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Abstraction and control by interconnection of linear systems: A geometric approach

Noorma Yulia Megawati [a,b,*], Arjan van der Schaft [a]

[a] Johann Bernoulli Institute for Mathematics and Computer Science, University of Groningen, 9747AG Groningen, The Netherlands
[b] Department of Mathematics, Universitas Gadjah Mada, Yogyakarta, Indonesia

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ABSTRACT

We consider the problem of constructing a controller achieving a desired linear specification, based on a linear abstraction system of the plant system. First, we extend the necessary and sufficient conditions for control by interconnection by bisimulation equivalence to the case of non-deterministic linear systems. Then we apply the controller constructed on the basis of the lower-dimensional abstraction system to the original plant system and show that the closed-loop system is simulated by the given specification system. We distinguish between two forms of abstraction of the plant system. In the first one, the set of variables available for controller interconnection remains the same. In the second, more general form, this is not anymore the case, and we show how an adapted form of interconnection of the controller system to the plant system yields the same result.

1. Introduction

A basic problem in system and control theory is to construct a controller such that the closed-loop system behaves exactly like a given specification system. This general problem has been explored in various frameworks, see e.g. [1–3] and the references quoted therein. When dealing with a large-scale plant system the controller system tends to become high-dimensional as well, posing severe problems for computation and implementation. One of the methods to address this complexity problem is to approximate the linear plant system by a *lower-dimensional* system. There are many methods for approximation. In this paper, we approximate the plant system in the sense that it is *simulated* by a lower-dimensional linear system. This is the idea of *abstraction* [4,5], and we call this lower-dimensional, but possibly non-deterministic, system an *abstraction system*. Loosely speaking, the abstraction system is a lower-dimensional linear system whose external behavior (with respect to a given set of input and output variables) *contains* the external behavior of the original system. Next, we want to apply the controller based on the abstraction system to the plant system in such a way that the closed-loop system *approximates* the specification system, where again approximation is formalized as simulation.

A similar problem setting was extensively studied in a number of papers, see e.g. [6,7] and the references quoted therein, with the main difference that the abstraction system in these papers is a *discrete transition system*. Instead, in the current paper the abstraction system is again a (lower-dimensional and non-deterministic) linear system. This allows us to remain completely within the framework of linear geometric control theory. In [6] the problem is studied of refining the controller for the discrete abstraction system in such a way that it can be applied to the plant system. Furthermore, the notion of *alternating simulation relation* is used to relate the plant system and the abstraction system. Examples in [7] show that the alternating simulation relations are not adequate for controller refinement whenever the controller has only quantized or symbolic state information, and the complexity of the refined controller exceeds the complexity of the controller for the abstraction system. Therefore, a novel notion of *feedback refinement relations* is proposed to resolve both issues. Moreover, [7] shows that feedback refinement relations are necessary and sufficient for controller refinement. The current paper shows that for linear abstraction systems the problem of applying the controller constructed for the abstraction system to the original plant system admits a direct and elegant solution within the framework of geometric control theory.

In this paper we consider linear plant systems with two types[1] of inputs f and u, and two types of outputs z and y. The first type of

* Corresponding author at: Johann Bernoulli Institute for Mathematics and Computer Science, University of Groningen, 9747AG Groningen, The Netherlands.
E-mail addresses: n.y.megawati@rug.nl, noorma_yulia@ugm.ac.id (N.Y. Megawati), a.j.van.der.schaft@rug.nl (A. van der Schaft).

[1] Note that in [6,7] the manifest and control variables of the abstraction system coincide, leading to a less general (and simpler) scenario.

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input f together with the output z describes the interaction of the system with its environment. We call (f, z) the *manifest variables*. The second type of input u and the output y are variables that are to be connected to the controller system. The variables (u, y) are called the *control* variables. Given a linear plant system we approximate it by a lower-dimensional linear abstraction system. A preliminary problem studied in this paper consists in finding a controller for the abstraction system such that the abstraction system interconnected with the controller system is bisimilar to a given specification system. This problem extends the 'control by interconnection' problem studied in [3] to linear systems involving *internal disturbances*. These internal disturbances are used for modeling the typical case of 'non-determinism' in the abstraction system. 'Non-determinism' means that the state of the system, starting from a given initial condition, and for a given input function may evolve into different time-trajectories.[2] Next we consider the problem of applying the controller system derived for the abstraction system to the original plant system. Here we make a distinction between the situation where the set of control variables of the abstraction system is equal to the set of control variables, and the more general situation where this is not anymore the case. In this last case we need to modify the interconnection of the controller to the original plant system. The main theorem consists of showing that the resulting interconnection of the original plant system and the controller system derived for the abstraction system is *simulated* by the specification system.

The paper uses throughout the framework of 'control by interconnection' and that of '(bi-)simulation' equivalence. In the control by interconnection framework the plant system is interconnected to a controller system via shared control variables. This does *not* always correspond to dynamical output feedback, as usually studied in control theory. On the other hand, certain physical control mechanisms, as exemplified by the *door closing mechanism* given in [9], are not of dynamical output feedback type but do correspond to control by interconnection. Furthermore, we will use the *canonical controller* introduced in [2], and further employed in [3] for control by interconnection by bisimulation equivalence. We will give more details about control by interconnection and the canonical controller in Sections 2 and 3, respectively.

With regard to the use of (bi-)simulation we note that there are many ways to define *equivalence* between systems. In the behavioral approach, two systems are equivalent if their behaviors are equal. In a linear input–output context, two systems are called equivalent if their transfer matrices are equal. Furthermore, two state space systems can be called equivalent if there exists an invertible state space transformation matrix linking the two systems. The notion of (bi-)simulation relation was first introduced in computer science [10], and later extended to continuous dynamical systems in [5]; see also [1]. In [4] it was shown how the notion of bisimulation relation directly extends the classical notions of transfer matrix equality and state space equivalence. This was recently extended to linear differential–algebraic (DAE) systems in [11], which also provides the natural context for (bi-)simulation using the canonical controller. We will give more details in Section 2.

This paper is organized as follows. In Section 2, we recall the basic notions that we need for the rest of the paper such as the definition of interconnection between systems and the notion of (bi)-simulation relation using geometric control theory. In Section 3, we deal with the problem of finding a controller for linear systems with internal disturbances. Then we address the main problem of applying the controller constructed on the basis of the abstraction system to the original plant system. We start with the case that the set of control variables of the abstraction system is equal to that of the plant system, and afterwards we show how this can be extended to the general case by making use of an adapted form of interconnection.

2. Preliminaries

In this section, we give definitions of the notions of *interconnection* and of *(bi-)simulation relations*.

2.1. Interconnection

Consider two linear systems

$$\Sigma_i : \begin{aligned} \dot{x}_i &= A_i x_i + B_i^u u_i + B_i^f f_i + G_i d_i, & x_i \in \mathcal{X}_i, u_i \in \mathcal{U}_i, f_i \in \mathcal{F}, d_i \in \mathcal{D}_i \\ y_i &= C_i^y x_i, & y_i \in \mathcal{Y}_i \\ z_i &= C_i^z x_i, & z_i \in \mathcal{Z} \end{aligned} \tag{1}$$

where $A_i \in \mathbb{R}^{q_i \times n_i}$, $B_i^u \in \mathbb{R}^{q_i \times k_i}$, $B_i^f \in \mathbb{R}^{q_i \times l}$, $G_i \in \mathbb{R}^{q_i \times s_i}$, $C_i^y \in \mathbb{R}^{p_i \times n_i}$ and $C_i^z \in \mathbb{R}^{r \times n_i}$; $\mathcal{X}_i, \mathcal{U}_i, \mathcal{F}, \mathcal{D}_i, \mathcal{Y}_i$ and $\mathcal{Z}$ are finite dimensional linear spaces, of dimension, respectively n_i, k_i, l, s_i, p_i and r. Here x_i denotes the state of the system, u_i, f_i are inputs, d_i is 'internal' disturbance and y_i, z_i are outputs. The set of allowed time functions $x_i : \mathbb{R}^+ \to \mathcal{X}_i, u_i : \mathbb{R}^+ \to \mathcal{U}_i, f_i : \mathbb{R}^+ \to \mathcal{F}, d_i : \mathbb{R}^+ \to \mathcal{D}_i, y_i : \mathbb{R}^+ \to \mathcal{Y}_i$ and $z_i : \mathbb{R}^+ \to \mathcal{Z}$, with $\mathbb{R}^+ = [0, \infty)$, will be denoted by $\mathfrak{X}_i, \mathfrak{U}_i, \mathfrak{F}, \mathfrak{D}_i, \mathfrak{Y}_i$ and $\mathfrak{Z}$, respectively. For simplicity of the notation, we will denote these time functions by x_i, u_i, f_i, d_i, y_i and z_i. The exact choice of function classes is for the purpose of this paper not really important as long as the state trajectories $x(\cdot)$ are continuous. For example, we can take all the functions to be piecewise $\mathcal{C}^\infty$.

Interconnection between two systems with respect to either manifest variables or control variables will be denoted by m and c, respectively. In order to allow for more general interconnections than the standard feedback one, we will use a permutation matrix Π as formalized in the following definition.

Definition 2.1. Let Σ_1 and Σ_2 be two systems of the form (1). Their interconnection through the manifest variables via a permutation matrix Π, denoted by $\Sigma_1 \|_m^\Pi \Sigma_2$, is defined

$$\begin{aligned} \dot{x}_i &= A_i x_i + B_i^u u_i + B_i^f f_i + G_i d_i, \\ y_i &= C_i^y x_i, \\ z_i &= C_i^z x_i, \quad i = 1, 2, \\ \begin{bmatrix} f_1 \\ z_1 \end{bmatrix} &= \Pi \begin{bmatrix} f_2 \\ z_2 \end{bmatrix}. \end{aligned}$$

The interconnection system $\Sigma_1 \|_m^\Pi \Sigma_2$ is a differential–algebraic system with algebraic constraints on the state variables (x_1, x_2). The state space of this interconnected system, denoted by $\mathcal{X}_{\Sigma_1,\Sigma_2}$, is defined as

$\{(x_1, x_2) \in \mathcal{X}_{\Sigma_1} \times \mathcal{X}_{\Sigma_2} \mid \exists$ input functions $u_1, u_2, f_1, f_2,$ disturbance functions d_1, d_2 and $\exists$ solution trajectory$(x_1(\cdot), x_2(\cdot))$ such that $(x_1(0), x_2(0)) = (x_1, x_2)$ and $\begin{bmatrix} f_1(t) \\ z_1(t) \end{bmatrix} = \Pi \begin{bmatrix} f_2(t) \\ z_2(t) \end{bmatrix}, \ t \geq 0, \}$.

Similarly, the interconnection through the control variables and a suitable permutation matrix Π is denoted by $\Sigma_1 \|_c^\Pi \Sigma_2$, where the first set of equations is as in Definition 2.1, while the state space of the interconnected system is $\{(x_1, x_2) \in \mathcal{X}_{\Sigma_1} \times \mathcal{X}_{\Sigma_2} \mid \exists$ input functions $u_1, u_2, f_1, f_2,$ disturbance functions d_1, d_2 and $\exists$ solution trajectories $(x_1(\cdot), x_2(\cdot))$ such that $(x_1(0), x_2(0)) = (x_1, x_2)$ and $\begin{bmatrix} u_1(t) \\ y_1(t) \end{bmatrix} = \Pi \begin{bmatrix} u_2(t) \\ y_2(t) \end{bmatrix}, \ t \geq 0\}$.

We use the notation $(x_1(0), u_1, y_1, f_1, z_1, d_1) \in \Sigma_1$ to indicate that starting from an initial condition $x_1(0)$, by applying input

[2] We want to emphasize that the problem studied here is fundamentally different from the problem studied in [8] where the problem of (behavioral) control by interconnection of a plant system with *external disturbances* was treated.