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On zero-input stability inheritance for time-varying systems with decaying-to-zero input power

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ABSTRACT

Stability results for time-varying systems with inputs are relatively scarce, as opposed to the abundant literature available for time-invariant systems. This paper extends to time-varying systems existing results that ensure that if the input converges to zero in some specific sense, then the state trajectory will inherit stability properties from the corresponding zero-input system. This extension is non-trivial, in the sense that the proof technique is completely novel, and allows to recover the existing results under weaker assumptions in a unifying way.

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1. Introduction

Stability properties for systems with inputs find natural application in control systems. Input-to-state stability (ISS) [1–3], integral ISS (iISS) [4,5], converging-input converging-state (CICS) [6,7], uniformly bounded-energy input bounded state (UBEBS) [8], bounded-energy-input convergent-state (BEICS) [9,10] and L^p-input converging-state [11] are examples of such properties. Most of the existing analyses and characterizations of these properties apply to time-invariant systems. Analogous results for time-varying systems are very scarce. There exist some characterizations of the ISS property [12–14] and a recent result by the authors characterizing the iISS property [15]. In a more general setting, some asymptotic behaviour results exist for asymptotically autonomous differential equations [16,17], and some also dealing with weak invariance principles [18]. An asymptotically autonomous differential equation is one such that the function f_0 defining its dynamics $\dot{x} = f_0(t,x)$ approaches a time-invariant function g, i.e. $f_0(t,x) \to g(x)$ as $t \to \infty$, in some suitable sense.

A time-invariant system $\dot{x} = \bar{f}(x,u)$, with an input u that converges to zero can be interpreted as an asymptotically autonomous system $[f_0(t,x) := \bar{f}(x,u(t)) \to g(x) := \bar{f}(x,0)]$ under reasonable assumptions. By contrast, time-varying systems of the form $\dot{x} = f(t,x,u)$ do not in general allow such a possibility. An interesting result in the latter case is provided in [18], where the concept of weakly asymptotically autonomous system is introduced, which, loosely speaking, means that $\dot{x} = f_0(t,x)$ approaches the differential inclusion $\dot{x} \in F(x)$ as $t \to \infty$ in some appropriate sense. The latter can be employed in the time-varying case with $f_0(t,x) := f(t,x,u(t))$.

An iISS system has, inter alia, the property that inputs with bounded energy, where energy is measured according to the iISS gain, produce state trajectories that asymptotically converge to zero. The latter is the BEICS property [9]. The function that weighs the input in order to measure input energy, i.e. the iISS gain in the iISS setting, is extremely important in the sense that a system may be iISS for some iISS gains but not for others. Interesting examples of some perhaps counter-intuitive facts are given in [19] and [20], where globally asymptotically stable systems (exponentially in [20]) are destabilized by additive inputs of arbitrarily small energy (exponentially decaying in [20]). The main point we make is that the ensuing stability or instability depends on how input energy is measured.

This work relates to the CICS and BEICS properties. Roughly speaking, these properties entail that if the system input converges to zero in some specific manner, then the state will also converge to zero. These properties are of importance in stability analysis for cascade systems and also in ensuring stability robustness under certain types of disturbances. We consider time-varying systems with inputs and pinpoint specific input power 'measures' (see Section 2.3) so that solutions corresponding to inputs with decaying-to-zero power may inherit specific properties from the corresponding zero-input system. More precisely, suppose that the zero-input system has a uniformly locally asymptotically stable compactum C within an open set $\mathcal{G}$ contained in the "region of attraction" (see [21] for the latter concept in time-varying systems,

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and Section 2). Let x be a forward complete solution of $\dot{x} = f(t, x, u)$ corresponding to an input u having decaying-to-zero power. Then, one of the results that we prove is that if the ω-limit set of x has nonempty intersection with $\mathcal{G}$, then x approaches C.

In this context, our contribution is the following. First, we provide a convergence result for time-varying systems with inputs under very mild assumptions on the function f defining the system dynamics. Worthy of mention is that we do not require $f(t, x, u)$ to be continuous in t, nor locally Lipschitz in x. As a consequence, solutions are not necessarily unique. Second, we pinpoint input power 'measures' for which such convergence is possible. These 'measures' relate to specific bounds on f. Third, we extend some of the main results in [6,9] and [11] to time-varying systems, under weaker assumptions and in a unifying way. We emphasize that these extensions are novel and nontrivial, since existing results for time-invariant systems, such as those in [9] and [11], cannot be adapted to the current setting (the corresponding proofs rely on converse Lyapunov theorems that do not remain valid).

The remainder of this paper is organized as follows. In Section 2 we introduce the notation, definitions and main assumptions required. Our main result and explanations of how our result subsumes other existing results are contained in Section 3. Section 4 contains some secondary technical results and conclusions are drawn in Section 5.

2. Preliminaries

2.1. Notation and preliminary definitions

The reals, nonnegative reals, naturals and nonnegative integers are denoted $\mathbb{R}$, $\mathbb{R}_{\geq 0}$, $\mathbb{N}$ and $\mathbb{N}_0$, respectively. For $\xi \in \mathbb{R}^n$, $|\xi|$ denotes its Euclidean norm. For a given nonempty subset $A \subset \mathbb{R}^n$, $|\xi|_A$ denotes the distance from $\xi \in \mathbb{R}^n$ to A, that is $|\xi|_A = \inf\{|\xi - \zeta| : \zeta \in A\}$. Given $r \geq 0$, $A_r = \{\xi \in \mathbb{R}^n : |\xi|_A \leq r\}$ and $B_r(\xi) = \{\xi\}_r$ for every $\xi \in \mathbb{R}^n$. Thus, if $\xi \in \mathbb{R}^n$ and $r \geq 0$, the statements $\xi \in A_r$ and $|\xi|_A \leq r$ are equivalent. For $p \geq 1$ and $m \in \mathbb{N}$, $L^p_{m,loc}$ (L^p_m) denotes the set of all the Lebesgue measurable functions $v : \mathbb{R}_{\geq 0} \to \mathbb{R}^m$ such that $|v|^p$ is integrable on each finite interval $I \subset \mathbb{R}_{\geq 0}$ ($|v|^p$ is integrable on $\mathbb{R}_{\geq 0}$). When $m = 1$ we just write L^p_{loc} and L^p. For a Lebesgue measurable set $J \subset \mathbb{R}$, $|J|$ will denote its Lebesgue measure. Given a metric space (U, d) and an interval $I \subset \mathbb{R}$, we say that $v : I \to U$ is piecewise constant if there exists a partition $I_1, \dots, I_m$ of I such that I_i is an interval for every i and v is constant on I_i. The function $u : I \to U$ is Lebesgue measurable if there exists a sequence of piecewise-constant functions $u_k : I \to U$ such that $\lim_{k\to\infty} u_k(t) = u(t)$ for almost all $t \in I$, that is $|\{t \in I : \lim_{k\to\infty} u_k(t) \neq u(t)\}| = 0$. When U is separable, $u : I \to U$ is measurable if and only if $u^{-1}(V)$ is Lebesgue measurable for every open subset V of U (see Remark C.1.1. in [22]). A function $\omega : U \to \mathbb{R}$ is proper if for all $r \in \mathbb{R}$ the sublevel set $\omega^{-1}((-\infty, r])$ is compact. We write $\sigma \in \mathcal{K}$ if $\sigma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is continuous, strictly increasing, and $\sigma(0) = 0$. We write $\sigma \in \mathcal{K}_\infty$ if $\sigma \in \mathcal{K}$ and σ is unbounded.

2.2. Problem statement

This work deals with time-varying control systems of the general form

$$\dot{x} = f(t, x, u) \tag{1}$$

where $f : \mathbb{R}_{\geq 0} \times \mathcal{X} \times U \to \mathbb{R}^n$ with $\mathcal{X}$ an open subset of $\mathbb{R}^n$ and (U, d) a metric space. An input is a Lebesgue measurable function $u : \mathbb{R}_{\geq 0} \to U$ and $\mathcal{U}$ is the set of all the inputs. We suppose that U is nonempty and there exists $0 \in U$, where "0" is nothing but some element in U that we distinguish from the rest. For an arbitrary $\mu \in U$, we define $|\mu| := d(\mu, 0)$, i.e. $|\mu|$ is the distance between μ and 0. In the case in which $U \subset \mathbb{R}^m$, 0 denotes the origin of $\mathbb{R}^m$ and d will be the metric induced by Euclidean norm. The *zero input* is the map $\mathbf{0} \in \mathcal{U}$ such that $\mathbf{0}(t) \equiv 0$. With system (1) we associate the zero-input system

$$\dot{x} = f(t, x, \mathbf{0}) =: f_0(t, x). \tag{2}$$

Assumption 1. The function $f : \mathbb{R}_{\geq 0} \times \mathcal{X} \times U \to \mathbb{R}^n$ satisfies the following conditions.

(A1) (Carathéodory) $f(\cdot, \xi, \mu)$ is Lebesgue measurable for all $(\xi, \mu) \in \mathcal{X} \times U$ and $f(t, \cdot, \cdot)$ is continuous for every $t \geq 0$.

(A2) (Zero-input Lipschitzianity) $f_0(t, \xi)$ is locally Lipschitz in ξ uniformly in t in the following sense: for every compact subset $K \subset \mathcal{X}$ there exists a nonnegative function $L_K \in L^1_{\text{loc}}$ such that $\sup_{t\geq 0} \int_t^{t+T} L_K(s)\, ds < \infty$ for all $T > 0$ and

$$|f_0(t, \xi) - f_0(t, \xi')| \leq L_K(t)|\xi - \xi'| \ \forall t \geq 0,\ \forall \xi, \xi' \in K.$$

In view of Assumption 1, for each $t_0 \geq 0$ and $\xi \in \mathcal{X}$ there is a unique maximally defined (forward) solution $x(t) = \varphi(t, t_0, \xi)$ of (2) which verifies $x(t_0) = \xi$. We will denote by $[t_0, t_{t_0,\xi})$ its maximal interval of definition. It is well-known that in the case in which $\varphi(t, t_0, \xi)$ belongs to a fixed compact subset of $\mathcal{X}$ for all $t \in [t_0, t_{t_0,\xi})$, then $t_{t_0,\xi} = \infty$.

Let $C \subset \mathcal{G} \subset \mathcal{X}$ be such that C is nonempty and compact and $\mathcal{G}$ is open. In what follows we assume that C is uniformly asymptotically stable with respect to (2) and that $\mathcal{G}$ is contained in the region of attraction of C. These statements are made precise in the following assumption.

Assumption 2 (*Zero-input stability*)**.** There exist a nonempty compact set C and an open set $\mathcal{G}$ such that $C \subset \mathcal{G} \subset \mathcal{X}$ and

(B1) (uniform Lyapunov stability) for every $\varepsilon > 0$ there exists $\delta > 0$ such that for all $t_0 \geq 0$ and $\xi \in C_\delta$, $\varphi(t, t_0, \xi) \in C_\varepsilon$ for all $t \geq t_0$;

(B2) (uniform boundedness of solutions) for every compact set $K \subset \mathcal{G}$ there exists a compact set $\Gamma \subset \mathcal{X}$ such that for all $t_0 \geq 0$ and $\xi \in K$ we have that $\varphi(t, t_0, \xi) \in \Gamma$ for all $t \geq t_0$;

(B3) (uniform attractiveness) for every compact set $K \subset \mathcal{G}$ and every $\varepsilon > 0$ there exists $T = T(K, \varepsilon) \geq 0$ such that for all $t_0 \geq 0$ and $\xi \in K$ we have that $\varphi(t, t_0, \xi) \in C_\varepsilon$ for all $t \geq t_0 + T$.

Note that under the uniform Lyapunov stability in (B1) above, it follows that C is forward invariant under (2), i.e. for all $t_0 \geq 0$ and $\xi \in C$, $\varphi(t, t_0, \xi) \in C$ for all $t \geq t_0$. When $\mathcal{G} = \mathcal{X} = \mathbb{R}^n$, then C is globally uniformly asymptotically stable with respect to (2).

Remark 1. When the zero-input system (2) is time-invariant, i.e. $f_0(t, \xi) \equiv f_0^*(\xi)$, Assumption 2 is satisfied with any compact set $C \subset \mathcal{X}$ which is asymptotically stable with respect to (2) [that is (i) C is stable: for every $\varepsilon > 0$ there exists $\delta > 0$ such that for all $\xi \in C_\delta$, $\varphi(t, 0, \xi) \in C_\varepsilon$ for all $t \geq 0$ and (ii) C is attractive: there exists $\delta_0 > 0$ such that $|\varphi(t, 0, \xi)|_C \to 0$ for all $\xi \in C_{\delta_0}$] and with $\mathcal{G} = \mathcal{A}$, where $\mathcal{A} = \{\xi \in \mathcal{X} : |\varphi(t, 0, \xi)|_C \to 0\}$ is the region of attraction of C. □

The problem we address in this paper is the following:

Give conditions under which the property of convergence to C that applies to solutions of the zero-input system (2) is inherited by (i.e. also applies to) solutions of (1).

Remark 2. Some solutions to this problem are given for time-invariant systems in [6,11] and [9]. The results in this paper extend these in different directions, as will be explained in more detail in Section 3. □

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