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Nonlinear norm-observability and simulation of control systems[☆]

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ABSTRACT

The (bi)simulation relation has recently been attracting growing interest in the study of nonlinear control systems, in the hope that through such a relation, the behaviors and properties of a nonlinear system can be inferred from those of another system which is easier to handle. In this paper, we consider the propagation of the property of nonlinear norm-observability through a simulation relation. Given two control systems that are related by a graph simulation relation, we derive conditions under which the norm-observability of the simulating system implies the norm-observability of the simulated system. The obtained results are given in terms of set-valued functions. Several examples are included to illustrate various applications of our results.

1. Introduction

In many cases, high fidelity models to accurately represent a dynamical system may be too intricate for use in system analysis and control design. It is therefore desirable to have a methodology that relates "complex" models (for example, models with high nonlinearity) to "simple" ones (for example, systems being linear or mildly nonlinear), while preserving certain properties of interest relevant for analysis or synthesis. In the past decades, approaches based on (bi)simulation relations have been introduced in the study of controlled dynamical systems, exploring the possibility of connecting a system with another system whose behaviors and/or properties are easier to understand (see, e.g., [1–4]). (Bi)simulation relations are natural and important objects in control systems theory. Loosely speaking, a simulation between two dynamical systems defines a relation with the property that every trajectory of the first system can be associated with a trajectory of the second system. If the association is bidirectional, then one obtains a bisimulation relation between the two dynamical systems. The notions of simulation and bisimulation relations provide a potentially useful tool for classifying linear and nonlinear systems [2,4]. They also have interesting connections with other fundamental concepts in nonlinear systems theory such as controlled invariance [1,3,5] and feedback transformations [6]. As already stated, an important motivation for studying (bi)simulation relations is to hope to reason about certain properties across related systems. Some pertinent work includes studies on reasoning about controllability of (C-related) linear systems [7], reasoning about stability properties of hybrid systems [8], and the propagation of controllability properties through a simulation relation for nonlinear systems [9].

Observability is certainly one of the key concepts in control theory. In the context of nonlinear systems, various observability definitions have been proposed in the literature in order to capture the relationship between the state, the output, and the input of a system (see, e.g., [10]). The notion of norm-observability was introduced in [11] and [12]. Rather than inferring the precise value of the state, the norm-observability properties describe the ability to determine an upper bound on the norm of the state using the output and the input. As pointed out in [12], such observability properties have close ties to the important concept of input–output-to-state stability in nonlinear systems analysis [13]. The problem of determining whether a system is norm-observable, besides being interesting in itself, is particularly relevant in the context of switched nonlinear systems, as it is strongly related to the stability and supervisory control of the systems (see, e.g., [12,14,15]).

In this paper, we focus on the notion of norm-observability and examine the extent to which the norm-observability properties of nonlinear systems are preserved by simulation relations. More specifically, given two control systems that are connected by a simulation relation, our main objective is to determine conditions that allow us to propagate the norm-observability properties from

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the simulating system to the simulated system, suggesting that an observability analysis of the simulating system can shed light on the norm-observability properties of the simulated system. Currently, the main tool used to test norm-observability for nonlinear systems in the literature, to our knowledge, is the Lyapunov-like method [12]. We demonstrate by example that our results offer a new possibility for the norm-observability analysis of nonlinear systems. The notion of simulation relation embraces many different types, such as exact simulation relations, approximate simulation relations [16–19], alternating simulation relations [20, Chapters 4.3 and 9.2], contractive simulation relations [21], and graph simulation relations [6,9]. Depending on the context, some relations may be more appropriate to use than others. The simulation relations considered in the paper are the so-called graph simulation relations. As will be seen, such relations are the right tool to use to reason about nonlinear norm-observability.

Organization: The notions of norm-observability and graph simulation are presented in Section 2. Main results, establishing the conditions that propagate norm-observability, are proposed in Section 3. Then, several illustrative examples are given in Section 4, and a brief conclusion is drawn in the final section.

Notation and terminology: We use $|\cdot|$ to denote the standard Euclidean norm, and $\|z\|_I$ the essential supremum norm of a function $z(t)$ on an interval I. We write $B^n(r)$ for the closed ball in $\mathbb{R}^n$ with center 0 and radius $r > 0$. For a function $g : A \to B$, the *graph* of g, denoted by Graph(g), is defined as Graph$(g) = \{(a, g(a)) : a \in A\}$. Let X and Y be finite-dimensional Euclidean spaces. A *set-valued function* F from X to Y is a function that associates with any $x \in X$ a subset $F(x)$ of Y. If $K \subseteq X$ and if F is a set-valued function from X to Y, the image of the set K under F is given by $F(K) = \cup_{x\in K} F(x)$. A set-valued function F is said to be *bounded* if the image of any bounded set under F is bounded. We say that F is *upper semicontinuous* at $x \in X$ if for any open N containing $F(x)$ there exists a neighborhood M of x such that $F(M) \subseteq N$.

2. Preliminaries

2.1. Norm-observability notions

To make the paper reasonably self-contained, we briefly recall the definitions of norm-observability introduced in [12]. Consider the following system

$$\Sigma : \quad \dot{x} = f(x, u), \qquad y = h(x). \tag{1}$$

We assume that (see, e.g., [10]) the function $f : \mathbb{R}^n \times \mathbb{R}^m \to \mathbb{R}^n$ is so that $f(\cdot, u)$ is of class C^1 for each fixed $u \in \mathbb{R}^m$, f and $\partial f/\partial x$ are continuous on $\mathbb{R}^n \times \mathbb{R}^m$, and $f(0, 0) = 0$, and that $h : \mathbb{R}^n \to \mathbb{R}^p$ is continuous with $h(0) = 0$. By an input or control for (1), we mean a measurable function $u(\cdot) : \mathbb{R} \to \mathbb{R}^m$ which is essentially compact valued on compact intervals, i.e., for every compact interval $I \subseteq \mathbb{R}$ there exists a compact subset $K \subseteq \mathbb{R}^m$ such that $u(t) \in K$ for almost all $t \in I$ [6,9]. We denote by $\mathcal{U}^m_{\mathrm{cpt}}$ the set of all inputs. For any $u(\cdot) \in \mathcal{U}^m_{\mathrm{cpt}}$ and any $x_0 \in \mathbb{R}^n$, there exists a unique maximally extended solution of the initial value problem

$$\dot{x} = f(x, u(t)), \qquad x(0) = x_0.$$

Such a solution is defined on some open interval $(t^{\min}_{x_0,u}, t^{\max}_{x_0,u})$ containing 0. We assume that the system Σ has the unboundedness observability property [22], which means that for every initial state x_0 and input u such that $t^{\max}_{x_0,u} < \infty$, the corresponding output becomes unbounded as $t \to t^{\max}_{x_0,u}$. We recall that a function $\alpha : [0, \infty) \to [0, \infty)$ is said to be of class $\mathcal{K}_\infty$ if it is continuous, strictly increasing, unbounded, and $\alpha(0) = 0$.

Definition 1 (*[12]*).

(a) We say that the system Σ is *small-time norm-observable* if for every $\tau > 0$, there exist $\mathcal{K}_\infty$ functions γ and χ such that for every $x_0 \in \mathbb{R}^n$ and for every $u \in \mathcal{U}^m_{\mathrm{cpt}}$, it holds that

$$|x_0| \le \gamma(\|y\|_{[0,\tau]}) + \chi(\|u\|_{[0,\tau]}). \tag{2}$$

(b) We say that Σ is *large-time norm-observable* if there exist $\tau > 0$ and two class $\mathcal{K}_\infty$ functions γ and χ such that (2) holds for any $x_0 \in \mathbb{R}^n$ and any input $u \in \mathcal{U}^m_{\mathrm{cpt}}$.

Remark 1. Roughly speaking, norm-observability imposes a bound on the norm of the initial state in terms of the norms of the output and the input. The principal difference between small-time norm-observability and large-time norm-observability is that the former requires the inequality (2) to hold for arbitrary τ, while the latter requires (2) to hold for at least one $\tau > 0$. It is clear from the definition that small-time norm-observability implies large-time norm-observability. Note that the converse is, in general, not true. However, for linear systems these two notions are known to be equivalent and are both equivalent to the usual concept of observability [12].

Remark 2. Other equivalent definitions of small-time and large-time norm-observability can be achieved under the assumption of the unboundedness observability property for the system Σ and its reversed-time system; see [12] for more information.

2.2. Graph simulation relations

Consider the system Σ together with another system

$$\widetilde{\Sigma} : \quad \dot{z} = \tilde{f}(z, v), \qquad w = \tilde{h}(z). \tag{3}$$

Here, the function $\tilde{f} : \mathbb{R}^{\tilde{n}} \times \mathbb{R}^{\tilde{m}} \to \mathbb{R}^{\tilde{n}}$ is such that $\tilde{f}(\cdot, v)$ is a C^1 function for each fixed $v \in \mathbb{R}^{\tilde{m}}$, $\tilde{f}$ and $\partial\tilde{f}/\partial z$ are continuous on $\mathbb{R}^{\tilde{n}} \times \mathbb{R}^{\tilde{m}}$, and $\tilde{f}(0, 0) = 0$; and $\tilde{h} : \mathbb{R}^{\tilde{n}} \to \mathbb{R}^{\tilde{p}}$ is continuous and vanishes at 0. The following definition is patterned after that given in [6] and [9].

Definition 2. Given Σ and $\widetilde{\Sigma}$, a pair of relations $(\mathcal{S}, \mathcal{R})$, where $\mathcal{S} \subseteq \mathbb{R}^n \times \mathbb{R}^{\tilde{n}}$ and $\mathcal{R} \subseteq \mathbb{R}^p \times \mathbb{R}^{\tilde{p}}$, is called a *compact graph simulation relation* of Σ by $\widetilde{\Sigma}$ if the following conditions are satisfied:

(a) The relation $\mathcal{S}$ is the graph of a C^2 function $\Phi : \mathbb{R}^n \to \mathbb{R}^{\tilde{n}}$ with the following property: given any $x \in \mathbb{R}^n$ and any $u \in \mathbb{R}^m$, there exist open neighborhoods $X \subseteq \mathbb{R}^n$ of x and $U \subseteq \mathbb{R}^m$ of u, and a compact set $V \subseteq \mathbb{R}^{\tilde{m}}$ such that for every $x' \in X$ and $u' \in U$ there is some $v' \in V$ such that

$$\frac{\partial\Phi}{\partial x}(x)\Big|_{x=x'} f(x', u') = \tilde{f}(\Phi(x'), v').$$

(b) For every $x \in \mathbb{R}^n$ we have $(h(x), \tilde{h}(\Phi(x))) \in \mathcal{R}$.

We call $\widetilde{\Sigma}$ the *simulating* system and Σ the *simulated* system.

Note that this definition is slightly different from the one of [6] and [9] in that in condition (b) we only require the outputs $h(x)$ and $\tilde{h}(\Phi(x))$ to be related by a relation $\mathcal{R}$, rather than identical.

Remark 3. Intuitively, a simulation should specify that every trajectory of the simulated (or original) system can be matched by a trajectory of the simulating (or abstract) system. Certainly, one can define the concept of simulation relation by directly using this idea. But, in practice, such a definition may be inconvenient to check, especially for nonlinear systems, since it requires knowledge of the system trajectories. On the other hand, conditions (a) and (b) of Definition 2 are relatively easy to verify and suffice to guarantee that the simulating system has the capability of mimicking the behavior of the simulated system [6].