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The signal-to-noise ratio and a hidden symmetry of Hall plates

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ABSTRACT

In a Hall plate with finite size and contacts the Hall output voltage is given by the product of sheet resistance, input current, Hall mobility, magnetic flux density, and Hall geometry factor G_H. $G_H \in [0,1]$ accounts for the loss in signal due to the contacts. At weak magnetic field $G_H \to G_{H0}$ is a function of geometrical parameters only, which makes it the crucial point for layout optimization. We show how to express G_{H0} alternatively as a function of electrical parameters only, namely of input and output resistances over sheet resistance. This allows for an analytical optimization of signal-to-noise-ratio (SNR) without getting lost in the multitude of geometrical representations of equivalent Hall plates. In the course of this investigation we notice a hidden symmetry property of G_H, which we prove rigorously in the limit of small magnetic fields. The physical meaning of this symmetry in the case of Hall plates with equal input and output resistances is also explained.

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1. Introduction

Allthough the Hall effect has been discovered already in 1879 [1], the first attempts to calculate the output voltage of Hall plates rigorously started much later, shortly after World War Two [2]. The reason for this long idle period might have been to a certain extent the difficulties in this calculation but probably even more the lack of interest in an accurate quantitative description. This changed drastically with the advent of semiconductor industry in the 1950s (see also the introduction to [3]). Then Wick introduced the conformal mapping method to compute the Hall plate output voltage [3] and shortly afterwards van der Pauw applied the same technique to sheet resistance measurements on Hall plates at zero magnetic field [4,5]. In the same year Lippmann and Kuhrt coined the term *geometry function* (which we now call *Hall geometry factor*) and computed it for rectangular Hall plates [6] (the same information is found in [3], too). In the following decade the Hall geometry factor was computed analytically for various devices, yet due to computational difficulties one pair of contacts had to be small. De Mey reduced the complexity of the calculation by an expansion method for weak magnetic fields [7]. Häusler considered a very special circular Hall plate with four equal contacts which cover 50% of the circumference, for which he could express the Hall geometry factor in terms of beta-functions and hypergeometric functions [8]; an impressive achievement, however, the only free parameter left is the arbitrary Hall angle. Thus, it is not useful for device optimization. Luckily, we will identify exactly this type of Hall plates as the one with optimum signal to noise ratio (SNR), so that his formulae are invaluable in practice particularly at large magnetic field. Versnel considered rectangular Hall plates with finite contacts at arbitrarily large magnetic field [9]. He also focused on Hall and van der Pauw devices with 90°-symmetry [10,11]. His results contained ratios of sums of numerical integrals as functions of geometrical input parameters, which are adequate to obtain numerical values. However, one cannot derive any properties or optimization rules from his analytical formulae. From the mid-1980s onwards numerical computation of Hall potentials became possible, and this offers a lot of new possibilities (e.g. to consider inhomogeneous doping and inhomogeneous plate thickness, velocity saturation and 3D-effects in real devices [12,13]). They undoubtedly have their merits when we want to study the properties of specific device geometry, but they are less suited to get a survey over the global optimization landscape of all possible devices.

Already the early paper of Wick mentioned a tremendous benefit of the conformal mapping technique: if we find a conformal map from one device to another one, this implies that both devices have identical electrical properties and identical Hall output voltages. It means they have the same input resistance and the same output resistance, they give the same readings in a van der Pauw measurement [14,15], and they can be modeled by the same equivalent circuit. Therefore, there must be a unique relation between any set of electrical parameters and its respective Hall geometry factor. Hence, we need to do the difficult task of computing the Hall

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geometry factor as a function of electrical parameters only once for all device geometries (as long as they belong to the same family of conformal maps). Then we can do the simpler calculation of electrical parameters for all distinct shapes of devices as functions of their layout parameters. This second step is only interesting for the physical device engineer (layout engineer) – the electronic concept and circuit design engineers can even skip it, because they are only interested in the electrical parameters and in the magnetic sensitivity of the device, irrespective of its shape. The engineering community seems to have lost sight of this remarkable benefit of the conformal mapping method, because during ~1984 ... 2015 many papers showed Hall geometry factors or magnetic sensitivities of Hall plates related to specific device geometries. Only recently an analytical formula was given that relates the Hall geometry factor at small magnetic field to input and output effective numbers of squares (see Eq. (29a–c) and Fig. 7b in [16]). Although the formula is an integral over an incomplete elliptic integral, which (still) resists a closed form solution, it is compact enough to derive several symmetry properties of G_H and to find unique optimum parameters for maximum SNR of Hall plates.

In this paper we further simplify this integral formula, derive an expression for the SNR, plot it for all possible Hall plates with two perpendicular mirror symmetries, identify the parameters for maximum SNR, give the maximum magnetic detectivity of Hall plates in a given bandwidth, reveal a hidden symmetry property of G_H and give a physical interpretation of it.

2. The Hall geometry factor as a function of input and output resistances

In an infinite Hall-effect region the Hall electric field vector $\vec{E}_H$ is given by the product of resistivity ρ, current density $\vec{J}$, Hall mobility μ_H, and induction field $\vec{B}$: $\vec{E}_H = \rho\vec{J} \times (\mu_H\vec{B})$. In thin Hall plates with finite size and contacts the output voltage V_{out} is given by a similar product of equivalent integral quantities [6].

$$V_{out} = R_{sh} I_{in} \mu_H B_\perp G_H \tag{1a}$$

The output voltage of the Hall plate is V_{out}, the input current through the Hall plate is I_{in}. $R_{sh} = \rho/t_H$ is the sheet resistance (also called square resistance) of the Hall effect region with the homogeneous Hall plate thickness t_H. The sheet resistance can be measured for any contact size with the generalized method of van der Pauw without need to know specific details of the geometry [14,15]. The product of Hall-mobility μ_H times magnetic induction $B_\perp$ perpendicular to the Hall plate is the equal to the tangent of the Hall angle θ_H: $\mu_H B_\perp = \tan\theta_H$. θ_H is the angle between the electric field and the current streamlines $\theta_H = \angle(\vec{E},\vec{J})$. It is constant throughout the Hall-effect region irrespective of its shape. The Hall geometry factor G_H depends on the shape of the Hall plate and the size of its contacts – and it also depends on the Hall angle θ_H: at large Hall angle it tends to 1. In other words, at weak magnetic field the Hall geometry factor is smallest. There we have the largest influence of the finite Hall plate geometry on the Hall output voltage. That is one justification why we focus on the weak field limit of the Hall-geometry factor

$$G_{H0} = \lim_{B_\perp \to 0} G_H \tag{1b}$$

A second motivation for G_{H0} is that we want to clarify what is the ultimate magnetic resolution of Hall plates, and there of course the field is weak. In [16] we have computed G_{H0} for a large class of Hall plates, namely for all plates with two perpendicular mirror symmetries. Fig. 1 shows several embodiments of Hall plates that belong to this class. In [16] we chose plate # 1 as representative of this class of Hall plates to carry out the calculation, but the results are valid for the entire class of Hall plates, because they can be mapped onto each other by conformal transformation. In modern sensors Hall plates with a higher degree of symmetry (90° symmetry) are commonly used, because they are better suited for the spinning current Hall probe scheme. However, in the new emerging field of Vertical Hall effect devices one is forced to work with even less symmetry: they usually have only one mirror axis of symmetry. In [16] the result was

$$G_{H0} = \frac{1}{2}\lambda_p\lambda_f\left\{1 + \frac{CC_1}{K(f)K'(p)}\int_{t=-1}^{1}\frac{F(t,f)}{\sqrt{1-t^2}\sqrt{1+ft}\sqrt{1+CC_2 t}}\,dt\right\}$$
$$\text{with } 0 \leqslant f,p \leqslant 1 \tag{2a}$$

with the abbreviations

$$CC_1 = \sqrt{\frac{2(1-f^2)}{f(1-p)^2 + 1 + p(6+p)}} \tag{2b}$$

$$CC_2 = \frac{(1-p)^2 + f[1+p(6+p)]}{f(1-p)^2 + 1 + p(6+p)} \tag{2c}$$

Thereby we used the incomplete elliptic integral of the first kind which is commonly defined as

$$F(w,k) = \int_0^w (1-\alpha^2)^{-1/2}(1-k^2\alpha^2)^{-1/2}\,d\alpha \tag{2d}$$

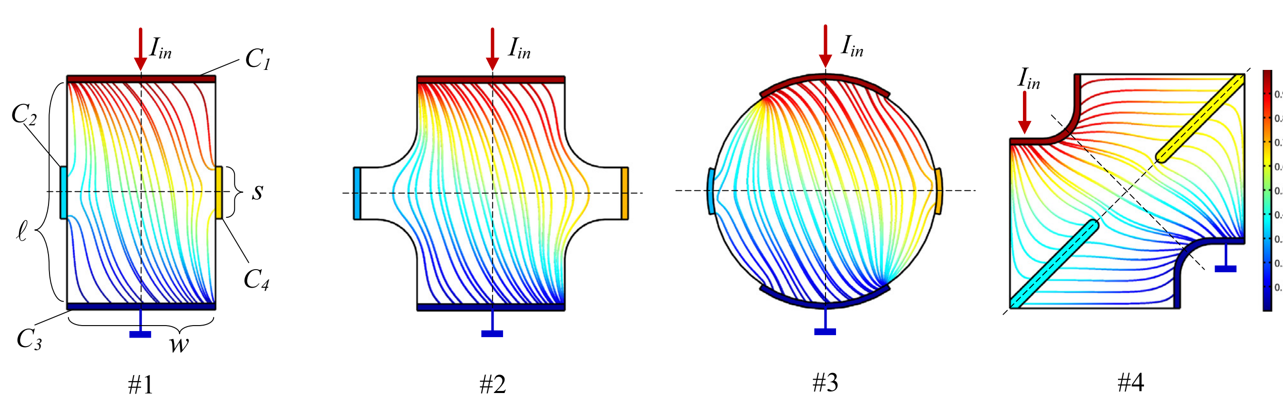

Fig. 1. Various embodiments of Hall plates with two perpendicular mirror symmetries to which the presented theory applies. The two dashed lines of mirror symmetry are shown. They go through the centers of opposite contacts C_1-C_3 and C_2-C_4, respectively. Device # 1 is studied in detail as *pars pro toto*, but the results are valid for the entire class of Hall plates. This class is characterized by three degrees of freedom (DoF) for each device and the DoFs can be measured with high accuracy by a generalized van der Pauw methode [15]. The figure shows current streamlines for a Hall angle of 45° and the color coding denotes electric potential (red means +1 V, blue means 0 V). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

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