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Two novel iterative learning control schemes for systems with randomly varying trial lengths[☆]

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ABSTRACT

This paper proposes two novel improved iterative learning control (ILC) schemes for systems with randomly varying trial lengths. Different from the existing works on ILC with variable trial lengths that advocate to replace the missing control information by zero, the proposed learning algorithms are equipped with a searching mechanism to collect useful but avoid redundant past tracking information, which could expedite the learning speed. The searching mechanism is realized by the newly defined stochastic variables and an iteratively-moving-average operator. The convergence of the proposed learning schemes is strictly proved based on the contraction mapping methodology. Two illustrative examples are provided to show the superiorities of the proposed approaches.

1. Introduction

In our daily lives, one could complete a given task and improve the performance gradually provided that the operation is repeated. Such process for human being is usually called the learning process. Inspired by this basic cognition, iterative learning control (ILC) theory is developed for systems that are able to complete tasks over a fixed time interval and perform them repeatedly. By synthesizing the control input from the previous control input and tracking error, the controller is able to learn from the past experience and improve the current tracking performance. Since first introduced by Arimoto in 1980s [1], ILC has attracted much attention from both scholars and engineers over the past three decades and many achievements have been made [2–13].

When considering learning, a basic premise is that the desired task should be performed under same conditions such as identical initial condition and identical trial length for all iterations. In fact, such premise has been assumed in most ILC literature. However, one may find that this assumption is commonly violated in many practical applications due to system uncertainties. That is, the trial lengths may vary in the iteration domain. For instance, [14–17] provided several practical systems that run repeatedly but the trial lengths are not identical due to the complex external environments. Specifically, [14] investigated the application of ILC to humanoid robots, where the gaits problems were divided into phases defined by foot strike times and the durations of the phases were usually not the same from iteration to iteration during the learning process. Moreover, two biomedical systems including functional electrical stimulation for upper limb movement and for gait assistance were introduced in [15–17]. Due to the unknown dynamics and related complex factors, the learning process might end earlier and start the next iteration. Another example is the trajectory tracking with output constraints on a lab-scale gantry crane given in [18]. When the output constraints were violated, the load was wound up and the trial was terminated, which results in variable pass lengths for ILC [18]. Motivated by these observations, ILC problem with iteration-varying trial lengths has attracted more and more attention in recent years.

In the existing literature, there are some works addressing ILC design problems with non-uniform trial lengths from different technical perspectives [16–24]. First, Li et al. proposed an ILC framework for both discrete-time linear and continuous-time nonlinear systems with randomly varying trial lengths by introducing a stochastic variable to describe the randomness of trial lengths in [19] and [21], respectively. In [19], to deal with the randomly varying trial lengths, an iteration-average operator of all historical data was employed in the ILC algorithm to reduce the effect of the lost tracking information. While in [21], instead of using all historical control information, an iteratively-moving-average operator is adopted in ILC law where only the most recent control information will be utilized for learning since 'older' control information would reduce the corrective action from the most recent trials. Moreover, to avoid the utilization of λ-norm, a lifted framework of ILC for discrete-time linear systems was provided

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in [20]. However, it is worthy noting that the convergence of the tracking errors in [19–21] is derived in the sense of mathematical expectation. Fortunately, there are some works showing stronger convergence properties of ILC with non-uniform trial lengths. For example, the almost sure and mean square convergence of a P-type ILC was established in [23,24]. Specifically, [23] considered a discrete linear system where the path statistic properties of the input error, namely, mathematical expectations and covariances, were first recursively calculated along the iteration axis. Based on the recursions of expectations and covariances of the input error, the convergence in the sense of expectation, mean square, and almost sure was derived in sequence. In [24], the ILC design problem was extended to a class of affine nonlinear systems, where the techniques used in [23] were no longer applicable for nonlinear systems. Thus, a modified λ-norm and a technical lemma were introduced to pave the way of showing the almost sure convergence of the tracking error. Furthermore, Seel et al. also contributed much on this topic, where the main focus lies in the monotonic convergence property [15,18,22] and practical applications [16,17]. A primary result is given in [15], where the authors presented the conditions of learning gain matrix for ensuring monotonic convergence. However, the calculations of the learning gain rely upon a completely known system model, which restricts the applicability of the proposed algorithm. A similar technique was applied to the trajectory tracking problem of a lab-scale gantry crane in [18]. The extended version of monotonic convergence with more detailed explanations was reported in a recent paper [22]. Additionally, [16,17] apply ILC with variable pass lengths in the Functional Electrical Stimulation (FES)-based treatment systems for strike patients. These two works also show that the addressed problem has great significance in real-time applications. However, it is worthwhile to highlight that a common feature of the works [18–24] on ILC with non-uniform trial lengths is to replace the missing tracking error information with zero. That is, when the tracking information is not available due to varying trial lengths, the lacked data is set to be zero. Therefore, how to develop a new ILC algorithm that is able to improve the control performance for systems with iteration-varying trial lengths, is an interesting and challenging problem.

Motivated by the above observations, in this paper, two novel improved ILC schemes are proposed for a class of discrete-time linear systems with randomly varying trial lengths. Different from the previous works on ILC with variable trial lengths that advocate to replace the missing tracking information by zero, the proposed learning algorithms are equipped with a searching mechanism to collect useful but avoid redundant past control information, which could expedite the learning speed. The searching mechanism is realized by introducing a new stochastic variable and an iteratively-moving-average operator.

The aim and main contribution of this paper is to reduce the impact of the randomly varying trial lengths to the learning control algorithm and to expedite the convergence speed. To achieve the objective, two ILC laws are proposed. More precisely, the first ILC scheme is proposed to reduce the redundant control information, which appears in the design of ILC laws in [19–21], while the second one is developed to make full use of the effective previous control information to further expedite the learning speed. In addition, the almost sure convergence for both ILC schemes is provided in a rigorous way.

The rest of the paper is organized as follows. Section 2 presents the problem formulation. Section 3 and 4 contribute to the controller design and convergence analysis. Furthermore, numerical simulations are given in Section 5 to verify the validation of the proposed control algorithms. Section 6 draws a conclusion of this work.

Notations. $\mathbf{R}$ is the real set and $\mathbf{R}^n$ is the n-dimensional space. $\mathbf{N}$ is the set of positive integers. $\|\cdot\|$ denotes the Euclidean norm of its indicated vector or matrix. Denote $\|\mathbf{f}(t)\|_\lambda \triangleq \sup_{t\in\{0,1,2,\ldots,T\}}\alpha^{-\lambda t}\|\mathbf{f}(t)\|$ and $\|\mathbf{f}(t)\|_s \triangleq \sup_{t\in\{0,1,2,\ldots,T\}}\|\mathbf{f}(t)\|$ the λ-norm and s-norm of a vector function $\mathbf{f}(t)$ respectively with $\lambda > 0$ and $\alpha > 1$.

2. Problem formulation

Consider the following discrete-time linear system

$$\begin{aligned} \boldsymbol{x}_k(t+1) &= A\boldsymbol{x}_k(t) + B\boldsymbol{u}_k(t), \\ \boldsymbol{y}_k(t) &= C\boldsymbol{x}_k(t), \end{aligned} \tag{1}$$

where $k \in \mathbf{N}$ is the iteration index, $t \in \{0, 1, 2, \ldots, T_k\}$ denotes the time instant, and T_k is the trial length at the kth iteration. Moreover, $\boldsymbol{x}_k(t) \in \mathbf{R}^n$, $\boldsymbol{u}_k(t) \in \mathbf{R}^p$, and $\boldsymbol{y}_k(t) \in \mathbf{R}^r$ denote the state, input, and output of the system (1), respectively. Furthermore, A, B and C are constant matrices with appropriate dimensions. It is worth to point out that the results and convergence analysis in this paper can be extended to linear time-varying systems straightforwardly, and thus we just consider the time-invariant case to clarify our idea. Let $\boldsymbol{y}_d(t), t \in \{0, 1, 2, \ldots, T_d\}$ be the desired output trajectory. Assume that, for any realizable output trajectory $\boldsymbol{y}_d(t)$, there exists a unique control input $\boldsymbol{u}_d(t) \in \mathbf{R}^p$ such that

$$\begin{aligned} \boldsymbol{x}_d(t+1) &= A\boldsymbol{x}_d(t) + B\boldsymbol{u}_d(t), \\ \boldsymbol{y}_d(t) &= C\boldsymbol{x}_d(t), \end{aligned} \tag{2}$$

where $\boldsymbol{u}_d(t)$ is uniformly bounded for all $t \in \{0, 1, 2, \ldots, T_d\}$ with T_d being the desired trial length.

The control objective is to tack the desired trajectory $\boldsymbol{y}_d(t)$, $t \in \{0, 1, 2, \ldots, T_d\}$ by determining a sequence of control inputs $\boldsymbol{u}_k$ such that the tracking error converges as the iteration number k increases.

Before addressing the controller design problem, the following assumptions are imposed.

A 1. *The coupling matrix CB is of full-column rank.*

A 2. *The initial states satisfy* $\|\boldsymbol{x}_d(0) - \boldsymbol{x}_k(0)\| \le \epsilon, \epsilon > 0$.

Remark 1. The initial state resetting problem is one of the fundamental issues in ILC field as it is a standard assumption to ensure the perfect tracking performance. In the past three decades, some papers have devoted to remove this condition by developing additional control mechanisms such as [25–27]. Under assumption A2, since the initial state is different from the desired initial state, it is impossible to achieve the perfect tracking. Instead, the ILC algorithms should force the system output to be as close as possible to the target.

Remark 2. It is worthy noting that, unlike the classic ILC theory that requires control tasks to repeat on a fixed time interval, the trial lengths T_k, $k \in \mathbf{N}$ are iteration-varying and may be different from the desired trial length T_d. For the case that the kth trial length is shorter than the desired trial length, both the system output and the tracking error information will be missing and cannot be used for learning. Thus, this paper aims to re-design ILC schemes to make up the missing signals by making full use of the previous available tracking information, and thus expedite the learning speed. Although some previous works have been published [19–21], a basic assumption is that the probability distribution of T_k is known prior. In this paper, the proposed ILC algorithms will be equipped with an automatic searching mechanism, thus the probability distribution of randomly varying trial lengths is no longer required.