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Density measurement sensitivity of micro-cantilevers influenced by shape dimensions and operation modes

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ABSTRACT

The coupled fluid-solid simulations have been developed to analyze the density measurement sensitivity (DMS) of resonant micro-cantilevers with different shapes, dimensions and operation modes. The analytical formulas have been established to illustrate the relationship of resonant frequency shifts and densities of working fluids. The coupled fluid-solid simulation analyses and experimental results have been used to study the effects of micro-cantilevers with different dimensions on the DMS under bending and torsion modes. The influences of micro-cantilevers with different shapes on the DMS have been also discussed and analyzed. It is concluded that the length and free end width of micro-cantilever as well as operation mode are the key factors affecting the DMS. In addition, the proposed coupled fluid-solid simulation method is an important instruction in the aspect of micro-cantilever design. At the last, the resonant frequency stability and resolution of sensor were analyzed to better understand the sensor's performance.

1. Introduction

Since the twentieth century, many scholars have studied different resonant micro structures to measure fluid density quickly and conveniently [1–6], such as cantilevers [1], channels [2], and plates [3]. Among these structures, micro-cantilever is most widely used for its simple structure, easy fabrication, and high degree of miniaturization for possible direct integration with other devices and functions [7–10]. Fluid density is measured by the variation of equivalent mass of the resonator that is submerged in the fluid to cause the resonance frequency shift. In this paper, the fluid density measurement sensitivity (DMS) is defined as the ratio of resonance frequency shift to the fluid density variation. Therefore, a large resonance frequency shift based on the same density variation results in a large DMS.

Previously, M. T. Boudjiet et al. used the micro-cantilever to measure the concentration of hydrogen in nitrogen [11]. Results shown that the wider and shorter rectangular-shaped cantilever had better sensitivity than either U- or T-shaped cantilevers with the similar dimensions, while the thickness of cantilever did not affect sensitivity. I. Dufour [12] studied the effect of hydrodynamic force on the vibration spectrum of micro-cantilever in the areas of chemical detections in liquid media or the detections of thermophysical properties of fluid, in order to improve the measurement sensitivity. H. Hocheng et al. [13] shown that low-aspect-ratio cantilever with high-aspect-ratio inner cut could achieve high sensitivity. A. Loui et al. [14] revealed that high-aspect-ratio micro-cantilever with longer length was the optimal design for point-loading applications such as microscopy and force measurements, while low-aspect-ratio cantilever with shorter length was the optimal design for surface stress-loading occurring in biological and chemical applications. S. Morshed et al. [15] found that large clamping width or reduced effective mass at the free end of the cantilever could enhance sensing sensitivity. P. S. Waggoner et al. [16] shown that device geometry had little or no effect on sensitivity. Most of these aforementioned micro-cantilevers operated under the bending mode without considering other vibration modes. Russell Cox et al. [17] studied the effect of liquid viscosity and density on the characteristics of laterally excited micro-cantilevers with different ratios of width to thickness, but the lateral resonance is also another type of bending resonance that is vibrating in a large dimension plane.

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Sensitivity improvement is very important to pursue the measurements with high precision and high signal-to-noise ratio. So different micro-cantilevers with various lengths and free end widths are analyzed, and a lot of experimental researches are implemented in order to enhance the sensing sensitivity of MEMS resonance density sensor. The coupled fluid-solid simulations are also developed to validate the theory results. Based on the simulation analyses and experimental results, the DMS could be improved by several methods such as optimizing micro-cantilever shape, reducing its length, and using higher operation mode.

2. Theory

The stiffness of a typical micro-cantilever in the bending mode has the minimum value than that in other resonant modes. Therefore, the first order resonant mode of a micro-cantilever is always the first order bending mode [18]. This study focuses on the relationships of structural parameters and the DMS under different resonant modes.

When the *Re* (Reynolds number) of a micro-cantilever immersed in the fluid is larger than 1, the effect of viscous damping is much smaller than that of inertial damping of fluid [17]. Then, the effect of fluid viscosity can be neglected. Based on the Euler-Bernoulli beam, when the length of a slender rectangular micro-cantilever is five times larger than its width, the resonant frequency of the micro-cantilever immersed in the fluid has the relationship with fluid density as follows [19]:

$$\frac{f_{\rm fluid}}{f_{\rm vac}} = \left(1 + \frac{\pi\rho_{\rm f} w}{4\rho_{\rm c} d}\right)^{-1/2} \tag{1}$$

where $f_{\rm fluid}$ and $f_{\rm vac}$ are the resonant frequencies of micro-cantilever in the fluid and vacuum, respectively; $\rho_{\rm f}$ is the density of fluid; $\rho_{\rm c}$ is the density of micro-cantilever; w and d are width and thickness of micro-cantilever, respectively.

Through mathematical conversion, the density of fluid can be expressed by:

$$\rho_{\rm f} = \frac{4\rho_{\rm c} d}{\pi w}\left(\frac{f_{\rm vac}^2}{f_{\rm fluid}^2} - 1\right) \tag{2}$$

Considering different micro-cantilevers with various shapes, a parameter α denoting the shape of micro-cantilever is introduced into Eq. (2) as follows [20]:

$$\rho_{\rm f} = \frac{4\alpha\rho_{\rm c} d}{\pi w}\left(\frac{f_{\rm vac}^2}{f_{\rm fluid}^2} - 1\right) \tag{3}$$

For the rectangular micro-cantilever, the parameter α is equal to 1.

When Eq. (3) is differentiated on both sides, Eq. (4) is obtained as:

$$\Delta\rho_{\rm f} = \frac{-8\alpha\rho_{\rm c} d f_{\rm vac}^2}{\pi w f_{\rm fluid}^3}\Delta f_{\rm fluid} \tag{4}$$

In this study, the first order bending mode of micro-cantilever is considered, and the resonant frequency $f_{\rm f}^{\rm n}$ of the n^{th} order resonant mode in vacuum can be expressed as follows [21]:

$$f_{\rm f}^{\rm n} = \frac{d\phi_{\rm n}^2}{4\pi l^2}\sqrt{\frac{E}{3\rho_{\rm c}}} \tag{5}$$

where $\phi_{\rm n}$ is the n^{th} order positive root of the equation of $1+\cosh(\phi_{\rm n})\times\cos(\phi_{\rm n})=0$, and $\phi_1=1.8751$, $\phi_2=4.6941$, $\phi_3=7.8548$, $\phi_4=10.9955$ and $\phi_5=14.1372$ for the first five orders, and $\phi_{\rm n}=\pi(n-1/2)$ for the higher order, and l is the length of micro-cantilever, E is elastic modulus of micro-cantilever.

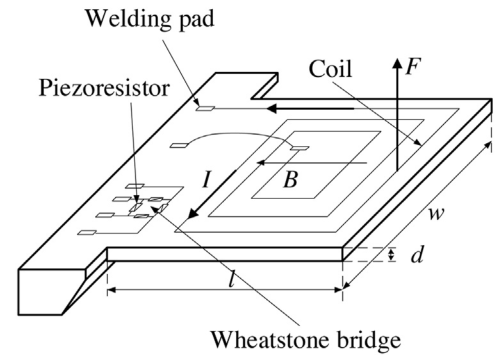

Fig. 1. The design of micro-cantilever chip.

The first order resonant frequency of micro-cantilever in vacuum is expressed as follows:

$$f_{\rm vac} = \frac{d\phi_1^2}{4\pi l^2}\sqrt{\frac{E}{3\rho_{\rm c}}} \tag{6}$$

The DMS can be derived by substituting Eqs (3) and (6) into Eq. (4):

$$S = \frac{\Delta f_{\rm fluid}}{\Delta\rho_{\rm f}} = \frac{-\phi_{\rm n}^2 w d^{3/2}\sqrt{E}}{4\sqrt{3}\alpha l^2(4\rho_{\rm c} d + \pi\rho_{\rm f} w)^{3/2}} \tag{7}$$

where S is the DMS, and it will be increased by reducing length l. However, the effects of width w and thickness d are more complicated. Therefore, this paper focused on the influences of micro-cantilever length and shape on the DMS.

3. Design

A rectangular micro-cantilever was taken as an example to illustrate its structure as shown in Fig. 1, where l, w and d denoted its length, width and thickness. There were coil, four piezoresistors to compose Wheatstone bridge, inside leads and welding pads. The micro-cantilever was fabricated using the Micro Electromechanical Systems (MEMS) technology [22]. The vibration of micro-cantilever was excited by Lorentz force which was generated by electrified coil in a magnetic field, and detected by the output of Wheatstone bridge.

Three different micro-cantilever chips with rectangular, trapezoidal, triangular structures were designed in this paper. The micro-cantilevers with variable cross-section were proposed in this study. We know that rectangle and triangle are two special structures. For a rectangle, the width of its free end is equal to that of its fixed end, and for a triangle, the width of its free end is equal to zero. The micro-cantilever chips with three different shapes had three different lengths of 1.5 mm, 1.7 mm and 1.9 mm, respectively, and the thicknesses of these chips were all 25 μm. Fig. 2 was the scanning electron microscope (SEM) photos of three chips with different shapes.

Because the deflection of micro-cantilever under third order resonant mode was so small, the output voltage of Wheatstone bridge was too weak and could not be detected to determine the resonant frequency in this research. Therefore, both the simulation and experiment sections were all concentrated on the first two order modes.

4. Simulation of DMS

4.1. The conditions of simulation

A coupled fluid-solid simulation was implemented using ANSYS multi-physics software to analyze a rectangular micro-cantilever in