new strategy is proposed for mode classification to reduce noise and remove the background of a fringe pattern. The RPSEMD generates different sinusoids adaptively in different stages of decomposition to solve the MMP in all IMFs. The new noise-signal-background classification strategy determines k_1 and k_2 correctly for the resulted mono-component IMFs. Both the RPSEMD method and the classification strategy are automatic. The proposed method is robust to cope with a variety of complex situations for fringe pattern analysis in optical metrology. Experiments also show high accuracy and efficiency of the proposed method.

2. The RPSEMD algorithm used for analyzing fringe patterns

2.1. The mode-mixing problem

EMD is a data-driven method that aims to iteratively decompose a signal into a series of mono-component IMFs. We call each iteration, namely, the process producing an IMF, as a stage of decomposition. So $\mathrm{IMF}_k(t)$ in Eq. (2) is the product after the *kth* stage of decomposition.

Each stage of EMD starts from detecting the extremum points of the tested signal [8]. Based on the detected extrema, $\mathrm{IMF}_k(t)$ is determined. We define an mono component of a signal as an intrinsic mode (IM) represented as $\mathrm{IM}(t)=a(t)\cos[2\pi f(t)]$, where $a(t)$ and $f(t)$ mean the instantaneous amplitude and instantaneous frequency respectively [31]. Then the obtained result in the kth stage can be written as $\mathrm{IMF}_k(t) = \sum_{j=1}^{J} a_j(t)\cos\,[2\pi f_j(t)t]$, where J is the number of IMs. If the detected extrema belong to a single IM, namely, $J=1$, then $\mathrm{IMF}_k(t)$ is an ideal result. On the contrary, if the detected extrema belong to multiple IMs, namely, $J>1$, the decomposed IMF contains more than one IM, and the MMP occurs.

2.2. Solutions to the MMP and the RPSEMD algorithm

EEMD [16] is a powerful method for solving the MMP, which can adjust the extrema of each scale in a signal by adding a large series of white noise because the white noise has scales populated throughout the time-frequency space. However, the ensemble size of white noise is required to be very large to make sure the added white noise is finally canceled out by average.

Inspired by EEMD, we have proposed to add a designed sinusoid as “a noise” to the signal of a fringe pattern [22,23]. The methods successfully solve the MMP in high-frequency IMFs but they ignore the possible MMP in low-frequency IMFs. Moreover, in [23], we design the amplitude and frequency of the added “noise” only through a simple average method. Thus an advanced technique is needed to robustly cope with signals with high complexity.

As a response to the above requirement, we developed the RPSEMD algorithm recently [32]. Within each stage of decomposition, a sinusoid is designed and added to make the extrema uniformly distributed in order to avoid the MMP. The procedure of RPSEMD is summarized as follows:

1. Apply EMD to a signal $I(t)$ to get initial IMFs and design a sinusoid $s(t|\,a, f, \theta_p)$ where a, f, and θ_p are amplitude, frequency and an initial phase of the sinusoid, respectively. The amplitude and frequency are determined according to the initial IMFs, while the initial phase is 0;
2. Apply EMD to $I(t)+s(t|\,a, f, \theta_p)$ only to get a temporary $\mathrm{IMF}_1(t)$;
3. Repeat **Step 2** with θ_p increased by $2\pi/P$ for another $(P-1)$ times and get the final first IMF as $c_1(t) = \left[\sum_{p=1}^{P} IMF_1(t)\right]/P$;
4. Remove $c_1(t)$ from $I(t)$, i.e., $I(t)\leftarrow I(t)\text{-}c_1(t)$, then repeat **Steps 1-3** on the residue $I(t)$ to get all other IMFs. The final residue is denoted as $r(t)$.

The result of RPSEMD can also be represented by Eq. (2), but this time, the MMP is largely resolved. Details of each step can be

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