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Robust distributed model predictive control of constrained dynamically decoupled nonlinear systems: A contraction theory perspective

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ABSTRACT

This paper considers the robust distributed model predictive control (MPC) of a group of dynamically decoupled nonlinear systems cooperating via the cost function subject to control constraints. Inspired by the contraction theory, we develop the robust distributed MPC scheme assuming that the dynamics of the cooperating systems satisfy the contraction property (they are contracting in a tube centered around a nominal state trajectory). Compared to conventional robust distributed MPC which uses the Lipschitz continuity property, the proposed method features the following aspects: (1) it can tolerate larger disturbances; and (2) it is feasible for a larger prediction horizon and could enlarge the feasible region accordingly. The paper evaluates the maximum disturbance which the nonlinear system can tolerate when controlled using the proposed method and derives sufficient conditions for the recursive feasibility of the optimization and for the practical stability of the closed-loop system. The effectiveness of the proposed method is illustrated using a simulation example.

1. Introduction

Cooperative control has been implemented in applications ranging from multi-vehicle systems [1], large-scale chemical systems [2], transportation systems [3], energy systems [4,5], and so on. However, state and control constraints are ubiquitous in physical systems and should be accounted for in the controller design. Model predictive control (MPC) is one of the few techniques that can explicitly handle such constraints. A direct implementation of MPC in cooperative control leads to centralized MPC [6–8]. Sometimes, centralized MPC has limited practical value because of its computational complexity and communication constraints. In such cases, decentralized MPC strategy is a natural choice. In decentralized MPC, each subsystem designs its own controller without considering the dynamics of other subsystems. As shown in [9], the performance degrades or even instability may arise when the couplings among subsystems are strong. Alternatively, distributed MPC can be adopted to make use of the neighbors' information.

In distributed MPC, each subsystem exchanges information with its neighbors (a subset of the other subsystems), and solves a local optimization which incorporates the state constraints, the control constraints and the neighbors' information. Compared to centralized MPC, distributed MPC has: (1) reduced computational complexity because it distributes an overall large-scale optimization into a set of small-scale optimizations; (2) smaller communication burden, because each subsystem only needs to exchange information with its neighbors, instead of with all other subsystems. However, the recursive feasibility and closed-loop stability results developed for centralized MPC cannot be simply extended to the distributed MPC setting.

Distributed MPC strategies have been developed both for dynamically coupled subsystems and for dynamically decoupled subsystems. In the following, the distributed MPC of dynamically decoupled subsystems, which is pertaining to the problem investigated in this paper, is reviewed.

In distributed MPC of dynamically decoupled subsystems, the cooperation among the subsystems is achieved via coupling state and/or control constraints and/or coupling cost function. When the coupling is implemented through state and/or control constraints, the main challenge is on how to satisfy the constraints. One way to guarantee recursive feasibility and closed-loop stability is to solve the local optimization associated with each subsystem sequentially [10–12]. Another technique is to utilize tightening constraints [13,14], i.e., to design robust positively invariant tubes around the ideal trajectory, and then to employ the feedback

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control to satisfy the coupling constraints. When the coupling is implemented through a coupled cost function [15,16], the recursive feasibility of the optimization can be guaranteed [15] if each subsystem does not deviate too much from its previously predicted state trajectory. The algorithm in [15] is implemented in a leader–follower stabilizing formation of unmanned aerial vehicles (UAVs) [17]. The distributed MPC for systems coupled through state constraints and cost function [18] simultaneously requires small enough prediction error and fast updating frequency. For systems satisfying the controllability condition [19], an easily-verifiable constraint can be imposed in the optimization solved by each subsystem. A recent novel distributed MPC [20] introduces a robustness constraint in the optimization. However, all the aforementioned algorithms assume and use the Lipschitz continuity of the system dynamics [15,17,20,21] to evaluate an upper bound on the deviation of the actual state trajectory from the predicted state trajectory, so they compute an upper bound which grows exponentially with the prediction horizon. Consequently, they can offer recursive feasibility and closed-loop stability guarantees only for small disturbances and for short prediction horizon, essentially leading to conservative results.

It is expected that MPC strategies which use additional properties of the system dynamics can have better performance for certain classes of nonlinear systems than general MPC methods which exploit only Lipschitz continuity. This paper exploits the contractive dynamics of such a class of nonlinear systems. The contraction theory has been firstly introduced in [22] inspired by fluid mechanics, and has been extended to nonlinear distributed systems [23], resetting hybrid systems [24] and stochastic incremental systems [25]. The application of contraction theory to mechanical systems can be found in [26] and to networked systems in [27]. Centralized MPC using contraction theory has been designed in [28]. For the class of nonlinear systems comprising subsystems with decoupled control constraints, and the dynamics of each subsystem are decoupled and contracting in tube-like regions along their nominal state trajectories, this paper designs a novel robust distributed MPC strategy with cooperative cost function, which exploits the contraction property within these tubes. The contributions of this paper are three-fold.

- It derives an upper bound on the deviation between the actual and the nominal state trajectories when both trajectories are in the contraction region.
- It establishes sufficient conditions for the recursive feasibility of the local optimization solved by each subsystem, namely the nominal and the actual state trajectories should lie in the contraction region such that the contraction property can be exploited. Then, the feasibility of the optimization at time t_k indicates the feasibility of the optimization at t_{k+1}.
- It shows that the recursive feasibility of all subsystem optimizations is sufficient for the closed-loop stability of the overall system. Also, the proof of the feasibility and stability exploits the contraction property rather than the decreasing of the cost function, which allows a stronger cooperation among subsystems.

The remainder of this paper is organized as follows. Section 2 presents the distributed MPC problem formulation for the large-scale nonlinear systems and derives the upper bound of the deviation between the actual and predicted state trajectories for each subsystem. Section 3 establishes sufficient conditions for the recursive feasibility of the local optimization solved by each subsystem. Provided that all local optimizations are recursively feasible, Section 4 shows that the closed-loop system is practically stable, i.e., the closed-loop system state enters a neighboring set of the origin in finite time. Section 5 illustrates the proposed robust distributed MPC algorithm using a simulation example. Section 6 summarizes the work presented in this paper.

The notations used in this paper are standard. The superscript "T" stands for matrix transposition; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; for a matrix P, $P > 0$ ($P \geq 0$) means that P is real symmetric positive definite (positive semidefinite); for a vector $x \in \mathbb{R}^n$, $\|x\|_2$ is its Euclidean norm and $\|x\|_P = \sqrt{x^{\mathrm{T}}Px}$ is its P-norm.

2. Problem formulation and preliminaries

Consider a system consisting of S decoupled nonlinear subsystems $\mathcal{A}_i$, $i = 1, 2, \ldots, S$. The dynamics of each subsystem $\mathcal{A}_i$ are

$$\dot{x}_i(t) = f_i(x_i(t), u_i(t)) + \omega_i(t),\ \ x_i(t_0) = x_i^0,\ \ t \geq 0, \tag{1}$$

where $x_i(t) \in \mathbb{R}^{n_i}$ is the subsystem state, $u_i(t) \in \mathbb{R}^{m_i}$ is the constrained control input and $\omega_i(t) \in \mathbb{W}_i \subset \mathbb{R}^{n_i}$ is the bounded additive disturbance satisfying $\rho_{\max} = \max_t \|\omega_i(t)\|_M$, where $M > 0$ will be defined in Lemma 1. f_i is continuous. The sets $\mathcal{U}_i$ and $\mathcal{W}_i$, in which the control signal and the additive disturbance are bounded, include the respective origins in their interiors.

The nominal dynamics of the subsystem $\mathcal{A}_i$ in (1) are

$$\dot{\bar{x}}_i(t) = f_i(\bar{x}_i(t), \bar{u}_i(t)). \tag{2}$$

Let the origin be an equilibrium point of the nominal dynamics in (2). The following standard assumption [15,29–31] characterizes the dynamical property of each subsystem around the origin.

Assumption 1. The linearization around the origin of the nominal dynamics in (2) is stabilizable, i.e., matrices K_i with appropriate dimensions exist such that $(A_i + B_iK_i)$ are stable for $i = 1, 2, \ldots, S$, where $A_i = \frac{\partial f_i}{\partial \bar{x}_i}|_{(0,0)}$ and $B_i = \frac{\partial f_i}{\partial \bar{u}_i}|_{(0,0)}$.

Assumption 1 indicates that, around the origin, a stabilizing controller exists for each nominal dynamics in (2). Specifically, it leads to the following lemma [32].

Lemma 1. *A control positively invariant set $\Omega_{\alpha_i} = \{x : \|x\|_M \leq \alpha_i, M > 0\}$ can be constructed for the dynamics in (2) with a feasible control signal $\bar{u}_i(t) = K_i\bar{x}_i(t)$.*

Instead of using the Lipschitz continuity property of the dynamics in (1) and (2) as in [15,20,33], this paper exploits the contraction property (Definition 1 in Appendix) satisfied by a class of nonlinear systems. The following assumption characterizes the class of nonlinear systems for which this paper designs a novel robust distributed MPC strategy.

Assumption 2. For the nominal dynamics in (2) with the initial state $x_i(t_0)$, there exist a feasible control signal $\bar{u}_i(t_0 + \tau; t_0)$, $\tau \in [0, T]$ and a tube-like region $\Theta_l =: \{\bar{x} : \|\bar{x} - \bar{x}_i(s; t_0)\|_M \leq l, \exists s \in [t_0, t_0 + T]\}$ such that: (1) the control signal steers the nominal dynamics in (2) to the positively invariant set Ω_{α_i} along the trajectory $\dot{\bar{x}}_i(t_0 + \tau; t_0) = f_i(\bar{x}_i(t_0 + \tau; t_0), \bar{u}_i(t_0 + \tau; t_0))$, $\bar{x}_i(t_0; t_0) = x_i(t_0)$; (2) Denote $\frac{\partial f_i}{\partial \tilde{x}_i(s;t_0)} := \frac{\partial f_i}{\partial \tilde{x}_i(s;t_0)}|_{(\tilde{x}_i(s;t_0),\bar{u}_i(s;t_0))}$, where $\tilde{x}_i(s; t_0)$ satisfies $\|\tilde{x}_i(s; t_0) - \bar{x}_i(s; t_0)\|_M \leq l$. The following inequality holds

$$\frac{\partial f_i}{\partial \tilde{x}_i(s; t_0)}^{\mathrm{T}} M + M \frac{\partial f_i}{\partial \tilde{x}_i(s; t_0)} \leq -\beta M,\ \ s \in [t_0, t_0 + T] \tag{3}$$

where $\beta > 0$, $l > 0$, and Ω_{α_i}, M are given in Lemma 1.

Assumption 2 indicates that, around the state trajectory $\bar{x}_i(t_0 + \tau; t_0)$, $\tau \in [0, T]$, there exists a tube-like region within which Inequality (3) holds. Furthermore, if Assumption 2 holds, then based on [22] (see Theorem 3 in Appendix), any nominal state trajectory starting in the tube Θ_l remains in this tube and converges exponentially to $\bar{x}_i(t_0 + \tau; t_0)$.