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Consensus in time-delayed multi-agent systems with quantized dwell times[☆]

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ABSTRACT

This paper investigates the effects of quantized dwell times in solving consensus problems in time-delayed multi-agent systems with the asynchronous property of unpredictable quantization jumps in inter-agent communications. The quantized dwell times can be introduced on purpose or be inherent from quantizing devices. They beneficially avoid undesirable fast chattering of quantizers' outputs and mathematical difficulties in convergence analysis. The main result shows that the largest eigenvalue of the interaction graph Laplacian well characterizes the dependence of maximum allowable time delay on quantized dwell times and quantizer parameters under the assumption of heterogeneous time-varying transmission delays. Simulations are presented to show the effectiveness of the main result.

1. Introduction

In multi-agent systems, numerous studies have shown the significant effects of information flows on the methodology for coordinating inter-connected autonomous agents [1], and particular effort has been put into the networks with finite capacity or bandwidth constraints [2,3,4]. One popular topic concerning this issue is the consensus protocol design with only quantized information. A quantizer digitizes incoming information and usually gives piecewise constant output. So at any instant, only finite bits of channel capacity are required for the transmission of quantized data [5,6,7,8]. Typical quantizers include uniform, logarithmic [9,10,11], and probabilistic quantizers [12,13]. Several fundamental and interesting quantization principles have been presented for the gossip consensus of discrete-time systems [5,12,14,15,16,17,18]. More general settings of time-varying topologies in discrete-time systems were studied and tight polynomial bounds on convergence times were provided in [19]. However, for continuous-time systems, quantized networks also give rise to the possibility of infinite switchings of quantizer outputs within a finite period of time [6]. This chattering behavior increases the burden of data transmission and it theoretically leads to the non-existence of solutions in the traditional framework of differential equations. Certainly, the concept of differential inclusions can be used to tackle the encountered mathematical difficulties [10,11,20,21]. To prevent infinitely frequent chattering, in [22], the mechanism of hysteretic quantizers was successfully presented for quantized average consensus. This paper explores the feasibility of quantized consensus of multi-agent systems when quantized dwell times are direct introduced to avoid chattering problems.

Due to decentralized interactions, quantized multi-agent systems also exhibit inherently the feature of asynchronous encoding of quantizers and asynchronous information transmission. Asynchronous consensus was formally addressed for continuous-time and discrete-time models in [23,24,25]. Examples of asynchronous problems include independent way-point updates of agents [23], asynchronous state updates [25,26], asynchronous data-sampling schedules [27], and asynchronous computation [28]. For analysis purpose, asynchronous events were modeled by an iteration graph with infinite vertices and were dealt with by nonlinear paracontractions theory in [25]. They can also be simply modeled by a sequence of interaction graphs, together with additional joint or periodical connectivity conditions. In such a setup, the concept of "analytic synchronization" and Lyapunov methods were used in the consensus analysis in [23,24] and [26,27], respectively.

This paper modifies the popular quantizers, such as uniform and logarithmic quantizers, by the introduction of quantized dwell times, and studies the effectiveness of these quantizers in the distributed consensus control of multi-agent systems. Specifically, a dwell time is given after each output change of quantizers; during dwell times, the involved quantizers have no response to

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the processed data. Consequently, any two consecutive output jumps of any quantizer are separated by a time interval of at least the dwell-time length, and the output chattering of quantizers is successfully avoided. This approach is different from the one based on hysteretic quantizers proposed in [22], which avoids output chattering by constructing overlapping level sets of quantization. Technically, by our approach, the minimum event interval is obviously not less than the dwell times; whereas, with hysteretic quantizers, the minimum event interval depends on both agent states and switching sequences of quantization events.

This paper tries to consider a general setting of data transmission delays, which can be fixed, time-varying, or random, and are only required to be upper bounded. However, because of the dynamic complexity induced by asynchronous quantization as well as delays, we only present the complete and interesting results on the average consensus problem in a bi-directional network of single-integrators. In the consensus analysis, we devise a new method with the help of incidence matrices and two groups of time-dependent diagonal matrices to describe the unstructured delay dependence of states and the asynchronous relationship between each pair of quantizers. The evolution of states is considered on a joint sequence of quantization times, which is similar to the idea from the method of "analytic synchronization" [23,24]. Under the assumption of heterogeneous time-varying delays on information channels, we show that the largest eigenvalue of the graph Laplacian well measures the robustness of the network with respect to asynchronous quantization and time delays. This conclusion is consistent with the classic result given in [29] for time-delayed continuous-time systems without quantization.

This paper is organized as follows. In Section 2, the model is presented and the motivation for studying quantized dwell times is given; In Section 3, we perform the convergence analysis and discuss the delay robustness of the studied protocol with quantized dwell times; In Section 4, simulations are given to show the correctness of the main result. The paper is concluded in Section 5.

2. Problem formulation

This section first gives the individual models and then defines the quantizers with quantized dwell times. Finally, the problem is formulated equivalently in the form of edge dynamics.

2.1. Individual dynamics and information flow

This paper considers the following multi-agent system with n single-integrators:

$$\dot{x}_i(t) = u_i(t), \quad i = 1, 2, \ldots, n, \tag{1}$$

where $x_i \in \mathbb{R}$ denotes the state of agent i and $u_i(t)$ is a state feedback, designed based on the local information received from the neighbors of agent i. $u_i(t)$ is formally called *protocol or algorithm* in the consensus control for multi-agent systems [29,30].

Assume that the information flow among agents is bidirectional and modeled by an undirected simple graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ with vertex set $\mathcal{V}$ and edge set $\mathcal{E}$, and assume that $\mathcal{V}$ consists of n vertices v_i, $i = 1, 2, 3, \ldots, n$, and $\mathcal{E}$ contains no self-loops. Vertex v_i represents agent i. Edge $(v_i, v_j) \in \mathcal{E}$ if and only if there exists an information link connecting agents i and j. The neighbor set of agent i is denoted by $\mathcal{N}_i \triangleq \{j : (v_j, v_i) \in \mathcal{E}\}$.

In this paper, we study the validity of the following protocol in solving consensus problems based on the quantized relative states:

$$u_i(t) = -\sum_{j\in\mathcal{N}_i} Q^h(x_i(t-\tau^{ij}(t)) - x_j(t-\tau^{ij}(t))). \tag{2}$$

Here, $Q^h(\cdot)$ is the quantizer, which will be given later; $\tau^{ij}(t)$ is the information transmission delay over link (v_i, v_j) at time t and it is assumed that $\tau^{ij}(t) = \tau^{ji}(t)$. Note that each relative state, like $x_i - x_j$, as a whole, is measurable in many scenarios, such as formation control and attitude alignment. So in (2), we assume that x_i and x_j share the same time delay $\tau^{ij}(t)$ [29], which is also a necessary condition in the average consensus control. We propose the following assumption for the time delays:

Assumption 1. For any information link (v_i, v_j), $t - \tau^{ij}(t)$ is an increasing function of time t.

The above assumption implies that the transmitted data keeps new at quantizers. It can be ensured by $\mathrm{d}\tau^{ij}(t)/\mathrm{d}t < 1$ if $\tau^{ij}(t)$ is differentiable.

Lemma 1. *Let $\kappa(t) = (1/n)\sum_{i=1}^{n} x_i(t)$. If $Q^h(\cdot)$ is an odd function, then $\mathrm{d}\kappa(t)/\mathrm{d}t = 0$; i.e., the state average is time-invariant.*

Proof. Noticing that the information flow between any pair of connected agents is symmetric, we have

$$\begin{aligned}
\frac{\mathrm{d}\kappa(t)}{\mathrm{d}t} &= -\frac{1}{n}\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} Q^h(x_i(t-\tau^{ij}(t)) - x_j(t-\tau^{ij}(t))) \\
&= -\frac{1}{2n}\Big(\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} Q^h(x_i(t-\tau^{ij}(t)) - x_j(t-\tau^{ij}(t))) \\
&\quad + \sum_{j=1}^{n}\sum_{i\in\mathcal{N}_j} Q^h(x_i(t-\tau^{ij}(t)) - x_j(t-\tau^{ij}(t)))\Big) \\
&= -\frac{1}{2n}\Big(\sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} Q^h(x_i(t-\tau^{ij}(t)) - x_j(t-\tau^{ij}(t))) \\
&\quad + \sum_{i=1}^{n}\sum_{j\in\mathcal{N}_i} Q^h(x_j(t-\tau^{ji}(t)) - x_i(t-\tau^{ji}(t)))\Big) \\
&= 0. \quad \square
\end{aligned}$$

2.2. Quantizers with quantized dwell times

In consensus coordination, the following quantizers are usually used to digitize data in the networks with band-limited channels:

(1) uniform quantizer:

$$Q_u(\xi) = \mathrm{sign}(\xi)\lfloor \frac{|\xi|}{\sigma} \rfloor \sigma;$$

(2) logarithmic quantizer:

$$Q_{\log}(\xi) = \begin{cases} 0, & \text{if } \xi = 0 \\ \mathrm{sign}(\xi)\sigma^{\lfloor \log_\sigma |\xi| \rfloor}, & \text{otherwise} \end{cases}$$

where $\lfloor\cdot\rfloor$ denotes the floor function giving the nearest integer less than or equal to the involved variable, $\mathrm{sign}(\cdot)$ is the sign function, and $\sigma > 0$ and $\sigma > 1$, respectively.

The application of the above quantizers in protocol (2) results in the possibility of infinite occurrences of quantizer jumps in a finite period of time [21]. In [22], the authors showed that the Carathéodory solutions may not be complete in quantized control. In the next subsection, we will give an example to show the case of the non-existence of the traditional solutions and the case of the existence of the Zeno behavior.

To deal with the above problem, our approach is to add a fixed dwell time h after each output jump of quantizers. Let $Q(\cdot)$ be the interested quantizer, which can be the uniform quantizer $Q_u(\cdot)$ or the logarithmic quantizer $Q_{\log}(\cdot)$. The employed quantizer $Q^h(\cdot)$ in (2) is defined in the following recursive way:

(1) let $t' = 0$;
(2) let $Q^h(\xi(t')) = Q(\xi(t'))$;