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A polarisation based approach to model the strain dependent permittivity of dielectric elastomers

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ABSTRACT

A wide-spread lumped parameter model describing the electrostatic pressure present in dielectric elastomer actuators is presented by Pelrine et al. in 1998. In Pelrine's model, the electrostatic pressure is affected by the relative permittivity of the material, also known as dielectric constant. However, many researchers found that the dielectric constant of dielectric elastomers is not constant at all, but decreasing with increasing pre-stretch of the material. This holds especially for acrylic materials such as VHB 4910 from 3M. From a physical point of view, polarisation within the dielectric material is responsible for the material's permittivity and in general, polarisation is deformation dependent. In this work, an alternative modelling approach is presented, explaining the stretch dependent electrostatic pressure. It is shown that Pelrine implicitly assumes that the polarisation of the material is linear in the imposed electric field strength. If this assumption is modified to allow for a more general polarisation field that is based on invariants of the electromechanically coupled problem, a new polarisation based lumped parameter model is obtained. It is shown that this new model fits experimental data found in literature quite well.

1. Introduction

Dielectric elastomer actuators (DEAs) are composed of an elastic dielectric material that is sandwiched between two compliant electrodes, as illustrated in Fig. 1. When the electrodes are charged by applying an electric potential, charges with opposite signs attract each other, leading to a contractive force also known as electrostatic pressure [1]. When several DEA cells are stacked on top of each other, resulting in a pile-up configuration, the electrostatic pressure provides macroscopically useful displacements [2]. Stacked DEAs are also referred to as artificial muscles, because they bear analogy to the behaviour of human muscles in terms of contracting in length direction when stimulated.

The idea of using artificial muscles as sophisticated actuators for humanoid robots offers a broad variety of potential applications [3]. The elastic structure acts as an energy storage and allows for dynamic motion [4,5] and safe human interaction. Compared to commonly used electrical drives, no gearbox is necessary and the operation is noiseless [6]. Due to the high efficiency, artificial muscles allow to build autarkic systems in contrast to pneumatic or hydraulic systems. However, the use of elastic actuators is also accompanied by new control challenges. Advanced control strategies need to avoid unwanted oscillations, bring the system as quickly as possible into its steady state and follow prescribed trajectories as close as possible.

Other applications of dielectric elastomers include bending membranes [7,8] or balloon and tubular shaped actuators [9–12]. A multilayer bending actuator equipped with so called chucking electrodes allows for variable stiffness as shown in [13]. More recent works consider the high frequency operation of spring loaded circular dielectric elastomer membranes [14] that can also be utilised as micropumps [15,16].

The behaviour of artificial muscles can generally be described by considering coupling forces between the applied electric field (whose distribution has to fulfil the Maxwell equations for electrostatics) and the deformation gradient (that is characterised by the mechanical momentum balance) as shown by Dorfmann et al. in 2005 [17]. In 2007, Vu et al. [18] solve the equations proposed by Dorfmann for arbitrary geometries in the static case, numerically simulated using the finite element method. The finite element framework is used to solve optimisation problems with dielectric film inclusions in [19]. The static formulation of Vu is extended by inertia terms that allow for dynamic motion and structure preserving time integration by Schlögl et al. in 2016 [20]. Even though

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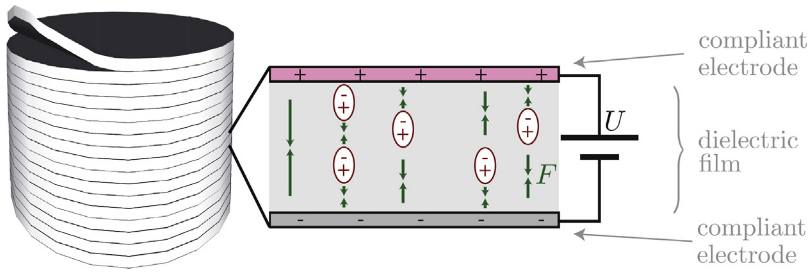

Fig. 1. Stacked dielectric elastomer actuator with functional principle. When a voltage is applied, attracting charges lead to a contractive force.

these models provide a powerful tool to solve electromechanically coupled and dynamic problems of arbitrary geometry, the computational cost is quite demanding. To find solutions for complex control problems where a multibody system is actuated by several muscles at the same time as in [21], it is necessary to make use of lumped parameter models that reduce the computational cost.

A wide-spread lumped parameter model describing the electrostatic pressure p is presented by Pelrine et al. in 1998 [1] and given as

$$p = \varepsilon_0 \varepsilon_r E^2, \tag{1}$$

where ε_0 and ε_r are the vacuum and relative permittivity, respectively, and E is the electric field. The fact that the electrostatic pressure present in a DEA is twice the pressure in a rigid plate capacitor is explained by the repelling of like charges within the electrodes. Because the elastomer is incompressible, the electrode surface area increases when the actuator contracts, releasing additional electric energy. In 2007, Wissler et al. [22] confirm this presumption by evaluating two-dimensional finite element simulations, finding electrostatic forces in 'in-plane' and 'out-of-plane' direction.

The electrostatic pressure given by Eq. (1) is affected by the relative permittivity ε_r of the material, also known as dielectric constant. As increasing the electric field strength E to gain a large electrostatic pressure is limited by manufacturing constraints [23,24] as well as dielectric strength and instabilities [25,26], the dielectric constant is of great importance. Remarkably enough, many researchers found that the dielectric constant ε_r is not constant at all, but decreasing with increasing pre-stretch of the material [27–30,22]. From a physical point of view, polarisation within the dielectric material is responsible for the materials permittivity and in general, polarisation is deformation dependent [31].

In this work, an alternative modelling approach is presented, replacing Eq. (1) and explaining the stretch dependent actuation pressure. In Section 2, the commonly used models including their derivations and assumptions are reviewed. Based on these findings, in Section 3 the assumptions are modified to allow for a more general polarisation field, leading to a polarisation dependent electrostatic pressure. Moreover, a general constitutive polarisation model that is based on invariants of the electromechanically coupled stress tensor is introduced. In Section 4, the new model is compared to measurement data found in literature, based on the VHB 4910 acrylic tape from 3M.

2. Common modelling approach and inconsistencies

This section discusses the various assumptions that are implicitly and explicitly made in commonly used models and compares them to the actual requirements of DEAs. First, the consistency between three-dimensional Maxwell stress models and one-dimensional lumped parameter models is shown. Then, a possible derivation of Eq. (1), based on the principle of virtual work, is presented in detail to provide a basis for the following modifications.

2.1. Maxwell stress, electrostriction and electrostatic pressure

The Maxwell stress tensor $\boldsymbol{\sigma}^{\text{elec}}$ describes the three-dimensional stress state within a dielectric material that is caused by the application of an electric field. Yamwong et al. [32] give this Maxwell stress as

$$\boldsymbol{\sigma}^{\text{elec}} = \varepsilon_0 \left(\frac{2\varepsilon_r - a_1}{2}\right) \boldsymbol{E} \otimes \boldsymbol{E} - \varepsilon_0 \boldsymbol{E} \cdot \boldsymbol{E} \left(\frac{\varepsilon_r + a_2}{2}\right) \mathbf{1}, \tag{2}$$

where $\boldsymbol{E}$ is the spatial electric field vector, $\mathbf{1}$ is the identity matrix and $a_{1/2}$ are electrostrictive components. Electrostriction relates electrical and mechanical stored energy and hence is the reason for electromechanical coupling. Even though the main part of the cited work from Yamwong is about polar rubber (which is based on a different functional principle compared to DEAs), this basic equation is generally valid.

The electrostrictive components $a_{1/2}$ in Eq. (2) arise from a few assumptions that are discussed in [31]. If the relationship between the electric displacement field and the electric field vector is linear and the material is homogeneous, the polarisation can be replaced by a tensor valued dielectric permeability. If it is further assumed that the displacement vector is small, neglecting higher order terms, the two electrostrictive coefficients of Eq. (2) $a_{1/2}$ are obtained. Whereas some works consider the electrostrictive coefficients to be of importance [33,34], in many cases it is further (implicitly) assumed that the material is isotropic, resulting in only the scalar dielectric permittivity $\varepsilon = \varepsilon_0 \varepsilon_r$ introduced in Eq. (1), without any further coefficients [1,29,30,35].

If Eq. (2) is evaluated without electrostrictive components ($a_{1/2} = 0$) and for a unidirectional electric field $\boldsymbol{E} = \begin{pmatrix} 0 & 0 & E \end{pmatrix}^T$ acting in z-direction only, the Maxwell stress becomes

$$\boldsymbol{\sigma}^{\text{elec}} = \begin{pmatrix} -\frac{1}{2}\varepsilon_0\varepsilon_r E^2 & 0 & 0 \\ 0 & -\frac{1}{2}\varepsilon_0\varepsilon_r E^2 & 0 \\ 0 & 0 & \frac{1}{2}\varepsilon_0\varepsilon_r E^2 \end{pmatrix}. \tag{3}$$

Because the DEA material is incompressible, its deformation state is independent of a superimposed hydrostatic pressure state. In other words, the atmospheric pressure has no effect on the material behaviour [36]. The stress tensor

$$\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma}^{\text{elec}} + \hat{p}\,\mathbf{1}, \tag{4}$$

with arbitrary pressure $\hat{p}$ leads to the same deformation state as $\boldsymbol{\sigma}^{\text{elec}}$ alone. If $\hat{p}$ is chosen such that $\hat{p} = \frac{1}{2}\varepsilon_0\varepsilon_r E^2$, Eq. (4) becomes

$$\hat{\boldsymbol{\sigma}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \varepsilon_0\varepsilon_r E^2 \end{pmatrix}. \tag{5}$$