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When is a linear multi-modal system disturbance decoupled?

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ABSTRACT

In this paper we study the question under which conditions a linear multi-modal system is disturbance decoupled. We establish necessary and sufficient geometric conditions from which the existing results on switched linear systems and conewise linear systems can be recovered as special cases. Also, we apply these conditions to a class of linear complementarity systems in order to obtain a more crisp characterization.

1. Introduction

Annihilating or reducing the effects of disturbances is of major importance virtually in every real-life control problem. Designing feedback laws that decouple the disturbances from a certain to-be-controlled output constitute the well-known disturbance decoupling problem. The study of this problem for linear systems led to the development of geometric control theory [1–3] which provided solutions to numerous control problems as well as a deep understanding of the dynamics of linear systems [4–6] and (smooth) nonlinear systems [7,8].

Geometric approach to linear systems is among many fields on which Jan Willems made a major impact. Examples are his seminal work on almost invariant subspaces [9,10], (almost) disturbance decoupling [11,12], and application of geometric ideas to singular optimal control problems [13].

In this paper, we focus on a class of hybrid dynamical systems and provide necessary and sufficient geometric conditions under which the disturbances are decoupled from pre-specified to-be-controlled outputs. Within the hybrid systems, the results on disturbance decoupling problem are limited to jumping hybrid systems [14], continuous piecewise affine systems [15], a class of linear complementarity systems [16], and switched linear systems [17–22]. The results presented in these papers very much resemble those for the linear systems although their derivation is much harder, in particular, in the presence of state-dependent switching [15,16].

Within this paper, we first lay a general framework that contains a particular class of switched linear systems, linear complementarity systems, and the so-called conewise linear systems (as well as combinations of these) as particular cases. Later, we investigate necessary and sufficient conditions for disturbances to be decoupled within this general framework. In addition, we show that all the existing results for the hybrid systems mentioned above can be recovered from the presented results as special cases.

Furthermore, we study a class of linear complementarity systems in detail in order to find novel necessary and sufficient conditions for this kind of systems to be disturbance decoupled.

The organization of the paper is as follows. We start with some preliminaries and notation in Section 2. We introduce general linear multi-modal systems in Section 3 and discuss a few special cases. In Section 4 we give the definition of the property of being disturbance decoupled for a linear multi-modal system, and present our main results. In Theorem 8 we give a necessary condition and in Theorem 9 a sufficient condition for a linear multi-modal system to be disturbance decoupled. In Corollary 11 we show that in some cases the necessary condition and the sufficient condition coincide. We apply these results in Section 5 to the special cases that we introduced in Section 3. For one type of linear complementarity systems, this will lead to novel results, stated in Theorem 14. The paper closes with the conclusions and discussions of possible future work in Section 6.

2. Preliminaries and notation

For a vector v we write n_v as its dimension. For two vectors v and w, we let $\mathrm{col}(v, w)$ denote the column vector that is obtained by stacking v and w. A cone is a subset of a vector space that

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is closed under multiplication by positive scalars. The relative interior of a set $S \subseteq \mathbb{R}^n$ is defined as

$$\mathrm{rint}(S) := \{x \in S : \exists \varepsilon > 0, N_\varepsilon(x) \cap \mathrm{aff}(S) \subseteq S\},$$

where $N_\varepsilon(x)$ is an ε-neighborhood of x and $\mathrm{aff}(S)$ is the affine hull of S.

Consider the linear system $\Sigma = \Sigma(A, B, C, D)$ given by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{1a}$$

$$y(t) = Cx(t) + Du(t) \tag{1b}$$

where the input u, state x, and output y have dimensions n_u, n_x and n_y respectively. In what follows, we quickly introduce some of the fundamental notions of geometric control theory of linear systems for the sake of completeness. We refer to [16] for more details.

The *controllable subspace* of Σ is the smallest A-invariant subspace containing $\mathrm{im}\, B$. We will denote it by

$$\langle A \mid \mathrm{im}\, B\rangle := \mathrm{im}\ B + A\,\mathrm{im}\ B + \cdots + A^{n_x-1}\,\mathrm{im}\, B. \tag{2}$$

A subspace $\mathcal{T} \subseteq \mathbb{R}^n$ is called an *input containing conditioned invariant subspace* of Σ if

$$\begin{bmatrix} A & B \end{bmatrix}\left((\mathcal{T} \times \mathbb{R}^{n_u}) \cap \ker \begin{bmatrix} C & D \end{bmatrix}\right) \subseteq \mathcal{T}.$$

It is well-known that a subspace $\mathcal{T}$ is an input containing conditioned invariant subspace if and only if there exists a matrix $L \in \mathbb{R}^{n \times p}$ such that

$$(A + LC)\mathcal{T} \subseteq \mathcal{T} \quad \text{and} \quad \mathrm{im}\,(B + LD) \subseteq \mathcal{T}. \tag{3}$$

The *strongly reachable subspace* of Σ is the smallest (with respect to the subspace inclusion) input containing conditioned invariant subspace and will be denoted by $\mathcal{T}^*(\Sigma)$.

It follows from (3) with the choice of $L = 0$ that the controllable subspace is an input containing conditioned invariant subspace. Hence, we have

$$\mathcal{T}^*(\Sigma) \subseteq \langle A \mid \mathrm{im}\ B\rangle. \tag{4}$$

Let K and L be $m \times n$ and $n \times p$ matrices, respectively. Also let $\Sigma_{K,L}$ denote the system $\Sigma(A+BK+LC+LDK, B+LD, C+DK, D)$. It can easily be verified that

$$\mathcal{T}^*(\Sigma_{K,L}) = \mathcal{T}^*(\Sigma). \tag{5}$$

A subspace $\mathcal{V} \subseteq \mathbb{R}^n$ is called an *output nulling controlled invariant subspace* of Σ if

$$\begin{bmatrix} A \\ C \end{bmatrix}\mathcal{V} \subseteq \left(\mathcal{V} \times \{0\}\right) + \mathrm{im} \begin{bmatrix} B \\ D \end{bmatrix}.$$

The *weakly unobservable subspace* of Σ is the largest (with respect to the subspace inclusion) output nulling controlled invariant subspace and will be denoted by $\mathcal{V}^*(\Sigma)$.

It is well-known that the transfer matrix $D + C(sI - A)^{-1}B$ is *right invertible* as a rational matrix if and only if

$$\mathcal{V}^*(\Sigma) + \mathcal{T}^*(\Sigma) = \mathbb{R}^{n_x} \quad \text{and} \quad \mathrm{rank} \begin{bmatrix} C & D \end{bmatrix} = n_y.$$

Straightforward linear algebra arguments show that these conditions are equivalent to

$$\mathrm{im}\ D + C\mathcal{T}^*(\Sigma) = \mathbb{R}^{n_y}. \tag{6}$$

3. Linear multi-modal systems

In this paper we consider linear multi-modal systems given by the differential inclusion

$$\dot{x}(t) \in Ax(t) + Ed(t) + \Phi(y(t)) \tag{7a}$$

$$y(t) = Cx(t) + Fd(t) \tag{7b}$$

$$z(t) = Jx(t) \tag{7c}$$

where x is the state, d is the disturbance, y is the selection output, z is the to-be-controlled output, A, C, E, F and J are matrices of appropriate sizes and $\Phi : \mathbb{R}^{n_y} \rightarrow \mathbb{R}^{n_x}$ is a set-valued map satisfying

$$\Phi(y) = \{M_i y \mid i \in \mathcal{I}, y \in \mathcal{Y}_i\},$$

where $\mathcal{I}$ is a finite index set, $\{\mathcal{Y}_i\}_{i \in \mathcal{I}}$ is a collection of cones in $\mathbb{R}^{n_y}$, and $\{M_i\}_{i\in\mathcal{I}}$ is a collection of $n_x \times n_y$ matrices. The cones $\mathcal{Y}_i$ are not necessarily *solid* (i.e., n_y-dimensional). Moreover, the cones may overlap and their union does not have to be equal to $\mathbb{R}^{n_y}$. Without loss of generality we can assume that the matrix $\begin{bmatrix} C & F \end{bmatrix}$ has full row rank.

Let $T > 0$. For a given initial state x_0 and an integrable disturbance d we call an absolutely continuous function $x : [0, T) \rightarrow \mathbb{R}^{n_x}$ a solution on $[0, T)$ of system (7) if (7a) holds for almost all $t \in [0, T)$ and $x(0) = x_0$. If $T = +\infty$, we simply say that x is a (complete) solution of (7). In the sequel, we will allow multiple solutions for a given initial state and disturbance but make two assumptions regarding the existence of solutions.

The first assumption we make is that *local* solutions can be extended to complete solutions.

Assumption 1. If the system (7) admits a local solution x_T on $[0, T)$ for some $T > 0$, initial state x_0, and disturbance d, then there exists a complete solution x for the same initial state x_0 and disturbance satisfying $x(t) = x_T(t)$ for all $t \in [0, T)$.

The second assumption regarding the existence of solutions requires that the disturbances are not restricted by the dynamics of the system.

Assumption 2. If the system (7) admits a complete solution for some initial state and disturbance, then there exists a complete solution for the same initial state and for any disturbance.

Later on, we will elaborate on these assumptions when we discuss specific classes of systems that fall into the framework of (7).

We say that an initial state is *feasible* if for all locally integrable disturbances d there exists a complete solution of (7). The set of all feasible states will be denoted by $\mathcal{X}_0$.

To simplify the notation, we define

$$A_i = A + M_i C, \qquad E_i = E + M_i F \tag{8}$$

and rewrite system (7) as

$$\dot{x}(t) \in \{A_i x(t) + E_i d(t) \mid y(t) \in \mathcal{Y}_i\} \tag{9a}$$

$$y(t) = Cx(t) + Fd(t) \tag{9b}$$

$$z(t) = Jx(t). \tag{9c}$$

We will work mainly with this form of the linear multi-modal system in the rest of the paper.

Examples of systems that fall into this framework include switched linear systems, conewise linear systems, and linear complementarity problems, which we discuss next.

Example 3 (*Switched Linear Systems*). We consider the following particular class of linear switched systems

$$\dot{x}(t) = A_{\sigma(t)}x(t) + E_{\sigma(t)}d(t) \tag{10a}$$

$$z(t) = Jx(t), \tag{10b}$$

where σ is a switching signal from $\mathbb{R}_{\geqslant 0}$ to a finite index set $\mathcal{I}$. By taking $A = A_j$ and $E = E_j$ for some $j \in \mathcal{I}$, we can rewrite (10) in the form of a multi-modal system as

$$\dot{x}(t) \in Ax(t) + Ed(t) + \Phi(y) \tag{11a}$$

$$y(t) = \mathrm{col}(x(t), d(t)), \tag{11b}$$

$$z(t) = Jx(t), \tag{11c}$$

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