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Consensus protocol for multiple delta operator systems ☆

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ABSTRACT

This paper considers the consensus problem for multiple delta operator systems. The delta operator system is used for describing continuous time processes at rapid rate sampling. Moreover, it has been known to have a better model description than that of shift-operator-based approach. In this work, the consensus problem is first solved for multiple delta operator systems by using a distributed protocol. To achieve consensus, several important sufficient conditions are proposed for the fast sampling systems based on the parameters of sampling period and consensus strength. In addition, two distributed consensus protocols are constructed for both leaderless multiple delta operator systems and leader–follower multiple delta operator systems. Finally, the effectiveness of the theoretical analysis is illustrated by applying distributed consensus protocol to multi-agent systems.

1. Introduction

Multi-agent systems are the interconnection of multiple small agents for performing cooperative missions. The structure in the multi-agent systems can be described as a graph, in which edges represent transmission lines and nodes represent small agents. Many real world scenarios could benefit from having multiple small nodes with each node operated in a cooperative way [1–12].

It is well known that multi-agent systems consensus is one of the most important behaviors which attracts considerable research efforts; see [1,2]. In [1], the linear Vicsek model was theoretically studied for coordination problem by using algebraic graph theory. In [2,3], sufficient conditions were derived for consensus of multi-agent systems under the condition that the network is jointly connected frequently enough as the network evolves with time. Some necessary and sufficient conditions have been examined for second-order consensus of multi-agent systems in [4]. In [5], the partial information based tracking problem was investigated for two-target sensor networks. The work of [6] studied practical consensus problem for multi-agent systems under uniform quantized delayed input signals, and the authors proved that the consensus converges to different consensus sets with respect to different values of quantization precisions. In [12], the distributed adaptive consensus protocols were proposed to overcome the fault and uncertainty effect in multi-agent systems. In [11], distributed synchronization criteria for dynamical networks were proposed with stochastic coupling. Nowadays, the development of digital control, signal processing and wide band communications requires fast discrete-time procedures that can operate on data obtained by rapid sampling [13]. Dynamical systems need to process these fast-sampled signals in estimating the communication channels or identifying system dynamics [13], and this will generate a discrete system. However, for a discrete system, as pointed out by [14], the control cannot be obtained from their continuous counterpart by means of simple equivalence. Especially, by using traditional shift operator, the fast sampling period will lead to a lot of unstable phenomena in finite word length computer due to the fact that the poles are located in the boundary [14]. The same problem will appear in considering the discrete multi-agent systems with fast sampling in finite word length computer system. To overcome this problem, delta operator systems are constructed in [15] to replace the traditional shift operator for sampling continuous systems in computers. This motivates us to consider the multiple delta operator systems as a research object for consensus.

Recently, a lot of interesting work related to delta operator system have been published [13,14,16–23]. The robust state feedback control using delta operator approach for linear fractional uncertain discrete-time systems was investigated in [16] based on Lyapunov–Krasovskii functional in delta domain. Using delta operator approach, the robust sliding-mode control for time-delay systems with mismatched parametric uncertainties was discussed in [14]. The results unified some related results of continuous and

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discrete systems into delta operator systems framework. Considering exogenous disturbance, the robust adaptive sliding mode controller was stated for delta operator systems in [17]. The H_∞ filtering problem for norm-bounded uncertain discrete-time systems was investigated by using delta operator approach in [18]. The strictly positive real control problem was presented for delta operator system [24], in which the low frequency strictly positive real controller was stated for the stabilizing the closed-loop delta operator system. In the book [13], the analysis and synthesis of delta operator system were discussed in detail. For stabilizing the closed-loop delta operator system, a gain scheduling controller is constructed for compensating the time delays, and then the optimal controller was also developed in [19].

It is worth noting that although some effective schemes related to delta operator system have been developed, there are still only very few works considering multiple delta operator systems. Furthermore, the distributed protocol for consensus in multiple delta operator systems is another useful, yet unsolved, problem. Consequently, under the delta operator framework, the following problems will be encountered.

(Q1) Can a distributed consensus protocol be designed for multiple delta operator systems?

(Q2) Can a traditional Lyapunov stability method be applied to analyze the consensus in multiple delta operator systems?

(Q3) Can sufficient conditions be found with respect to the delta sampling period and consensus strength under the consensus requirement of multiple delta operator systems?

To answer the above non-trivial questions is a challenging task. Therefore, in this paper, we aim to find a suitable consensus protocol for multiple delta operator system. By using the delta domain analysis for delta operator systems, the consensus problem has been solved successfully. In summary, the main contributions of this paper are as follows:

(1) Compared with existing consensus results, this paper addresses issues in multiple delta operator systems, which are useful for those systems with fast sampling period. This problem has not been discussed in previous literature.

(2) Consensus protocols are proposed for both leaderless and leader–follower multiple delta operator systems, which potentially contributed to both theoretical research and applications in finite word length computer system.

(3) Sufficient conditions with respect to the sampling period and consensus strength are given, which can make multiple delta operator systems achieve consensus.

The remainder of this paper is organized as follows: In Section 2, some preliminaries are proposed. In Section 3, distributed consensus protocol for multiple delta operator systems without leader is given. In Section 4, the consensus protocol for leader–follower multiple delta operator systems is investigated. In Section 5, consensus problem is analyzed to illustrate the effectiveness of the theoretical results. Section 6 concludes this paper.

Notation. $\mathcal{G} = (V, \mathcal{E}, \mathcal{A})$ represents a weighted undirected graph, where V represents vertex set, $\mathcal{E}$ represents an edge set. A path between nodes v_{i_1} and node v_{i_l} in $\mathcal{G}$ is a sequence of edges $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \dots, (v_{i_{l-1}}, v_{i_l})$[25]. A graph $\mathcal{G}$ is connected if for any pair of distinct nodes v_i and v_j in $\mathcal{G}$, there exists a path between v_i and v_j, $(i, j = 1, 2, \dots, N)$. The Laplacian matrix $L = (l_{ij}) \in \mathcal{R}^{N\times N}$ is defined by $l_{ii} = \sum_{j\neq i} w_{ij},\ l_{ij} = -w_{ij}, (i \neq j)$, where w_{ij} is the adjacency element. For a Laplacian matrix, we have the following fact $0 = \lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$, where $\lambda_i, i = 1, 2, \dots, N$ are the eigenvalues of the Laplacian matrix L [3].

2. Preliminaries

In this paper, a group of N delta operator systems indexed by the set $\{1, 2, \dots, N\}$ are considered. Each delta operator system is given by the following form.

$$\delta x_i(t_k) = Ax_i(t_k) + Bu_i(t_k), \tag{1}$$

where $x_i(t) \in \mathcal{R}^n$ is the state variable, $u_i(t) \in \mathcal{R}^p$ is the consensus protocol, $\delta x_i(t_k)$ is the delta operator of $x_i(t_k)$[15], which is defined by

$$\delta x_i(t_k) = \begin{cases} \dfrac{dx_i(t)}{dt}, & h = 0; \\ \dfrac{x_i(t+h) - x_i(t)}{h}, & h \neq 0. \end{cases} \tag{2}$$

Detailed definition of delta operator system can be found in Appendix.

Definition 1 (*[16]*). The conditions for the asymptotic stability of a delta operator system hold:
(a) $V(x(x(t_k))) \geq 0$, with equality if and only if $x(t_k) = 0$,
(b) $\delta V(x(x(t_k))) = [V(x(t_k + h)) - V(x(t_k))]/h < 0$,
where $V(x(x(t_k)))$ is a Lyapunov functional in δ-domain.

One can find the relation of the Lyapunov functional in δ-domain, z-domain, and s-domain in Appendix.

Lemma 1 (*[16]*). *For any of the time functions $x(t_k)$ and $y(t_k)$:*

$$\delta(x(t_k)y(t_k)) = x(t_k)\delta(y(t_k)) + \delta(x(t_k))y(t_k) + h\delta(x(t_k))\delta(y(t_k)). \tag{3}$$

In this work, we consider the problem of designing distributed protocol $u_i, i = 1, 2, \dots, N$, such that the multiple delta operator systems achieve consensus. For the multiple delta operator systems (1), we say that consensus is achieved if, for each delta operator system $i \in \{1, 2, \dots, N\}$, there exists consensus protocol u_i such that

$$\lim_{t\to\infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j = 1, 2, \dots, N \tag{4}$$

for any initial condition $x_i(0), i = 1, 2, \dots, N$.

3. Distributed consensus protocol design: leaderless case

In this section, a distributed consensus protocol design for system (1) is considered. Based on the local state information of neighboring delta operator systems, the following distributed consensus protocol is applied to each system.

$$u_i = \alpha K \sum_{j\in\mathcal{N}_i} w_{ij}(x_i - x_j), \tag{5}$$

where α is the protocol gain for the multiple delta operator systems, $K \in \mathcal{R}^{p\times n}$ is the feedback gain matrix.

The following result presents a sufficient condition for designing consensus protocol (5).

Theorem 1. *Suppose that graph $\mathcal{G}$ is connected. The feedback gain matrix of (5) is designed as $K = -B^TP$, where $P > 0$ is the unique solution to the algebraic Riccati equation (ARE):*

$$A^TP + PA - PBB^TP + I = 0, \tag{6}$$

and there exists a solution α which satisfies the following inequalities:

$$\frac{1}{2\lambda_2} \leq \alpha \leq \frac{1}{2\lambda_N\lambda_{max}(BB^T)}\lambda_{min}\left(-P^{-1}P^{-1} + BB^T + \frac{4}{h}P^{-1}\right) \tag{7}$$