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Consensus of adaptive multi-agent systems

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ABSTRACT

This paper studies a leader–follower consensus problem for a multi-agent system composed of identical adaptive agents. We propose a distributed adaptive protocol with which a successful consensus is guaranteed if inter-agent communication is purely bidirectional. When unidirectional communication exists, the closed loop census system employing the proposed protocol is locally stable near a consensus state. A numerical example is presented to illustrate our theoretical findings and properties of our adaptive consensus system.

1. Introduction

Control problems of networked multi-agent systems attracted much attention in science and engineering. One of key topics which has been widely studied in literature is how to change collective behaviors of all agents in a given networked system by controlling only a small number of agents and/or by assigning a certain distributed protocol to which each agent is subject.

In tackling this problem one generally meets a dilemma that a dynamic model of an isolated agent should be general enough to adequately describe the behavior of each agent but even slight complexity in the agent dynamics can be hugely amplified in the process of inter-agent networking and consequently the resulting overall dynamics of a networked system might be unmanageably complicated, discouraging rigorous analytic approaches. This explains why a simple integrator model has been widely adopted as an agent model and most research work on networked systems assumed the agent homogeneity.

Meanwhile, *adaptiveness* is a natural and essential capability of living creatures and many man-made systems, and there are many networked systems composed of those adaptive agents. Presumably, to say nothing of peoples in social networks, swarming animals and insects may retain some level of autonomy when they serve as members of a group and thus can be better described as adaptive agents, rather than homogeneous agents following a fixed dynamics. This observation motivates us to consider a consensus problem of networked multi-agent systems with adaptive agents in this paper.

The term *adaptiveness* have been used for different meanings in the field of networked systems. For instances, authors of [1–6] proposed *adaptive protocols* for time-invariant homogeneous agents where agent protocols include some adaptive parameters. In network theory, *adaptive network* seems to mean that both node (agent) dynamics and network topology change adaptively, e.g., see [7]. Authors of [8,9] dealt with a networked set of *adaptive estimators* and studied a consensus problem between estimators. In this paper we will consider a scalar version of the same networked system in [8,9] but adaptive estimators are regarded as *adaptive agents* trying to follow a leader state.

The adaptive estimator in [8] assumed an unrealistic all-to-all connectivity between agents. In addition, the adaptive estimator proposed in [9] required that firstly each agent should measure not only the states but also adaptive parameters of its neighbor agents, and that secondly every agent should directly measure a leader state. Those two strong requirements are removed in our consensus protocol to be presented below.

In the field of adaptive control, there were some attempts to extend adaptive control theory to inter-connected systems [10,11]. A basic idea of those approaches is to construct a set of local adaptive subsystems such a way that a resulting networked system can maintain stability against communication between subsystems. This viewpoint that subsystem communication is a disturbance that may hinder global stability, sharply contrasts with our view in this work that inter-agent communication can be beneficial to overall stability of a networked system. This paper will show that a consensus of networked adaptive system can be achieved owing to inter-agent communication even when some agents cannot directly measure a leader (model) state.

We assume in this paper that a distributed adaptive consensus law can be forced to all follower agents in multi-agent systems from a control viewpoint. However our adaptive consensus law to be presented below is a physically plausible policy that followers

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with limited information but seeking a consensus with its neighbors would naturally choose. In this sense our results in the paper can provide useful insights on dynamic behaviors of multi-agent systems with adaptive consensus-preferring agents.

2. Problem definition

Consider a dynamical multi-agent system composed of *adaptive* agents satisfying the following scalar dynamics; for $k = 1, \ldots, n$,

$$\dot{x}_k = a_k x_k + b_k u \tag{1}$$

where $a_k = a_k(t) \in \mathbb{R}, b_k = b_k(t) \in \mathbb{R}$ are time-varying parameters and $u = u(t) \in \mathbb{R}$ is an exogenous input. We assume $n_\ell (< n)$ *leader* agents among n agents, are selected and they share fixed parameters $a_k(t) = \alpha < 0$ and $b_k(t) = \beta$. Thus leader agents satisfy a time-invariant dynamics

$$\dot{x} = \alpha x + \beta u \quad (\alpha < 0). \tag{2}$$

In contrast, each *follower* agent changes its parameters a_k, b_k comparing its state x_k with those of neighbor agents. We assume that a follower agent k can observe a *local consensus error*

$$z_k := \sum_{j \in N_k} w_{kj}(x_k - x_j), \quad w_{kj} \in \mathbb{R} \tag{3}$$

and the exogenous input u in (1), in addition to its own state x_k. In (3), the set $N_k \subset \{1, \ldots, n\}$ denotes the neighbor agents that agent k can directly observe, and $w_{kj} > 0$ is a weighting factor.

Remark 1. Notice that, as $a_k(\cdot), b_k(\cdot)$ are functions of time, our follower dynamics (1) is not a simple integrator for which various leader–follower consensus algorithms have been proposed in literature.

Depending on whether or not the set N_k includes any leader agents, followers can be divided into two groups; *direct followers* which are directly connected to at least one leader agent and *indirect followers* which are not.

An interesting problem to be studied in this paper is how each follower agent k should update its parameters a_k, b_k based on local information $\{z_k, x_k, u\}$ in order to achieve a global state consensus defined below:

Definition 1. A multi-agent system with adaptive agents of the form (1) is said to achieve a *consensus* if $\lim_{t\to\infty} |x_i(t) - x_j(t)| = 0$ for all $i, j \in [1, n]$.

Our problem can be stated more rigorously as follows;

Problem 1. Find a distributed adaptive protocol (functions f and g)

$$\begin{aligned} \dot{a}_k &= f(x_k, z_k, u), \\ \dot{b}_k &= g(x_k, z_k, u) \end{aligned} \tag{4}$$

such that the associated multi-agent system achieves a consensus.

Notice that if agent k is an indirect follower, then the local error z_k in (3) does not include a leader state. If there are no indirect followers in a given networked system, then every follower can see at least one leader, and a standard adaptive law, e.g., see [12] (p. 102),

$$\dot{a}_k = -e_k x_k, \quad \dot{b}_k = -u e_k, \quad e_k := x_k - x \tag{5}$$

works for each follower–leader pair where x denotes the leader state. Furthermore it is expected that inter-agent communication between those pairs will not significantly hinder the overall consensus because every follower state approaches to the same leader state.

The existence of indirect followers makes our problem challenging. Indirect followers never know actual leader (reference) state and at a specific time they may receive contradictory information on the leader state from multiple direct followers to which they are connected simultaneously. A key contribution of this work is to show that a natural generalization of the classical adaptive law (5) can solve the consensus problem stated in Problem 1.

3. Main results

3.1. Adaptive consensus protocol

A network topology of a multi-agent system will be described as a mathematical graph $G = (V, E)$ in which agents and inter-agent connections represent a vertex set $V = \{1, \ldots, n\}$ and edges $E \subset \{(i, j) : i, j \in V\}$, respectively. A directed graph (digraph) is a graph where edges are directed, i.e., the edge (i, j) is an ordered pair starting at vertex i and ending at vertex j. Graphs without ordered edges are called undirected graphs. Weighted graphs are graphs in which a positive weight is associated to each edge. The adjacency matrix A of a (di)graph is defined as $A = (a_{ij})$ where $a_{ij} = w_{ij} > 0$ if and only if (i, j) is an edge with weight w_{ij} and $a_{ij} = w_{ij} = 0$ otherwise. We assume all diagonal elements of A are zeros, i.e., graphs have no self-loops. The outdegree a vertex is the sum of the weights of edges starting from that vertex. The Laplacian matrix of a (di)graph is defined $L := D - A$ where A is the adjacency matrix and D is the diagonal matrix of vertex outdegrees. Explicitly, we have

$$L = (L_{ij}), L_{ij} = \begin{cases} \displaystyle\sum_{j=1, j\neq i}^{n} w_{ij}, & i = j \\ -w_{ij}, & i \neq j. \end{cases} \tag{6}$$

By construction, $L\mathbf{1}_n = \sum_j L_{ij} = 0$ holds for all i where $\mathbf{1}_n := [1 \cdots 1]'$ is the so-called all-one vector, i.e., $\mathbf{1}_n$ is a (right) eigenvector of L with zero eigenvalue. In addition, every Laplacian matrix is a *diagonally dominant* matrix in the sense that

$$|L_{ii}| \geq \sum_{j \neq i} |L_{ij}| \tag{7}$$

holds for every i. In fact the equality holds for every Laplacian matrix L.

For notational simplicity, for a given multi-agent system, we label agents such a way that, in a sequence $k = 1, \ldots, n$, the first n_i agents are indirect followers and the next n_d agents are direct followers and the last n_ℓ agents are leaders; $n_f := n_i + n_d$ and $n_f + n_\ell := n$. This labeling gives the state vectors of followers, leaders and all agents as

$$x_f := \begin{bmatrix} x_1 \\ \vdots \\ x_{n_f} \end{bmatrix}, x_\ell := \begin{bmatrix} x_{n_f+1} \\ \vdots \\ x_n \end{bmatrix}, x := \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} x_f \\ x_\ell \end{bmatrix}. \tag{8}$$

Given a multi-agent system, from a partition of an associated Laplacian matrix L, the local error $z := [z_1 \cdots z_{n_f}]'$ of followers in (3) can be represented as

$$\left[\begin{array}{c} z \\ \hline * \end{array}\right] = L \begin{bmatrix} x_f \\ x_\ell \end{bmatrix} := \left[\begin{array}{c|c} L_z & B \\ \hline * & * \end{array}\right] \begin{bmatrix} x_f \\ x_\ell \end{bmatrix} \tag{9}$$

where $L_z \in \mathbb{R}^{n_f \times n_f}, B \in \mathbb{R}^{n_f \times n_\ell}$ and the symbol "$*$" stands for terms irrelevant to the following developments.

It is an easy task to confirm that the submatrix L_z in (9) can be decomposed into

$$L_z = L^f + \mathrm{diag}(\overbrace{0, \ldots, 0}^{n_i}, \overbrace{\delta_{n_i+1}, \ldots, \delta_{n_f}}^{n_d}) \tag{10}$$