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Output tracking for a class of nonlinear systems with mismatched uncertainties by active disturbance rejection control[☆]

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ABSTRACT

In this paper, we apply active disturbance rejection control, an emerging control technology, to achieve practical output tracking for a class of nonlinear systems in the presence of vast matched and mismatched uncertainties including unknown internal system dynamic uncertainty, external disturbance, and uncertainty caused by the deviation of control parameter from its nominal value. The total disturbance influencing the performance of controlled output is refined first and then estimated by an extended state observer (ESO). Under the assumption that the inverse dynamics of the uncertain systems are bounded-input-bounded-state stable, a constant high gain ESO based output feedback is constructed to guarantee that the state is bounded and the output tracks practically a given reference signal. A time-varying gain ESO is also discussed to reduce the peaking value near the initial stages of ESO caused by constant high gain. Numerical simulations are presented to demonstrate the effectiveness of the proposed output-feedback control scheme.

1. Introduction

Dealing with uncertainty is a key issue in modern control theory since the inception of the modern control theory in the later years of 1950s, seeded in [1] where it is stated that the control operation "must not be influenced by internal and external disturbances" [1, p. 228]. Many methods have been developed since 1970s to cope with uncertainty like robust control [2], high-gain control [3], internal model principle [4–6], adaptive control [7], among them, the robust control is a remarkable paradigm shift in modern control theory [8]. However, most of these control methods are based on the worst case scenario, which makes the controller designed rather conservative. Very different strategy is the estimation/cancellation strategy which can be found in adaptive control and internal model principle for dealing with almost known uncertainty.

The idea of estimation/cancellation strategy is carried forward by known as active disturbance rejection control (ADRC) to this day, proposed by Han [9] in later 1980s. ADRC lumps vast uncertainty into "total disturbance" which may include the coupling between unknown system dynamics, external disturbance, the superadded unknown part of control input, or even if whatever the part of hardly to be dealt with by practitioner. This spans significantly the concept of "disturbance". The key idea of ADRC is that the "total disturbance", as a signal of time, no matter it is state-dependent or free, time invariant or variant, linear or nonlinear, is reflected entirely in the observable measured output and can hence be estimated. The estimation of total disturbance as well as state is realized through a device called extended state observer (ESO). The "total disturbance" is then compensated in the feedback loop by its estimate. This estimation/cancellation nature of ADRC makes it capable of eliminating the uncertainties before it causes negative effect to control plant.

In the last few years, some progresses have been made leading to theoretical foundation of ADRC in [10–21], among many others. The convergence of linear ESO, which is proposed in [22] in terms of bandwidth, is discussed in [17,21]. Linear ADRC has been addressed for different systems like those for control and disturbance unmatched systems [14], lower triangular systems [16], and the system without known nominal control parameter [13]. In addition, linear ADRC with adaptive gain ESO is investigated in [15]. The convergence of nonlinear ADRC for SISO systems is proved firstly in [10] and extended secondly to MIMO system in [11], and then to lower triangular system in [19,20], and to system with stochastic disturbance in [12]. The convergence of nonlinear ADRC with time-varying gain ESO is discussed in [18].

On the other hand, most of aforementioned literatures mainly address ADRC for essential-integral-chain systems with matched

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uncertainties, and very little attention is paid to systems with uncertainties that are not in the control channel. Actually, systems with non-integral chain form and mismatched uncertainties are more general and widely exist in practical engineering systems. For example, in flight control systems, the lumped disturbance torques caused by un-modeled dynamics, external winds, parameter perturbations, etc., always influence the states directly but not through the input channels [23]. To this end, a generalized ESO based control approach was proposed for general systems with mismatched uncertainties and non-integral chain form in [14], whose feasibility and validity are mainly demonstrated by numerical and application design examples. The stability analysis in [14] is addressed under strong conditions that the mismatched uncertainties are bounded, independent of states, and have constant values in steady state. In addition, [16] addresses ADRC to achieve desired performance for a class of MIMO lower-triangular nonlinear systems with vast mismatched uncertainties by state feedback.

In this paper, we address ADRC approach to output tracking for lower triangular nonlinear systems with more general mismatched uncertainties without restrictive conditions like that in [14], and output feedback control instead of state feedback like that in [16] is concerned.

The remainder of the paper is organized as follows. In the next section, Section 2, the total disturbance that affects the output of the system is first determined. We then design a constant high gain ESO to estimate the total disturbance in real time, and finally a constant gain ESO based output feedback control is designed. It is shown that the output feedback control law can guarantee the boundedness of the state of the closed-loop and the output tracks practically a given reference signal. In Section 3, a time-varying gain ESO is briefly discussed to reduce the peaking value near the initial stage of ESO caused by constant high gain. Finally, in Section 4, we present some numerical simulations for illustration of the performance of closed-loop and the peaking value reduction.

2. ADRC with constant gain ESO

In this paper, we consider output tracking problem for a class of uncertain nonlinear systems in lower triangular form described as follows:

$$\begin{cases} \dot{x}_1(t) = x_2(t) + h_1(x_1(t), \zeta(t), w(t)), \\ \dot{x}_2(t) = x_3(t) + h_2(x_1(t), x_2(t), \zeta(t), w(t)), \\ \qquad \vdots \\ \dot{x}_n(t) = f(t, x(t), \zeta(t), w(t)) + b(t, w(t))u(t), \\ \dot{\zeta}(t) = f_0(x_1(t), \zeta(t), w(t)), \\ y(t) = x_1(t), \end{cases} \tag{2.1}$$

where $x(t) = (x_1(t), \dots, x_n(t)) \in \mathbb{R}^n$ and $\zeta(t) \in \mathbb{R}^m$ are system states with $\zeta(t)$ the zero dynamics, $y(t) \in \mathbb{R}$ the measured output, $u(t) \in \mathbb{R}$ the control input, $w(t) \in \mathbb{R}$ the unknown exogenous signal or external disturbance, and $b(\cdot) : [0, \infty) \times \mathbb{R} \to \mathbb{R}$ the control coefficient which is not exactly known yet has a nominal value $b_0(t)$ sufficiently closed to $b(\cdot)$. The functions $h_i(\cdot) : \mathbb{R}^{i+m+1} \to \mathbb{R}$ $(i = 1, 2, \dots, n-1)$, $f(\cdot) : [0, \infty) \times \mathbb{R}^{n+m+1} \to \mathbb{R}$, and $f_0(\cdot) : \mathbb{R}^{m+2} \to \mathbb{R}^m$ are generally unknown. So system (2.1) allows nonlinear uncertainties in all channels, not only in the control channel as considered in existing literature. As indicated in [9], the key point in application of ADRC is how to reformulate the problem by lumping various known and unknown quantities that affect the system performance into “total disturbance”. This is a crucial step in transforming a complex control problem into a simple one. A natural requirement is that the total disturbance can be identified from the measured output. The idea of addressing ADRC for deterministic systems with mismatched uncertainties is originated from [24] where no theoretical proof is given. Motivated by [24], we set

$$\begin{cases} \bar{x}_1(t) = x_1(t), \\ \bar{x}_2(t) = x_2(t) + h_1(x_1(t), \zeta(t), w(t)), \\ \bar{x}_i(t) = x_i(t) + \displaystyle\sum_{j=1}^{i-1} h_{i-j}^{(j-1)}(x_1(t), \dots, x_{i-j}(t), \zeta(t), w(t)), \\ \quad 3 \le i \le n, \end{cases} \tag{2.2}$$

where $h_{i-j}^{(j-1)}(\cdot)$ represents the $(j-1)$-th derivative of $h_{i-j}(\cdot)$ with respect to time variable t. A straightforward computation shows that for all $i \ge 3$,

$$\begin{aligned} &\sum_{j=1}^{i-1} h_{i-j}^{(j-1)}(x_1(t), \dots, x_{i-j}(t), \zeta(t), w(t)) \\ &\quad = f_{i-1}(x_1(t), \dots, x_{i-1}(t), \zeta(t), w(t), \dots, w^{(i-2)}(t)) \end{aligned} \tag{2.3}$$

for some continuous function $f_{i-1}(\cdot)$ when $h_i(\cdot) \in C^{n+1-i}(\mathbb{R}^{i+m+1}; \mathbb{R})$, $f_0(\cdot) \in C^{n-1}(\mathbb{R}^{m+2}; \mathbb{R}^m)$, and $w(\cdot)$ is n-th continuously differentiable with respect to time variable t supposed in Assumption (A1) later. Equivalently, there are continuous functions $\phi_i(\cdot)$ $(i = 1, 2, \dots, n-1)$ such that

$$\begin{cases} x_1(t) = \bar{x}_1(t), \\ x_2(t) = \bar{x}_2(t) - h_1(\bar{x}_1(t), \zeta(t), w(t)) \triangleq \phi_1(\bar{x}_1(t), \bar{x}_2(t), \zeta(t), w(t)), \\ \qquad \vdots \\ x_n(t) \triangleq \phi_{n-1}(\bar{x}_1(t), \dots, \bar{x}_n(t), \zeta(t), w(t), \dots, w^{(n-2)}(t)). \end{cases} \tag{2.4}$$

Under the new state variable $\bar{x}(t) = (\bar{x}_1(t), \dots, \bar{x}_n(t))$, the x-subsystem of (2.1) is transformed into an essentially integral-chain system with control matched total disturbance as follows:

$$\begin{cases} \dot{\bar{x}}_1(t) = \bar{x}_2(t), \\ \dot{\bar{x}}_2(t) = \bar{x}_3(t), \\ \qquad \vdots \\ \dot{\bar{x}}_n(t) = \bar{x}_{n+1}(t) + b_0(t)u(t), \\ y(t) = x_1(t), \end{cases} \tag{2.5}$$

where the “total disturbance” $\bar{x}_{n+1}(t)$ is given by

$$\begin{aligned} \bar{x}_{n+1}(t) &= f(t, x(t), \zeta(t), w(t)) + (b(t, w(t)) - b_0(t))u(t) \\ &\quad + \sum_{j=1}^{n-1} h_{n-j}^{(j)}(x_1(t), \dots, x_{n-j}(t), \zeta(t), w(t)). \end{aligned} \tag{2.6}$$

Our control objective is to design an output feedback control so that for all initial states in given compact set, the state $(x(t), \zeta(t))$ is bounded and the output $y(t)$ tracks practically a given, bounded, reference signal $r(t)$ whose derivatives $\dot{r}(t), \ddot{r}(t), \dots, r^{(n+1)}(t)$ are supposed to be bounded. Let

$$(r_1(t), r_2(t), \dots, r_{n+1}(t)) = (r(t), \dot{r}(t), \dots, r^{(n)}(t)). \tag{2.7}$$

The key step of ADRC is to design an extended state observer (ESO) for x-subsystem of (2.1) to estimate the total disturbance, which can be reduced to design ESO for system (2.5). This is because these two systems have the same controlled output and there exist continuous invertible transformations between x-variable and $\bar{x}$-variable as shown in (2.2) and (2.4). The simplest ESO is linear one which takes advantage of simple turning parameter but it may bring the peaking value problem, slow convergence, and many other problems contrast to fast tracking and small peaking value indicated numerically in [25] by nonlinear ESO. By taking these points into account, we introduce a nonlinear ESO [10,19,20] with constant high gain tuning parameter for system