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Sampled-data boundary feedback control of 1-D linear transport PDEs with non-local terms

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ABSTRACT

The paper provides results for the application of boundary feedback control with Zero-Order-Hold (ZOH) to 1-D inhomogeneous, linear, transport partial differential equations on bounded domains with constant velocity and non-local terms. It is shown that the emulation design based on the recently proposed continuous-time, boundary feedback, designed by means of backstepping, guarantees closed-loop exponential stability, provided that the sampling period is sufficiently small. It is also shown that, contrary to the parabolic case, a smaller sampling period implies a faster convergence rate with no upper bound for the achieved convergence rate. The obtained results provide stability estimates for the sup-norm of the state and robustness with respect to perturbations of the sampling schedule is guaranteed.

1. Introduction

Sampled-data feedback control has been studied extensively for finite-dimensional systems due to the use of digital technology in modern control systems for the implementation of the controller (see for instance [1–7] and the references therein). However, sampled-data feedback control has been scarcely studied for infinite-dimensional systems. Most of the available results deal with delay systems (see [8–14]). For systems described by Partial Differential Equations (PDEs) the design of sampled-data feedback control faces major technical difficulties: even the notion of the solution of a PDE under sampled-data feedback control has to be clarified. Sampled-data controllers for parabolic systems were designed by Fridman and coworkers in [15–18] by using matrix inequalities. The works [19,20] provided necessary and sufficient conditions for sampled-data control of general infinite-dimensional systems under periodic sampling (see also [21,22] for the case of "generalized sampling"). Approximate models of infinite-dimensional systems were used in [23] for practical stabilization. A sampled-data feedback controller for hyperbolic age-structured models was proposed in [24].

In the linear finite-dimensional case, there are results that guarantee closed-loop exponential stability for continuous-time linear feedback designs when applied with Zero-Order-Hold (ZOH) and sufficiently small sampling period (see for instance [2,3,5,6]). The results deal with the case of globally Lipschitz right hand sides (which contains the linear case as a subcase) and the application of the continuous-time feedback under ZOH is called the "emulation" sampled-data feedback design.

A general robustness result that guarantees closed-loop exponential stability for continuous-time linear boundary feedback designs when applied with ZOH and arbitrary (not necessarily periodic) sampling schedules of sufficiently small sampling period is missing for the case of systems described by PDEs. In the recent work [25], efforts were made towards the development of such general results for linear parabolic PDE systems.

While the development of continuous-time boundary feedback controllers for hyperbolic PDE systems has progressed significantly during the last decade (see [26–31] for a single PDE and [32–34] for systems of PDEs), there are no results that guarantee stability properties for the sample-and-hold implementation of continuous-time controllers with arbitrary sampling schedules of sufficiently small sampling period. The present paper provides sampled-data, boundary feedback controllers for 1-D, first-order, linear, transport PDEs with non-local terms. The design is based on the emulation of the continuous-time boundary feedback design presented in [30]. It is proved that there is a sufficiently small sampling period, such that the closed-loop system preserves exponential stability under the sample-and-hold implementation of the controller (Theorem 2.2). In other words, we prove that emulation design works for the case of linear hyperbolic PDEs with boundary feedback. The derived exponential stability estimates are expressed in the sup-norm of the state and (conservative) upper bounds for the sampling period are derived. Finally, robustness with respect to the sampling schedule is established, exactly as in the finite-dimensional case.

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The methodology for proving the main result of the present work is very different from the corresponding methodology in the parabolic case. While both proofs of the main results in [25] exploit an eigenfunction expansion procedure, the proof of Theorem 2.2 relies on the representation of the solution of the closed-loop system by means of an Integral Delay Equation (IDE), as proposed in [29]. However, there is an additional important difference between the parabolic and the hyperbolic case. In the hyperbolic case (Theorem 2.2), by selecting a sufficiently small maximum allowable sampling period we can achieve an arbitrarily fast rate of convergence. This is not possible in the parabolic case. This important difference can be explained by the fact that the nominal continuous-time feedback law (proposed in [30]) achieves finite-time stability in the hyperbolic case, while the nominal continuous-time feedback laws in the parabolic case achieve exponential stability. The proof of Theorem 2.2 provides an estimate of how small the maximum allowable sampling period must be in order to achieve a given rate of convergence.

The structure of the present work is as follows: Section 2 is devoted to the presentation of the problem, the clarification of the notion of the solution for a hyperbolic PDE system under boundary sampled-data control, the statement of the main result (Theorem 2.2) and a discussion about the main result. The proof of the main result is provided in Section 3. A simple illustrating example is presented in Section 4. Finally, the concluding remarks are provided in Section 5.

Notations. Throughout this paper, we adopt the following notations.

* $\Re_+ := [0,+\infty)$. Z^+ denotes the set of all non-negative integers.
* Let $U \subseteq \Re^n$ be a set with non-empty interior and let $\Omega \subseteq \Re$ be a set. By $C^0(U;\Omega)$, we denote the class of continuous mappings on U, which take values in Ω. By $C^k(U;\Omega)$, where $k \geq 1$, we denote the class of continuous functions on U, which have continuous derivatives of order k on U and take values in Ω.
* Let $I \subseteq \Re$ be an interval. A function $f : I \to \Re$ is called right continuous on I if for every $t \in I$ and $\varepsilon > 0$ there exists $\delta(\varepsilon,t) > 0$ such that for all $\tau \in I$ with $t \leq \tau < t+\delta(\varepsilon,t)$ it holds that $|f(\tau)-f(t)| < \varepsilon$. A function $f : I \to \Re$ is called left continuous on I if for every $t \in I$ and $\varepsilon > 0$ there exists $\delta(\varepsilon,t) > 0$ such that for all $\tau \in I$ with $t \geq \tau > t-\delta(\varepsilon,t)$ it holds that $|f(\tau)-f(t)| < \varepsilon$. A function $f : I \to \Re$ is called piecewise continuous on I if for every compact $K \subseteq I$ there exists a finite set $B \subset I \cap K$ such that f is C^0 on $(I\cap K)\setminus B$ and furthermore, for every $t \in I$ all meaningful limits $\lim_{h\to 0^+}(f(t+h))$, $\lim_{h\to 0^+}(f(t-h))$ exist and are finite. Let $a \in \Re$ be a given real number. A function $f : [a,+\infty) \to \Re$ is called right differentiable on $[a,+\infty)$ if for every $t \geq a$ the limit $\lim_{h\to 0^+}\left(h^{-1}(f(t+h)-f(t))\right)$ exists and is finite. A function $f : I \to \Re$ is called piecewise C^1 on I if for every compact $K \subseteq I$ there exists a finite set $B \subset I\cap K$ such that f is C^1 on $(I\cap K)\setminus B$ and all meaningful limits $\lim_{h\to 0^+}\left(\dot f(t+h)\right)$, $\lim_{h\to 0^+}\left(\dot f(t-h)\right)$, $\lim_{h\to 0^+}(f(t+h))$, $\lim_{h\to 0^+}(f(t-h))$ exist for all $t \in I$ and are finite.
* Let $x : \Re_+ \times [0,1] \to \Re$ be given. We use the notation $x[t]$ to denote the profile of x at certain $t \geq 0$, i.e., $(x[t])(z) = x(t,z)$ for all $z \in [0,1]$.
* Let $I \subseteq \Re$ be an interval. $L^2(I)$ denotes the space of equivalence classes of measurable functions $f : I \to \Re$ which are square integrable. $L^\infty(I)$ denotes the space of equivalence classes of measurable functions $f : I \to \Re$ which are essentially bounded on I. $L^\infty_{loc}(I)$ denotes the space of equivalence classes of measurable functions $f : I \to \Re$ which are of class $L^\infty(K)$ for every compact $K \subseteq I$.
* We define $\phi_a(t) := \int_0^t \exp(-a(t-s))\,ds$ for all $t \geq 0$ and $a \in \Re$. Notice that $\phi_a(t) := \frac{1-\exp(-at)}{a}$ for $a \neq 0$ and $\phi_0(t) := t$.

2. Main results

We consider the control system

$$\frac{\partial y}{\partial t}(t,z) + \frac{\partial y}{\partial z}(t,z) = g(z)\,y(t,1) + \int_z^1 f(z,s)\,y(t,s)\,ds,$$
$$\text{for } (t,z) \in \Re_+ \times [0,1] \tag{2.1}$$

$$y(t,0) = u(t) - \int_0^1 p(s)\,y(t,s)\,ds, \quad \text{for } t \geq 0 \tag{2.2}$$

where $g \in C^0([0,1];\Re)$, $p \in C^1([0,1];\Re)$, $f \in C^0\left([0,1]^2;\Re\right)$ are given functions, $y[t]$ is the state and $u(t)$ is the control input. More specifically, we consider the solution of the initial–boundary value problem (2.1), (2.2) under boundary sampled-data control with ZOH:

$$u(t) = u_i, \ \text{for } t \in [\tau_i, \tau_{i+1}) \quad \text{and for all } i \in Z^+ \tag{2.3}$$

where $\{\tau_i \geq 0, i = 0,1,2,\ldots\}$ is an increasing sequence (the sequence of sampling times) with $\tau_0 = 0$, $\lim_{i\to+\infty}(\tau_i) = +\infty$ and $\{u_i \in \Re, i = 0,1,2,\ldots\}$ is the sequence of applied inputs and initial condition

$$y(0,z) = y_0(z), \ \text{for all } z \in (0,1] \tag{2.4}$$

where $y_0 : [0,1] \to \Re$ is a given function.

The motivation for the study of initial–boundary value problems of the form (2.1), (2.2), (2.4) comes from multiple sources and that is the reason that problems of the form (2.1), (2.2), (2.4), as well as the related problem (2.2), (2.4) and

$$\frac{\partial y}{\partial t}(t,z) + \frac{\partial y}{\partial z}(t,z) = \int_0^1 f(z,s)\,y(t,s)\,ds,$$
$$\text{for } (t,z) \in \Re_+ \times [0,1] \tag{2.5}$$

where $p \in C^1([0,1];\Re)$, $f \in L^2\left([0,1]^2\right)$ are given functions, have been studied extensively in the literature (see [26–31]). In [31] (Chapter 10), it was shown that the stabilization problem of a Korteweg–de Vries-like PDE leads to the stabilization problem for system (2.1), (2.2). Transport PDEs with non-local terms are used frequently in mathematical biology (see [24,35] and the references therein). It should be noticed that initial–boundary value problems of the form

$$\frac{\partial \phi}{\partial t}(t,z) + \frac{\partial \phi}{\partial z}(t,z) = a(z)\,\phi(t,z) + \tilde g(z)\,\phi(t,1)$$
$$+ \int_z^1 \tilde f(z,s)\,\phi(t,s)\,ds, \ \text{for } (t,z) \in \Re_+ \times [0,1] \tag{2.6}$$

$$\phi(t,0) = u(t) - \int_0^1 \tilde p(s)\,\phi(t,s)\,ds, \ \text{for } t \geq 0 \text{ and}$$
$$\phi(0,z) = \phi_0(z), \ \text{for all } z \in (0,1] \tag{2.7}$$

where $\tilde g \in C^0([0,1];\Re)$, $\tilde p \in C^1([0,1];\Re)$, $\tilde f \in C^0\left([0,1]^2;\Re\right)$, $a \in C^0([0,1];\Re)$ are given functions, can be transformed to the form (2.1), (2.2) by means of the transformation $y(t,z) = \exp\left(-\int_0^z a(s)\,ds\right)\phi(t,z)$. Transport PDEs are frequently encountered in heat and mass transfer phenomena when diffusion is negligible. In this case, non-local terms arise from quasi-static approximations (e.g., by applying a quasi-steady state approximation for the dynamics of the cooling medium temperature in a jacket surrounding a tube with a hot fluid moving with constant velocity).

Let X be the set of left continuous and piecewise C^1 functions $y : [0,1] \to \Re$. This is a linear normed space with