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Implications of dissipativity on stability of economic model predictive control—The indefinite linear quadratic case

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ABSTRACT

In contrast to the conventional model predictive control (MPC) approach to control of a given system where a positive–definite objective function is employed, economic MPC employs a generic cost which is related to the 'economics' of the process as the objective function in the regulation layer. Often, stability proofs of the closed-loop system are based on strict-dissipativity of the system with respect to this objective function. In this paper, we focus on linear systems with indefinite quadratic costs. We show that while strict–dissipativity guarantees stability of the closed–loop system, it is not required. Hence we formulate a necessary and sufficient condition that guarantees asymptotic stability of the closed loop system. This condition comes down to the existence of two distinct storage functions for which the system is dissipative.

1. Introduction

In conventional model predictive control (MPC) and linear quadratic regulator (LQR) approaches to the control of a given system, the optimal steady-state of the system with respect to the 'economic' cost is first computed then deviations from this optimal steady-state are minimized using a dynamic regulation layer. This dynamic regulation layer is usually referred to as the advanced process control layer (which uses MPC or LQR) . The cost function employed in the MPC plays a very important role for the stability of the closed-loop system. It has been established in both the MPC and LQR literature that under nominal operation, stability of the closed loop system (using MPC or LQR) can be guaranteed provided the system is stabilizable, the cost function is positive-definite and a suitable terminal cost is used [1–3].

Economic MPC (e–MPC) employs a different approach to predictive control. The 'economic' cost is used directly in the dynamic regulation layer. Since this cost is generic and not guaranteed to be positive-definite as in conventional MPC, proof of stability cannot be based on this property of the cost function. Strict-dissipativity, a property of the system with respect to the cost function, has often been used to overcome this limitation. This strict-dissipativity condition plays a central role in the analysis of economic MPC. The sufficiency of strict-dissipativity condition for optimality of steady-state operation was established in [4–6] while [4,7] further showed that this same strict-dissipativity condition guarantees stability of the closed-loop system obtained using the economic cost in the dynamic regulation layer. Thus, optimality of steady-state operation and stability of the dynamic regulation to this steady-state are both guaranteed by the same strict-dissipativity condition. It has also been proven that a less strict condition, dissipativity, also guarantees optimality of steady-state operation and is close to being necessary for steady state operation to be optimal (under some additional controllability assumption) [5,6]. Simulations however show that in some cases, dissipativity (and not strict-dissipativity) appears to be sufficient for stability in the dynamic regulation layer. It is therefore of interest to characterize the cases when dissipativity is sufficient for the stability of the closed loop system.

This work focuses on linear system with purely quadratic costs, without definiteness restrictions. Such purely quadratic costs arise, for instance, in ocean wave energy conversion where the objective is to maximize the absorbed power. The power extracted can be modelled as a product of the damping coefficient (constant factor), velocity of the buoy (state, $x(k)$) and the active forcing element ($u(k)$) [8–10]. This leads to an indefinite quadratic formulation of the economic objective function. Such indefinite quadratic costs are also encountered in process control where the economic objective of an isothermal continuous stirred-tank reactor is to maximize the production rate (of one of the outputs), modelled as a product of the concentration of the output (state) and the flow rate through the reactor (input) [6,11,12]. Another scenario is when there are conflicting objectives, for instance, minimizing the control effort (energy input) of steering an aircraft while trying to maximize the cruise speed (kinetic energy) of the aircraft [13,14].

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This can be achieved by minimizing a quadratic cost using a negative weighting ($Q < 0$) on the states and positive weighting ($R > 0$) on the inputs, which once again leads to an indefinite cost formulation.

Hence, we seek to establish conditions under which the optimal economic controller in such cases as these are also an asymptotically stabilizing controller for the system. By creating a link between dissipativity, existence of a control Lyapunov function for the closed–loop system and the optimal cost function, we establish a necessary and sufficient condition for stability of the closed–loop system based on dissipativity of the system with respect to the stage cost.

This paper is organized as follows: Section 2 investigates the link between dissipativity and the existence of a stabilizing optimal controller; Section 3 presents a discussion on the sufficiency of strict-dissipativity for closed–loop asymptotic stability and possibility of dissipativity to guarantee stability; Section 4 characterizes the necessary and sufficient condition under which dissipativity guarantees asymptotic stability; Section 5 presents some numerical examples and Section 6 concludes the paper.

Nomenclature.

The symbols $\mathbb{R}$ and $\mathbb{I}_{0:N-1}$ denote the sets of real numbers and $\{0, 1, \dots .N-1\}$ respectively. We denote $\rho(C)$ as the spectral radius of C, $C^{\dagger}$ as the Moore–Penrose generalized inverse of C and $Ker(C)$ as the Kernel of C.

2. On dissipativity and existence of stabilizing optimal controller

In this section, the necessity and sufficiency of dissipativity (of a system with respect to the given objective function) for the optimal controller to be stabilizing is investigated. To ease checking of the dissipativity condition, we focus on linear systems with quadratic cost functions without any restriction on the definiteness of the cost.

Given the linear discrete time system

$$x_{k+1} = Ax_k + Bu_k \tag{1}$$

and the stage cost

$$l(x_k, u_k) = x_k^T Q x_k + u_k^T R u_k + 2x_k^T S u_k, \tag{2}$$

we consider the following finite-horizon optimization problem

$$\begin{aligned} &\min_{\mathbf{u}} J_N(x, \mathbf{u}) \triangleq x_N^T P_N x_N + \sum_{k=0}^{N-1} l(x_k, u_k) \\ &\text{subject to } \begin{cases} x_{k+1} = Ax_k + Bu_k, \ k \in \mathbb{I}_{0:N-1} \\ x_k \in \mathbb{X}, u_k \in \mathbb{U}, \ k \in \mathbb{I}_{0:N-1} \\ x_0 = x(i), x_N \in \mathbb{X}_F \end{cases} \end{aligned} \tag{3}$$

where $\mathbb{X} \subseteq \mathbb{R}^n$, $\mathbb{U} \subset \mathbb{R}^m$ and $\mathbb{X}_F \subseteq \mathbb{X}$ is a compact terminal region chosen to ensure recursive feasibility. $x(i)$ is the measured state at time i and x_k the predicted value of state x at time step $i + k$ given measurement $x(i)$. Without loss of generality, the optimal steady–state, defined as the solution to the optimization problem

$$l(x_s, u_s) = \min_{x,u} l(x_k, u_k) \text{ s.t.}\{x_k = Ax_k + Bu_k, \ x_k \in \mathbb{X}, u_k \in \mathbb{U}\} \tag{4}$$

is assumed to be the origin, unique and lies in the interior of the constraint sets. Moreover, there is no restriction on the definiteness of the matrix $\begin{bmatrix} Q & S \\ S^T & R \end{bmatrix}$ and the terminal cost, P_N.

The optimization problem (3) is repeatedly minimized over the horizon N in a moving horizon manner. At each iteration i, (3) yields the optimal input sequence $\mathbf{u}^* = \{u_0^*, u_1^*, \dots, u_{N-1}^*\}$. The first element of the sequence is applied to the plant yielding the control law $u(i) = u_0^*$. We refer to this generated implicit control law as $u_0^* = -K_N(x(i))$ and the closed loop system is

$$x(i + 1) = Ax(i) - BK_N(x(i)). \tag{5}$$

If x_0 is in the set of states that can be admissibly steered to the origin in N steps (or less) and P_N is chosen such that it solves the Discrete Algebraic Riccati Equation (DARE)

$$A^T P_N A - P_N + Q - (S + A^T P_N B)K_N = 0 \tag{6}$$

where $K_N = (R + B^T P_N B)^{\dagger}(S^T + B^T P_N A)$ and a solution to (6) is assumed to exist, then the terminal cost is the same as the optimal linear quadratic cost, and hence the cost in (3) is equivalent to an infinite-horizon cost [15–17]. Thus the control law beyond the horizon becomes the linear law $u_0^* = -K_N x(i)$ and stability depends on the stabilizing properties of this feedback control law. The closed loop system (4) is thus asymptotically stable if $\rho(A - BK_N) < 1$ and marginally stable if $\rho(A - BK_N) \leq 1$where the eigenvalues with unit modulus have equal algebraic and geometric multiplicity. P_N is said to be the stabilizing solution for (DARE) (6) if P_N satisfies (6) and $\rho(A-BK_N) < 1$. Except where otherwise stated, it is assumed that (6) holds with the terminal cost P_N used in (3).

Assumption 2.1.

- (A, B) is stabilizable.
- $x_0 \in \mathbb{X}^0$ where $\mathbb{X}^0$ is the set of states that can be admissibly steered to $\mathbb{X}_F$ in N steps (or less).

Definition 1. System (1) is said to be dissipative [4,18,19] with respect to the stage cost (2) if there exists a quadratic storage function $x_k^T P_d x_k$ such that for all $k \geq 0$,

$$x_{k+1}^T P_d x_{k+1} - x_k^T P_d x_k \leq l(x_k, u_k). \tag{7}$$

Equation (7) is equivalent to the existence of a $P_d = P_d^T$ such that the dissipativity Linear Matrix Inequality (d–LMI)

$$\begin{bmatrix} A^T P_d A - P_d - Q & A^T P_d B - S \\ B^T P_d A - S^T & B^T P_d B - R \end{bmatrix} \leq 0 \tag{8}$$

is feasible. If (7) and (8) hold with strict inequality, the system is said to be strictly-dissipative.

We note that while the original definition of dissipativity [20,21] required P_d to be non-negative, recent definitions and usage in economic MPC (e–MPC) have removed this restriction [4,7,22,23].

Consider the DARE (6). Taking the Schur complement of (6) yields the LMI

$$\begin{bmatrix} A^T PA - P + Q & A^T PB + S \\ B^T PA + S^T & B^T PB + R \end{bmatrix} \geq 0 \tag{9}$$

with P_N being the maximum P for which (9) holds. This equivalence was established in [24] where it was shown that

- The set of strongly rank minimizing solutions of the discrete LMI coincide with the set of real symmetric solutions of the DARE associated with the LMI.
- Stabilizing and semi-stabilizing rank minimizing solutions of the discrete LMI are also strongly rank minimizing.
- The semi-stabilizing rank minimizing solution of the LMI, if it exists, is the largest solution of the LMI.

We also note that the existence of P that ensures feasibility of (9) is a necessary condition for the existence of P_N that