Systems & Control Letters 107 (2017) 44–48

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Improved results on transmission power control for remote state estimation[☆]

Junfeng Wu[a], Yuzhe Li[b,*], Daniel E. Quevedo[c], Ling Shi[d]

[a] College of Control Science and Engineering, Zhejiang University, Hang Zhou, China
[b] Department of Electrical and Computer Engineering, University of Alberta, Edmonton, Alberta, T6G 1H9, Canada
[c] Department of Electrical Engineering (EIM-E), Paderborn University, Germany
[d] Department of Electronic and Computer Engineering, Hong Kong University of Science and Technology, Hong Kong

ARTICLE INFO

Article history:
Received 24 June 2016
Received in revised form 7 July 2017
Accepted 10 July 2017
Available online 10 August 2017

Keywords:
Kalman filtering
Transmission power control
Packet loss
Majorization
Convex optimization

ABSTRACT

We consider a sensor transmission power control problem for remote state estimation. In this problem, a sensor sends its local estimate to a remote estimator over a wireless packet-dropping communication channel. The transmission power is determined by a recently proposed algorithm which uses the innovative information contained in the measurement. In the current paper, we focus on parameter optimization arising from the selection of design parameters for this power controller. The existing work obtained a suboptimal solution to the parameter optimization problem, while by using a vector rearrangement inequality argument and the vector majorization, we now show that there exists an optimal solution within a subset of the whole feasible set. By leveraging this property, we obtain an optimal solution via solving a convex optimization problem.

1. Introduction

Networked state estimation acts as a key component in a wide spectrum of applications, where sensors are deployed across a field, collecting physical data from distributed spots and sending data over a network to an estimator. Compared with wired sensors, wireless ones provide many advantages such as low cost and easy installation. However, an energy dilemma rises for state estimation when using wireless sensors: as wireless sensors are usually powered by on-board batteries, energy saving is critical for prolonging their lifetime; on the other hand, packet losses caused by channel interference and fading are inevitable, and high transmission power leads to good estimation performance due to an inverse relationship between the packet loss probability and the transmission power as revealed in classic communication theory and practice [1]. Motivated by this, the problem of how to balance transmission energy usage and estimation performance has been widely studied in recent years from different perspectives. Some works assumed communication cost to be constant and transmission power management is reduced to communication control. There are mainly two major types of communication control studied in the literature. The first type is known as time-based (offline) communication control, whereby the communication decisions are simply specified only according to the time. Informally, a purely time-based strategy is likely to lead to a periodic communication schedule, see [2–4]. The second type is known as event-based communication control, whereby the communication decisions are specified according to the system's realtime states. Event-based communication control has been extensively studied in existing works, e.g. [5–16]. In the papers [13–15], a sensor scheduling problem is considered, where the decision variable is sending the data or not. In practical applications, the sensor may choose the transmission power from a continuous level. In [15], a one-dimensional discrete time stochastic process is considered. We aims to extend the model to a more general framework with multi-dimensional process with noisy sensor measurements. Some other works have investigated state estimation over fading channels, in which it is more meaningful to manage sensors' transmission power for encountering the effect of time-varying channel fading. In [17], the authors proposed a predictive control algorithm, where power and codebook are determined in an online fashion based on the undergoing estimation error covariance and the channel gain predictions. Ref. [18] presented a design methodology for optimal transmission power allocation at a sensor equipped with energy harvesting technology, in which transmission power is allocated according to energy harvesting, channel fading and the expected estimation error covariance of the receiver's Kalman filter. In [19], system states are used to determine the transmission power. Packet losses signal the receiver of side information of system

☆ The work of L. Shi i supported by an HKUST KTH Partnership FP804.
* Corresponding author.
E-mail addresses: junfengw@kth.se (J. Wu), yuzhe2@ualberta.ca (Y. Li), dquevedo@ieee.org (D.E. Quevedo), eesling@ust.hk (L. Shi).

http://dx.doi.org/10.1016/j.sysconle.2017.07.008
0167-6911/

state. To avoid computation difficulty, the signaling information is discarded. Ref. [20] also suggests that managing transmission power based on real-time system information can improve state estimation performance, attributed to side information signaled by packet losses. More related works can been seen in [21–23].

Of particular interest to the current work is the approach of [20], where at each time instant, the transmission power used by a sensor to send a local estimate is determined according to a quadratic function of the "incremental innovation". The *a posterior* distribution of the system state is updated based on the outcome of the packet-dropping channel, using a Bayesian inference approach. Such a power control law was proved to preserve Gaussianity of the *a posteriori* distribution, which results in a closed-form expression of the MMSE estimator. Comparisons with non-data-driven controllers demonstrate significant performance improvement. In [20], the minimization of the expected estimation error at each time was addressed via the selection of design parameters. However, the resulting optimization problem in [20] was relaxed and solved only approximately; the performance loss due to the suboptimal solution *is neither known nor proved to be bounded*. Similar situations also occur in [13] and [24], where the parameter optimization problems are solved by minimizing an upper bound of the performance. As a result, only suboptimal solutions are obtained. In the current work, we investigate the parameter optimization problem posed in the above sensor transmission control problem. By using the unitary matrix decomposition and the vector rearrangement inequality arguments, we show that an optimal solution lies in a restricted subset of the feasible set, which can be computed via solving a convex optimization problem.

The remainder of this paper is organized as follows. In Section 2, we give the system model. In Section 3, we introduce the transmission power controller devised in [20] and review some preliminary results thereof. The main result is presented in Section 4. In Section 5 we provide a numerical example. Section 6 draws concluding remarks.

Notation: $\mathbb{N}$ (and $\mathbb{N}_+$) is the set of nonnegative (and positive) integers. $\mathbb{S}_+^n$ is the set of n by n positive semi-definite matrices. The pseudo-determinant of a matrix $X \in \mathbb{R}^{n\times n}$ is defined as the product of all non-zero eigenvalues of X. The Moore–Penrose pseudo-inverse is a generalized inverse of a matrix. For $X \in \mathbb{R}^{m\times n}$, the Moore–Penrose pseudo-inverse of X, denoted as $Y \in \mathbb{R}^{n\times m}$, always exists and is unique, satisfying the following four criteria: (i) $XYX = X$; (ii) $YXY = Y$; (iii) $(XY)^* = XY$; (iv) $(YX)^* = YX$. When X is a square matrix, by abuse of notations, we use $\det(X)$ and X^{-1} in case of a singular matrix X, to denote the pseudo-determinant and the Moore–Penrose pseudo-inverse. For a vector $x \in \mathbb{R}^n$, we use $x^{\downarrow}$ and $x^{\uparrow}$ to represent the vectors with the same entries, but re-ordered in decreasing and increasing order respectively. The symbol $\mathcal{N}(x, \Sigma)$ denotes a Gaussian distribution with mean x and covariance Σ. We introduce an operator $h : \mathbb{S}_+^n \to \mathbb{S}_+^n$, where

$$h(X) \triangleq AXA' + W,\ \ W \in \mathbb{S}_+^n.$$

2. System model

We are concerned with transmission power control for the remote state estimation scheme depicted in Fig. 1. In what follows, we will give the system description.

Consider a discrete-time linear time-invariant (LTI) system measured by a sensor:

$$\begin{aligned} x_{k+1} &= Ax_k + w_k, \\ y_k &= Cx_k + v_k, \end{aligned}$$

where $A \in \mathbb{R}^{n\times n}$ and $k \in \mathbb{N}$, $x_k \in \mathbb{R}^n$ is the system state, $y_k \in \mathbb{R}^m$ is the sensor's measurement, the state noise $w_k \in \mathbb{R}^n$ and observation noise $v_k \in \mathbb{R}^m$ are zero-mean i.i.d. Gaussian with $\mathbb{E}[w_k w_j'] = \delta_{kj}W$ $(W \succeq 0)$, $\mathbb{E}[v_k(v_j)'] = \delta_{kj}R$ $(R \succ 0)$, $\mathbb{E}[w_k(v_j)'] = 0$ $\forall j, k \in \mathbb{N}$. The initial state x_0 is a zero-mean Gaussian random vector, uncorrelated with w_k and v_k. The pair (A, C) is assumed to be detectable and (A, W) stabilizable.

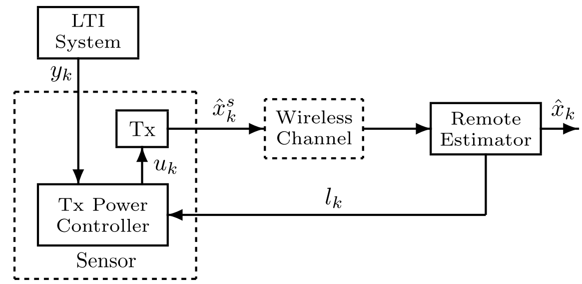

Fig. 1. Remote state estimation scheme.

As shown in Fig. 1, the sensor locally runs a Kalman filter and generates a local MMSE estimate. Then it transmits the local estimate to a remote estimator using power level u_k to be designed. Denote the sensor's local estimate and error covariance by $\hat{x}_k^s$ and P_k^s respectively, i.e., $\hat{x}_k^s \triangleq \mathbb{E}[x_k|y_1, \dots, y_k]$ and $P_k^s \triangleq \mathbb{E}[(x_k - \hat{x}_k^s)(x_k - \hat{x}_k^s)'|y_1, \dots, y_k]$. We assume that this local Kalman filter has entered steady state, that is, $P_k^s = \overline{P} \succeq 0,\ \forall k \in \mathbb{N}$.

The sensor sends data to a remote estimator over an additive white Gaussian noise (AWGN) channel suffering from channel fading [1]. The details and assumptions for the communication channel are provides in [20].

We use a random binary process $\{\gamma_k\}_{k\in\mathbb{N}}$ to describe communication success as follows:

$$\gamma_k = \begin{cases} 1, & \text{if } \hat{x}_k^s \text{ arrives error-free at time } k, \\ 0, & \text{otherwise,} \end{cases} \tag{1}$$

initialized with $\gamma_0 = 1$. Let $u_k \in [0, +\infty)$ be the transmission power for the QAM symbol at time k. From [1], the packet loss probability can be approximated by

$$\Pr(\gamma_k = 0|u_k, h_k) \approx \exp\left(\frac{-\alpha h_k u_k}{N_0 B}\right),$$

where N_0 is the AWGN power spectral density, B is the channel bandwidth, h_k is the channel power gain, and α is a constant depending on the specific modulation scheme used. Throughout this paper we will adopt (1) with equality.

3. Transmission power control and remote state estimation

We restrict our attention to one type of transmission power controllers that render the estimation problem linear and tractable. The benefit of a linear estimation process is that a closed-form recursive MMSE estimator can be derived. See [20] for the idea of preserving the linearity of the estimation process described below.

Let the incremental innovation contained in the sensor's local estimate compared to the latest reception instant be defined as follows:

$$z_k = \hat{x}_k^s - A^{\tau_k}\hat{x}_{k-\tau_k}^s,$$

where

$$\tau_k \triangleq k - \max_{1\leqslant t\leqslant k-1}\{t : \gamma_t = 1\}.$$

Consider a transmission power controller $f : \mathbb{R}^n \mapsto [0, \infty)$ of the following form:

$$u_k = f_k(z_k) \triangleq \frac{N_0 B}{2\alpha h_k} z_k' Q_k z_k + \varphi_k, \tag{2}$$