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Tangent vector field approach for curved path following with input saturation

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ABSTRACT

Path following is an indispensable function for autonomous vehicles. Desired paths may be of arbitrary shape, not just the mostly investigated straight lines and circles. This paper addresses the path following problem of arbitrary twice differentiable curves using vector-field-based approach. A tangent vector field is constructed through coordinate transformation, and a sufficient condition for its feasibility concerning the input saturation is given out. A saturated turning velocity controller is designed and its Lyapunov stability is discussed in detail. Numerical simulation results show us that the path following performance of the proposed approach is comparable with that of the literature while involving 5 less parameters to be set.

1. Introduction

For various autonomous vehicles, such as mobile robots, autonomous marine surface vehicles, autonomous underwater vehicles (AUVs), unmanned ground vehicles (UGVs), unmanned aerial vehicles (UAVs), space vehicles, etc., two essential motion control problems, trajectory tracking [1–5] and path following [4–11], have been paid much attention during the past few years. Compared with trajectory tracking, path following requires no time parametrization of the desired paths. In such application circumstances as mapping, border patrol, search and rescue and so on, path following is preferable to trajectory tracking [11].

Varieties of approaches have been proposed for the path following problem. In the recent survey paper [11], Sujit et al. classified these approaches into two categories, geometric methods and control techniques. Furthermore through a thorough performance comparison of five typical approaches, which are the carrot-chasing approach, the nonlinear guidance law (NLGL) approach, the pure pursuit with line-of-sight-based (PLOS-based) approach, the vector-field-based (VF-based) approach, and the linear quadratic regulator (LQR) approach, to follow the two most commonly considered paths, straight lines and circles, the authors concluded that the VF-based approach can achieve more accurate path following results than the other approaches, and also requires the least control effort [11].

The vector-field-based path following approach was developed by Lawrence [12,13], Frew [14], Nelson [15], Griffiths [16] et al. In [12–14] the authors proposed a named Lyapunov vector field for circular path following. Based on the global convergence of the reference trajectories determined by the Lyapunov vector field to the desired loiter circle, heading rate controllers consisting of a feedback term and a feedforward term were designed to track the vector field and finally achieve the circular path following objective. In [15] the authors constructed two respective vector fields for straight lines and circular paths, and designed sliding mode controllers accordingly to achieve these two types of path following. The global exponentially stabilities of the controllers were also discussed. In [16], the author further investigated the path following of arbitrary curved paths making use of a modified vector field.

Based on the Lyapunov vector field proposed in [14] and by further taking the heading error with respect to the vector field into account, an alternative feedforward term was exploited to obtain more accurate circular path following [17,18]. Recently Zhu et al. gave the rigorous proof of the global convergence of this circular path following approach [19]. By combining Lyapunov guidance vector field and a tangent vector field, Chen et al. further considered this problem temporally to get a theoretically shortest path for circular path following [20]. Other innovative results for the vector-field-based path following can be found in [21–24].

Till now except the work given in [16], the existing results concerning vector-field-based approach for path following only considered paths with particular forms, mostly lines and circles. Although a path following approach for arbitrary 2D curves was

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Nomenclature

$\mathbf{p}(t) = (\mathbb{x}(t), \mathbb{y}(t))^T$ The vehicle position at time t in XY frame
$\mathbf{p}^*(t) = (\mathbb{x}^*(t), \mathbb{y}^*(t))^T$ The vehicle position at time t in X*Y* frame
$\mathbf{p}_r = (x_r, y_r)^T$ Chosen reference point on the desired path
$\mathbf{w}_r = (w_{rx}, w_{ry})^T$ Tangent vector of the desired path at $\mathbf{p}_r$
θ_r Corresponding curve parameter at $\mathbf{p}_r$
$r = \|\mathbf{p} - \mathbf{p}_r\|$ Distance from $\mathbf{p}$ to $\mathbf{p}_r$
r_c Lateral distance, the directional distance from $\mathbf{p}$ to the tangent line of the desired path at $\mathbf{p}_r$
$v(t)$ Linear velocity (synthetical speed) of the vehicle
$\omega(t)$ Turning velocity of the vehicle
$(\dot{\mathbb{x}}_d, \dot{\mathbb{y}}_d)^T$ Vector field
$\varphi(t)$ Turning angle of the vehicle
φ_r Reference turning angle, the inclination of the tangent vector $\mathbf{w}_r$
φ_d Desired turning angle determined by the vector field
$\varphi_e = \langle \varphi - \varphi_d \rangle$ Turning angle error between the vector field and the vehicle
$\tilde{\varphi} = \langle \varphi_d - \varphi_r \rangle$ Turning angle error between the vector field and the desired path
φ_{e0} Initial turning angle error between the vector field and the vehicle
$\dot{\varphi}_r$ Changing rate of the reference turning angle
$\dot{\varphi}_d$ Changing rate of the desired turning angle after introducing φ_e

proposed in [16], it has the following two limitations. Firstly, the approach does not consider the input saturation explicitly. Secondly, the sliding mode controller involves 7 parameters and actually 4 conditions for them (see **Theorem III.1** therein), and it is not an easy work to set appropriate values for these parameters to satisfy these four conditions. This is because some of these parameters depend on a specific curved path (e.g., ρ and $\bar{d}$) and some of them are severely coupled (e.g., μ and $\bar{d}$). Moreover inappropriate parameter settings are apt to result in input saturations.

This paper focuses on the path following of arbitrary curves. Compared with the previous results given in [16], the main contributions are

- The derivation of an equivalent tangent vector field is given.
- Given the input saturation constraint, a sufficient condition for the tangent vector field to be feasible is presented.
- By considering the input saturation explicitly, a saturated turning velocity controller is designed. This controller includes only 3 parameters, two of them are common vector field parameters, and the third one is the feedback gain.
- The Lyapunov stability of the saturated controller is discussed in detail.

The remainder of this paper is structured as follows. The curved path following problem is formulated in Section 2. The tangent vector field is derived in Section 3, and its feasibility concerning the input saturation constraint is also studied. A saturated controller and its Lyapunov stability are discussed in Section 4. Section 5 gives some simulation examples to assess the proposed approach. And a short conclusion is given in Section 6.

2. Problem formulation

Denote $\mathbf{p}(t) = (\mathbb{x}(t), \mathbb{y}(t))^T$ as the instantaneous position of the vehicle at time t in the chosen inertial XY frame, and $\varphi(t)$ as the corresponding turning angle. In this paper we assume that the kinematics of the vehicle is modeled as

$$\begin{cases} \dot{\mathbb{x}}(t) = v(t)\cos\varphi(t) \\ \dot{\mathbb{y}}(t) = v(t)\sin\varphi(t) \\ \dot{\varphi}(t) = \omega(t) \end{cases} \tag{1}$$

where $v(t)$ and $\omega(t)$ stand for the linear velocity and the turning velocity respectively. (1) can describe the kinematic characteristic of a unicycle-type robot [2], a UAV [14,15], a lunar rover [25], etc. For instance, consider that the vehicle is a UAV. If no wind is present, then $v(t)$ represents the UAV airspeed, and $\omega(t)$ represents the heading rate [14]. And if wind disturbance exists, then $v(t)$ and $\omega(t)$ represent the groundspeed and the course rate respectively [15].

Saturation constraints generally exist for the velocities $v(t)$ and $w(t)$, which are given by

$$v(t) \le v_{\max}, \; |\omega(t)| \le \omega_{\max}. \tag{2}$$

For fixed-wing aerial vehicles, an additional stall speed constraint $v(t) \ge v_{\text{stall}} > 0$ should also be satisfied. For surface vehicles such as robots, v_{stall} can be set to 0.

In this paper we use "body speed" to denote the speed of the vehicle itself with no consideration of the effect of the external environment, e.g., the airspeed of a UAV. And correspondingly, "synthetical speed" is used to indicate the vehicle speed after considering the external environment, i.e., the speed of the vehicle with respect to the chosen inertial frame. An example of the synthetical speed is the groundspeed of a UAV and the linear velocity $v(t)$ is actually the synthetical speed of the vehicle.

In this paper we aim to design the controlling strategy for the turning velocity $\omega(t)$ to achieve the path following objective. We just assume that the body speed and further the synthetical speed (i.e., $v(t)$) of the vehicle are continuous, and for the synthetical speed, the constraint $0 < v_{\min} \le v(t) \le v_{\max}$ always satisfies.

The desired path $\mathcal{C}(\theta) : \{x = x(\theta), y = y(\theta)\}$ is assumed to be twice differentiable. Herein we use "twice differentiable" to mean that all the first- and second-order derivatives of x and y with respect to the curve parameter θ exist, i.e., all of x'_θ, $x''_{\theta\theta}$, y'_θ and $y''_{\theta\theta}$ exist, where the superscript $'$ denotes the derivative operator. Naturally we require that $(x'^2_\theta + y'^2_\theta)$ is not zero, since otherwise the desired path $\mathcal{C}(\theta)$ degenerates to an isolated point.

Define a directional lateral distance (or, directional cross-track distance) r_c as follows. Given the current vehicle position $\mathbf{p} = (\mathbb{x}, \mathbb{y})^T$, find an appropriate reference point $\mathbf{p}_r = (x_r, y_r)^T = (x(\theta_r), y(\theta_r))^T$ on the desired path. The parametric tangent vector of the desired path at this point is $\mathbf{w}_r = (w_{rx}, w_{ry})^T = (sx'_{\theta_r}, sy'_{\theta_r})^T$, where $s = \pm 1$ determines the vector direction. And the inclination of this tangent vector can be computed by

$$\varphi_r = \arctan2(w_{ry}, w_{rx}) = \arctan2(sy'_{\theta_r}, sx'_{\theta_r}) \tag{3}$$

where $\arctan2(\cdot,\cdot)$ denotes the four-quadrant inverse tangent function. We name φ_r reference turning angle. Then we can define r_c as the directional distance from $\mathbf{p}$ to this tangent line. This distance is computed as

$$r_c = (\mathbb{x} - x_r)\sin\varphi_r - (\mathbb{y} - y_r)\cos\varphi_r. \tag{4}$$

The reason why we name r_c as a directional distance is that r_c can be > 0, < 0, or, $= 0$. The magnitude of r_c represents the absolute lateral distance, and the sign of r_c determines on which side of the desired path the vehicle currently locates. The geometry illustration is given in Fig. 1.

Reference points $\mathbf{p}_r$ should be carefully chosen for different desired paths. The requirement is that the convergence of the directional lateral distance r_c to 0 can be well representative of the convergence of the actual distance $r = \|\mathbf{p} - \mathbf{p}_r\| = \sqrt{(\mathbb{x} - x_r)^2 + (\mathbb{y} - y_r)^2}$ to 0. The following two examples give respective typical guides for the reference point choosing of non-closed and closed desired paths.