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Quantizer design for asymptotic stability of quantized linear systems with saturations

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ABSTRACT

This paper considers continuous linear systems involving input saturations and quantized control laws. Based on spherical polar coordinates, a quantizer of infinite data rate is proposed with a definite relation between the quantized data and the corresponding quantization error. By the proposed quantizer, state feedback controllers are designed for input quantization case and state quantization case, respectively, to achieve both local asymptotic stability and larger stability regions of the systems. Further, a quantizer of finite data rate is proposed for the same problem.

1. Introduction

In this paper we formulate and solve a stabilization problem with a limited communication channel. Our task involves designing quantizers for continuous linear systems with input saturations and achieving local asymptotic stability of the systems.

The stabilization problem of quantized feedback systems motivated by numerous applications is a very active and expanding research area, where communication between the plant and the controller is limited due to capacity or security constraints; see, e.g., [1–10] and references therein. Since the systems in practice are subject to magnitude limitation in the input inevitably, which may reduce the performance of the closed-loop system or even lead to instability, much attention is paid to the systems with input saturations; see, e.g., [11–18] and references therein. Furthermore, some papers address quantized feedback control problem of the systems with saturations, for example, Cepeda and Astolfi [11] investigate the feedback stabilization problem for SISO linear uncertain control systems with saturating quantized measurements; Fridman and Dambrine [12] study quantized and delayed state-feedback control of linear systems with given constant bounds on the quantization error and on the time-varying delay; in Liberzon [15], stabilization of continuous-time systems subject to quantization is considered by a hybrid control strategy, the saturation and quantizer blocks are one and only block, and the saturation is a particular effect of the quantizer; in Tarbouriech and Gouaisbaut [18], the saturation nonlinearity and the quantization nonlinearity are disjoined in order to characterize them precisely, and convex optimization procedure is provided to design the state feedback gain and local uniform ultimate boundedness stability of the systems is obtained. In contrast to these works, we are concerned with local asymptotic stability of continuous linear systems with input saturations and larger stability regions of the systems.

The objective of this paper is to propose appropriate quantizers to achieve local asymptotic stability and larger stability regions of continuous linear systems with input saturations and quantized control laws. Two kinds of quantizers are proposed: one is of infinite data rate and the other finite data rate. Under the proposed quantizers, the state feedback control design problem is addressed for input quantization case and state quantization case, respectively. The proposed approach allows to characterize a set (stability region) such that the closed-loop trajectories initiated in the set converge toward the origin. Our work is related to the work of Tarbouriech and Gouaisbaut [18], of which the saturation nonlinearity condition is used in the present paper. Except this, the major difference is the quantizers with different quantization nonlinearity conditions. In Tarbouriech and Gouaisbaut [18], uniform quantizer under Cartesian coordinates is used, by which the quantization nonlinearity conditions are given, while the quantizers in this paper are proposed based on spherical polar coordinates, by which a new quantization nonlinearity condition is developed. The developed nonlinearity condition shows that the magnitude of the quantized data is proportional to an upper bound of the magnitude of the corresponding quantization error, The proposed quantizers bring benefits to the systems as follows:

(1) The closed-loop trajectories converge toward the origin instead of a set containing the origin. Since the magnitude of the quantized data is proportional to an upper bound of the magnitude of the corresponding quantization error, the

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quantization error will tend to null as the quantized data tends to the origin, by which the local asymptotic stability is achieved.

(2) The new developed quantization nonlinearity condition helps to obtain larger stability regions than the existing results, and the closed-loop trajectories initiated in the obtained stability regions converge toward the origin.

Notation: E and O denote respectively the identity matrix and the null matrix of appropriate dimensions. For a matrix $A \in \mathbb{R}^{m\times n}$, $A_{(i)}$ and A^T denote its ith row and its transpose, respectively. For two symmetric matrices, A and B, $A > B$ (resp. $A \geq B$) means that $A - B$ is positive definite (resp. semi-definite positive). $*$ stands for symmetric blocks in matrices. For two sets S_1 and S_2, $S_1 \setminus S_2$ denotes the set S_1 deprived of S_2. $\|\cdot\|$ denotes the Euclidean norm for a vector and the corresponding matrix induced norm for a matrix, $\delta_{\min}(\cdot)$ denotes the minimum singular value of a matrix and $\lceil\cdot\rceil$ denotes the ceiling function.

2. Problem statement

Consider the following continuous linear system:

$$\dot{x} = Ax + Bsat(u) \tag{1}$$

where $x \in \mathbb{R}^d$ and $u \in \mathbb{R}^m$ are the state and the input of the system. Matrices A, B are real constant matrices of appropriate dimensions. Given any vector $u \in \mathbb{R}^m$, the saturation map $sat(u) \in \mathbb{R}^m$ is classically defined from the symmetric saturation function which has the positive vector u_0 as level, that is, $sat(u_{(i)}) = sign(u_{(i)}) \min\{u_{0(i)}, |u_{(i)}|\}$, $i = 1, \ldots, m$.

The input of the system can be the result of a quantized control law. A vector x with appropriate dimension is quantized as $q(x)$, the estimate of x, with the same dimension, where $q(\cdot)$ is the quantizer function defined in Section 3.1. Two different quantized control laws are investigated: (i) the input quantization case $u(t) = q(Kx(t))$; (ii) the state quantization case $u(t) = Kq(x(t))$, where K is a real constant matrix of appropriate dimension. Let $\Xi(x) = q(x) - x$ be quantization error, a type of quantization nonlinearity.

In this paper, the problem we intend to solve in both cases can be summarized as follows:

Problem 2.1. Determine a quantizer and a stabilizing state feedback gain K, and characterize a set $\mathcal{S}$ such that for every initial state belonging to $\mathcal{S}$ the system (1) is asymptotically stable.

Thus, the key problem is to design a quantizer with a new quantization nonlinearity condition such that the state of the system (1) converges toward the origin instead of a set containing the origin and larger stability region is achieved for each case.

Remark 2.1. Since the quantization error induced by quantizer is a discontinuous isolated nonlinearity entering into the dynamics of the closed-loop system, the resulting closed-loop system is described by a discontinuous right-hand side differential equation and the notion of solutions should be then properly defined. In our paper, we have considered Caratheodory solutions and accordingly excluded some particular solutions like sliding motions on the boundaries between the quantization regions. If we aim at considering such dynamics, we need to extend the concept of solution and consider Krasovskii solutions, which include Caratheodory solutions and chattering phenomena resulting of sliding motions [19,20]. Following the works of Ceragioli et al. [3], this extension can be performed in our work using differential inclusion tools.

3. Quantizer based on spherical polar coordinates

The quantizer in this paper will be based on spherical polar coordinates. Let the vector $x = [x_1 \quad x_2 \quad \cdots \quad x_{d-1} \quad x_d]^{\mathrm{T}} \in \mathbb{R}^d$. Then we call the column $[x_1 \quad x_2 \quad \cdots \quad x_{d-1} \quad x_d]^{\mathrm{T}}$ as the Cartesian rectangular coordinate of x. The vector can also be represented using spherical polar coordinate

$$\begin{bmatrix} r \\ \theta_1 \\ \vdots \\ \theta_{d-2} \\ \theta_{d-1} \end{bmatrix} \in \mathbb{B}^d := \left\{ \begin{bmatrix} r \\ \theta_1 \\ \vdots \\ \theta_{d-2} \\ \theta_{d-1} \end{bmatrix} : 0 \leq r < \infty, 0 \leq \theta_1, \right.$$
$$\left. \theta_2, \ldots, \theta_{d-2} \leq \pi, 0 \leq \theta_{d-1} \leq 2\pi \right\}$$

via the coordinate transformation pair

$$\begin{aligned} x_1 &= r\cos\theta_1 \\ x_2 &= r\sin\theta_1\cos\theta_2 \\ &\vdots \\ x_{d-1} &= r\sin\theta_1\sin\theta_2\cdots\sin\theta_{d-2}\cos\theta_{d-1} \\ x_d &= r\sin\theta_1\sin\theta_2\cdots\sin\theta_{d-2}\sin\theta_{d-1} \end{aligned}$$

and

$$\begin{aligned} r &= \sqrt{(x_1)^2 + (x_2)^2 + \cdots + (x_d)^2} \\ \theta_1 &= \arccos\frac{x_1}{\sqrt{(x_1)^2 + \cdots + (x_d)^2}} \\ \theta_2 &= \arccos\frac{x_2}{\sqrt{(x_2)^2 + \cdots + (x_d)^2}} \\ &\vdots \\ \theta_{d-2} &= \arccos\frac{x_{d-2}}{\sqrt{(x_{d-2})^2 + \cdots + (x_d)^2}} \\ \theta_{d-1} &= \begin{cases} \arccos\dfrac{x_{d-1}}{\sqrt{(x_{d-1})^2 + (x_d)^2}}, & \text{if } x_d \geq 0, \\ 2\pi - \arccos\dfrac{x_{d-1}}{\sqrt{(x_{d-1})^2 + (x_d)^2}}, & \text{if } x_d < 0. \end{cases} \end{aligned}$$

For the objective of the paper, we propose a new quantizer based on spherical polar coordinates.

Definition 3.1. A quantizer of infinite data rate based on spherical polar coordinates (for abbreviation, a quantizer of infinite data rate) is a tritupple (L, a, M), where the real number $L > 0$ represents the radius of the support ball, the real number $a > 0$ regulates the proportional coefficient, and the positive integer $M \geq 2$ represents the number of the angles into which the angle of radian π is equally partitioned. This quantizer partitions the support

$$\Lambda = \left\{ x \in \mathbb{R}^d : r \leq L \right\}$$

into quantization blocks as follows:

the sets $\{x \in \mathbb{R}^d : \frac{L}{(1+2a)^{i+1}} < r \leq \frac{L}{(1+2a)^i}, j_n\frac{\pi}{M} < \theta_n \leq (j_n+1)\frac{\pi}{M}, n = 1, \ldots, d-2, s\frac{\pi}{M} < \theta_{d-1} \leq (s+1)\frac{\pi}{M}\}$, indexed by $(i, j_1, \ldots, j_{d-2}, s)$, $i = 0, 1, 2, \ldots, j_n = 0, \ldots, M-1$ for $n = 1, \ldots, d-2$, and $s = 0, \ldots, 2M-1$.

Since there are infinite quantization blocks in the support by Definition 3.1, the quantizer needs infinite data rate. The quantizer of infinite data rate is adopted firstly for highlighting the main results. In Section 4.4, a quantizer of finite data rate will be proposed.