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Adaptive finite-time bipartite consensus for second-order multi-agent systems with antagonistic interactions

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ABSTRACT

This paper is concerned with the adaptive finite-time bipartite consensus problems for networked second-order multi-agent systems with antagonistic interactions and unknown external disturbances. A signed undirected graph is used to describe the interactions among agents. For the leaderless case, continuous nonlinear distributed protocols with adaptive update laws are given, which can not only guarantee the bipartite steady-state errors of any two agents converge to a small region in finite time, but also eliminate the chattering problem. Then, the proposed algorithm is extended to solve the adaptive finite-time bipartite consensus tracking problem for leader–follower case by designing distributed finite-time estimator. Simulation example is included to show the effectiveness of the presented methods.

1. Introduction

In the last decade, the consensus problem of multi-agent systems has drawn a lot of attention due to its potential applications in multi-vehicles formation, sensory networks, distributed computation, and so forth [1,2]. The consensus means that all agents reach an agreement on a state under a designed protocol based only on local relative information between neighboring agents. Many consensus algorithms have been reported in [3–9] by synthesizing algebraic graph theory and control theory.

The above studies on the consensus problem only consider the multi-agent systems with cooperative interactions and the edges of graph have nonnegative weights. However, the multi-agent networks with antagonistic interactions may also exist in some real networks, such as social network [10], and the networks with antagonistic interactions are usually represented by signed graphs, in which the weights of edges are either positive or negative. Note that the bipartite consensus is an important problem for networks over signed graph, and it means that all agents in networks converge to a value that is the same for all in modulus but not in sign [11]. [11,14,15] considered the bipartite consensus problems for multi-agent systems with different dynamics, but the protocols proposed in there are with asymptotically convergence rate, which means the bipartite consensus can only be achieved in infinite settling time.

It should be pointed out that the design of distributed control laws such that all states of agents achieve consensus in finite-time is very important for multi-agent systems with fully cooperative interactions [24–35], because the finite-time control not only provides a faster convergence rate but also guarantees the systems have better disturbance attenuation performances [19–23]. For example, [31–34] considered the finite-time control for multi-agent systems with first-order and second-order dynamics, respectively. Recently, [16,17] considered the finite-time bipartite consensus protocols design for multi-agent systems with first-order dynamics over signed graph. But many real systems are with second-order dynamics, such as the manipulators and spacecrafts, so the finite-time bipartite consensus problems of multi-agent systems with second-order dynamics and antagonistic interactions should be further studied. It should be pointed out that the existing finite-time consensus protocols [24–30,33–35] can only be applicable to second-order multi-agent networks with fully cooperative interactions.

On the other hand, the effects brought by disturbances and noises are unavoidably for multi-agent systems in real physical environment [26–28,34,35]. It is of vital importance to consider the finite-time consensus for multi-agent systems with external disturbances, and some efforts have been made on this issue, such as [26,27,35] used the terminal sliding mode (TSM) method and the adding a power integrator method to address the finite-time control problems for second-order multi-agent systems with external disturbances, respectively. However, the upper bounds of

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the external disturbances are required to be known in [26,27,35], which are difficult to obtain in practice. Moreover, the sign function is used to design robust controller to attenuate the influences of disturbances in [26,27,35], which may lead to chattering problem. To mitigate these shortcomings, the adaptive control technique presents a good approach [14], but how to achieve adaptive finite-time consensus is rather challenging, especially for multi-agent systems with antagonistic interactions. As far as we know, the adaptive finite-time bipartite consensus for multi-agent systems with antagonistic interactions have not been studied yet, and this may be of interest as concluded in [11].

Based on the above discussion, in this paper, we will consider the adaptive finite-time bipartite consensus problems for multi-agent systems with second-order dynamics and external disturbances, and a signed undirected graph will be used to describe the interactions among agents. Compared with the existing related works, this work differs in two aspects: (1) the bipartite consensus is achieved for second-order multi-agent systems with cooperative and antagonistic interactions simultaneously by using finite-time control technique; (2) the upper bounds of external disturbances are not required in control law design. The main contributions of this paper are stated as follows. Firstly, new adaptive finite-time distributed control algorithms are established for second-order multi-agent systems with unknown external disturbances over signed undirected graphs by using the adding a power integrator technique and adaptive control technique, which can provide finite-time convergence, robustness, and high control precision. Secondly, the proposed distributed protocols in light of novel adaptive control architecture are continuous and chattering-free.

The rest of the paper is organized as follows. Section 2 gives some preliminaries on graph theory. The main theoretical results are discussed in Section 3. In Section 4, a numerical example is given to illustrate the theoretical results. Section 5 gives the conclusions at last.

2. Problem formulation

2.1. Graph theory

A weighted signed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ is used to describe the network with antagonistic interactions, where $\mathcal{V} = \{1, \ldots, n\}$ is the node set, $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set, and $\mathcal{A} = [a_{ij}] \in \Re^{n\times n}$ is the adjacency matrix of the signed weights of $\mathcal{G}$, where $a_{ij} \neq 0 \Leftrightarrow (j, i) \in \mathcal{E}$. Assume that no self-loops exist in $\mathcal{G}$, i.e., $a_{ii} = 0$, $\forall i \in \mathcal{V}$. $(j, i) \in \mathcal{E}$ means that there is an edge from node j to node i, and $N_i = \{j : (j, i) \in \mathcal{E}\}$ denotes the set of neighbors of node i. For undirected $\mathcal{G}$, if $(j, i) \in \mathcal{E}$, we also have $(i, j) \in \mathcal{E}$, so $\mathcal{A}$ is a symmetric matrix. $\mathcal{G}$ is said to have a path if there exists a finite sequence nodes $i_1, \ldots, i_m$ such that $(i_o, i_{o+1}) \in \mathcal{E}, \forall o = 1, \ldots, m-1$. Furthermore, $\mathcal{G}$ is said to be strongly connected if any two nodes are linked with a path. Note that the strongly connected implies connected for undirected $\mathcal{G}$. To analyze the bipartite consensus, an important concept called structurally balanced is proposed in [11] for signed graph $\mathcal{G}$, which describes that the nodes of $\mathcal{G}$ have a bipartition $\{\mathcal{V}_1, \mathcal{V}_2\}$, in which $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$ and $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$, such that $a_{ij} \geq 0$ for $\forall i, j \in \mathcal{V}_l (l \in \{1, 2\})$ and $a_{ij} \leq 0$ for $\forall i \in \mathcal{V}_l, j \in \mathcal{V}_q, l \neq q (l, q \in \{1, 2\})$.

Consider another signed graph $\bar{\mathcal{G}} = \{\bar{\mathcal{V}}, \bar{\mathcal{E}}, \bar{\mathcal{A}}\}$ to describe the interaction network involving leader agent 0, where $\bar{\mathcal{V}} = \mathcal{V} \cup \{0\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. The interaction network for the followers is described by signed graph $\mathcal{G}$. Denote $B = \mathrm{diag}\{b_1, b_2, \ldots, b_n\} \in \Re^{n\times n}$ as the connection matrix between the follower and the leader, and if the ith follower node is connected to the leader, then $b_i > 0$; otherwise, $b_i = 0$.

The Laplacian matrix of signed graph $\mathcal{G}$ is defined as $L = C - \mathcal{A}$, where C is the connectivity matrix, and the elements of L are

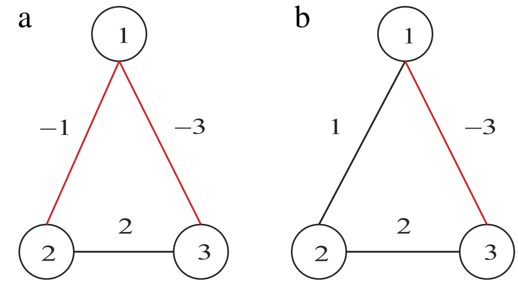

Fig. 1. Signed undirected connectivity graphs. (a): Example 1: structurally balanced; (b): Example 2: structurally unbalanced.

$l_{ij} = \begin{cases} \sum_{j\in N_i} |a_{ij}|, j = i \\ -a_{ij}, j \neq i \end{cases}$ [11]. For a signed undirected graph $\mathcal{G}$, $\forall x = [x_1, \ldots, x_n]^T \in \Re^n$, we establish the Laplacian potential as

$$\Phi(x) = x^T L x = \frac{1}{2}\sum_{i=1}^{n}\sum_{j\in N_i} |a_{ij}|(x_i - \mathrm{sign}(a_{ij})x_j)^2. \tag{1}$$

Let $\lambda_1(L) \leq \lambda_2(L) \leq \cdots \lambda_n(L)$ as the eigenvalues of L, then from $\Phi(x) \geq 0$, we have $\lambda_1(L) \geq 0$. Different with the case of nonnegative $\mathcal{A}$ for the graph with the weights of edges that are nonnegative, $-L$ is no longer a Metzler matrix in general, and its row sum and column sum need not be zero [11]. Compared with the standard theory of nonnegative adjacency matrices, where the coupling weights a_{ij} are nonnegative, the major difference is that L can be positive definite. We will give the following example to show this point.

Example: Consider the signed undirected connectivity graph in Fig. 1(a), we can see that it is structurally balanced and the adjacency matrix is

$$\mathcal{A}_1 = \begin{bmatrix} 0 & -1 & -3 \\ -1 & 0 & 2 \\ -3 & 2 & 0 \end{bmatrix}$$

so its Laplacian matrix is $L_1 = \begin{bmatrix} 4 & 1 & 3 \\ 1 & 3 & -2 \\ 3 & -2 & 5 \end{bmatrix}$ and has eigenvalues $\mathrm{sp}(L_1) = \{0, 4.2967, 7.7321\}$, which means that L_1 is semipositive definite. Next, Consider another signed undirected connectivity graph in Fig. 1(b), we can see that it is structurally unbalanced and the adjacency matrix is

$$\mathcal{A}_2 = \begin{bmatrix} 0 & 1 & -3 \\ 1 & 0 & 2 \\ -3 & 2 & 0 \end{bmatrix}$$

so its Laplacian matrix is $L_2 = \begin{bmatrix} 4 & -1 & 3 \\ -1 & 3 & -2 \\ 3 & -2 & 5 \end{bmatrix}$ and has eigenvalues $\mathrm{sp}(L_2) = \{1.1944, 2.3868, 8.4188\}$, which means that L_2 is positive definite.

2.2. Some lemmas

The following lemmas will be used in the sequel.

Lemma 1 ([17]). *For a connected signed graph $\mathcal{G}$, if $\mathcal{G}$ is structurally balanced, then a diagonal matrix $D = diag\{\sigma_1, \ldots, \sigma_n\}$ is existed such that DAD is nonnegative, where $\sigma_i \in \{-1, 1\}, i \in \mathcal{V}$, and the following performances are satisfied: 1. $\Phi(x)$ is semipositive definite, and $\Phi(x) = 0$ means $x = D\mathbf{1}_n c$ for some $c \in \Re$; 2. $\lambda_1(L) = 0$, $\lambda_2(L) > 0$, and $\Phi(x) \geq \lambda_2(L)x^T x$ for all x that satisfies $\mathbf{1}_n^T Dx = 0$.*

Lemma 2 ([19]). *Suppose $V(x)$ is a C^1 smooth positive-definite function (defined on $U \subset \Re^n$) and $\dot{V}(x) + \lambda V^{\alpha}(x)$ is a negative semi-definite function on $U \subset \Re^n$ and $\alpha \in (0, 1)$, then there exists an area*