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# A flocking algorithm with individual agent destinations and without a centralized leader

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ABSTRACT

We present a multi-agent control method that addresses the combined problem of flocking and destination seeking. The method is completely decentralized, that is, each agent's controller relies on local sensing to determine the relative positions and velocities of nearby agents but does not rely on a centralized flock leader. Each agent has double-integrator dynamics and a potentially unique destination (i.e., position) that the agent must reach. We demonstrate that the flocking-and-destination-seeking control method accomplishes 2 objectives: (i) if an agent is far from its destination, then that agent flocks with nearby agents, and (ii) if an agent is close to its destination, then that agent approaches its destination. The flocking-and-destination-seeking algorithm is demonstrated with several numerical examples.



## 1. Introduction

Multi-agent systems have many exciting applications such as distributed sensing, formation flying, cooperative surveillance, and point-to-point mail delivery. For example, autonomous aircraft or spacecraft can fly in formations for distributed sensing [1,2]. Coordinated aircraft could be used in a forest-fire scenario to measure wind velocities and thus, predict fire movement. In the agricultural industry, coordinated aircraft could conduct crop surveys. All of these applications require decentralized methods for coordinating and controlling groups of autonomous agents [3].

For coordinated control, each agent relies on sensing to determine the relative positions and velocities of nearby agents. Then, each agent uses these measurements combined with other information such as mission objectives to accomplish tasks, which can include: cohesion, collision avoidance, velocity matching, and guidance. Cohesion attracts an agent to nearby agents, whereas collision avoidance repels an agent from nearby agents (or obstacles). Velocity matching causes nearby agents to approach a consensus velocity, and guidance causes an agent or agents to follow a leader agent or approach a desired destination.

Cohesion and collision avoidance can be addressed using position-formation methods [4–6] or distance-formation methods [7–15]. Position-formation approaches force agents into a configuration using desired relative-position vectors between pairs of agents. In contrast, distance-formation methods induce a configuration using only a desired distance between adjacent agents. In this case, the agents autonomously determine their configuration based on the desired interagent distance and initial conditions. A common approach for distance formation is to use potential functions that create attractive forces when nearby agents are too far away and repulsive forces when nearby agents are too close [7–14]. A survey of multi-agent formation methods is presented in [16]. Consensus algorithms [17–20] are used to achieve velocity matching. Approaches that use distance-formation methods for cohesion and collision avoidance, and consensus for velocity matching lead to formations called *flocks* [10–15].

Agent guidance is often addressed using leader–follower methods [2,5,7–13] or destination-seeking methods [6,21–23]. Leader–follower approaches rely on a centralized leader, who can be an actual or virtual member of the formation and whose real-time position and velocity are known by all agents [8–13] or at least by some [2,5,7]. Each agent uses knowledge of the centralized leader and measurements of nearby agents to induce a formation and follow the leader. In contrast, destination-seeking methods (e.g., [6,21–23]) cause agents to approach desired destinations. The flocking algorithms with leader–follower guidance in [5,7–13] do not address destination seeking, and the destination-seeking methods in [6,21–23] do not address flocking. In contrast to [2,4–16,21–23], this paper addresses the combined problem of flocking and destination seeking.

The flocking-and-destination-seeking control objective is twofold—if an agent is far from its destination, then it flocks with nearby agents, but ultimately each agent approaches its destination. The flocking-and-destination-seeking algorithm in this paper uses a distance-formation approach for cohesion and collision avoidance, a consensus algorithm for velocity matching, and a destination-seeking method for guidance. The main analytic results in this paper examine the formation properties of agents

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and provide sufficient conditions for agents to converge to their destinations. Flocking and destination seeking is also considered in [24]; however, the analysis in [24] considers only a single destination and does not examine formation properties.

The flocking-and-destination-seeking algorithm that we present is completely decentralized, namely, each agent's controller does not incorporate a centralized leader and relies on only local sensing to determine the relative positions and velocities of nearby agents. Each agent has knowledge of its own destination but does not require knowledge of other agents' destinations. The controller in this paper achieves multiple objectives, that is, flocking and destination seeking, and thus, extends the work of [2,4–16,21–24].

The combined flocking-and-destination-seeking problem has applications such as point-to-point passenger transport and point-to-point mail delivery. For example, consider a group of autonomous ground vehicles on a highway, where each vehicle has a unique destination that it needs to reach. While traveling to the destination, it is beneficial for a vehicle to reduce wind resistance and energy expenditure by drafting off neighboring vehicles. When a vehicle gets close to its destination, it leaves the flock and approaches the destination. The remaining vehicles then form a new flock and repeat the process until all vehicles reach their destinations. Example 2 in Section 7 applies the flocking-and-destination-seeking algorithm to a vehicles-on-a-highway problem.

## 2. Problem formulation

Let the positive integer $n$ be the number of agents, and define $\mathcal{I} \triangleq \{1, 2, \ldots, n\}$, which is the agent index set. For each $i \in \mathcal{I}$, consider the double-integrator dynamics

$$\dot{q}_i(t) = p_i(t), \tag{1}$$

$$\dot{p}_i(t) = u_i(t), \tag{2}$$

where $t \geq 0$; $q_i(0)$ and $p_i(0)$ are the initial conditions; and $q_i(t) \in \mathbb{R}^m$, $p_i(t) \in \mathbb{R}^m$, and $u_i(t) \in \mathbb{R}^m$ are the position, velocity, and control of the $i$th agent, respectively. Define $\mathcal{P} \triangleq \{(i, j) \in \mathcal{I} \times \mathcal{I} : i \neq j\}$, which is the set of ordered pairs, and let $\|\cdot\|$ denote the Euclidean norm.

The rules for flocking are that agents stay close to one another, avoid collisions, and match velocities [25]. We use these rules to define flocking. Let $T$ be a connected subset of $[0, \infty)$. Then, the agents in $\mathcal{I}$ *flock with radius* $d > 0$ *over the interval* $T$ if the following conditions hold:

(F1) For all $(i, j) \in \mathcal{P}$ and all $t \in T$, $q_i(t) \neq q_j(t)$.
(F2) For all $(i, j) \in \mathcal{P}$ and all $t \in T$, $\|p_j(t) - p_i(t)\| \approx 0$.
(F3) For all $i \in \mathcal{I}$ and all $t \in T$, $\max_{j\in\mathcal{I}\setminus\{i\}} \|q_j(t) - q_i(t)\| \leq d(n-1)$.
(F4) For all $i \in \mathcal{I}$ and all $t \in T$, $\min_{j\in\mathcal{I}\setminus\{i\}} \|q_j(t) - q_i(t)\| \approx d$.

Condition (F1) states that no agents occupy the same position at the same time. Condition (F2) states that all agents have approximately the same velocity. Condition (F3) states that each agent is at most a distance $d(n-1)$ away from its farthest neighbor. Condition (F4) states that each agent maintains a distance of approximately $d$ from its nearest neighbor.

We address not only flocking but also destination seeking. For all $i \in \mathcal{I}$, let $\xi_i \in \mathbb{R}^m$ be the $i$th agent's destination. Let $r_\beta \geq 0$, and for each $i \in \mathcal{I}$ and each $t \geq 0$, we say the *$i$th agent is far from its destination* if $\|\xi_i - q_i(t)\| > r_\beta$. Assume there exists $t_f > 0$ such that for all $t \in [0, t_f)$, the agents in $\mathcal{I}$ are far from their destinations. In this case, we consider 2 objectives:

(O1) *Flocking:* The agents in $\mathcal{I}$ flock with radius $d > 0$ over a connected subset of $[0, t_f)$.
(O2) *Destination seeking:* For all $i \in \mathcal{I}$, $\lim_{t\to\infty} q_i(t) = \xi_i$ and $\lim_{t\to\infty} p_i(t) = 0$.

Objective (O1) states that if agents are far from their destinations, then they flock. Objective (O2) states that each agent approaches its destination asymptotically. Unless otherwise stated, all statements in this paper that involve the subscript $i$ are for all $i \in \mathcal{I}$.

## 3. Review of Algorithm 1 from [10]

We review Algorithm 1 from [10], which is a flocking method for agents with double-integrator dynamics. Let $\epsilon > 0$, and consider $\|\cdot\|_\epsilon : \mathbb{R}^m \to [0, \infty)$ defined by

$$\|x\|_\epsilon \triangleq \tfrac{1}{\epsilon}\left(\sqrt{1 + \epsilon\|x\|^2} - 1\right). \tag{3}$$

Note that $\|\cdot\|_\epsilon$ is continuously differentiable on $\mathbb{R}^m$, but $\|\cdot\|_\epsilon$ is not a norm on $\mathbb{R}^m$. Define $\sigma_\epsilon : \mathbb{R}^m \to \mathbb{R}^m$ by

$$\sigma_\epsilon(x) \triangleq \left(\frac{\partial}{\partial x}\big[\|x\|_\epsilon\big]\right)^{\mathrm{T}} = \frac{x}{1 + \epsilon\|x\|_\epsilon}. \tag{4}$$

Next, let $h \in (0, 1)$, and define $\rho_h : [0, \infty) \to [0, 1]$ by

$$\rho_h(\eta) \triangleq \begin{cases} 1, & \text{if } \eta \in [0, h), \\ \frac{1}{2} + \frac{1}{2}\cos\pi\frac{\eta - h}{1 - h}, & \text{if } \eta \in [h, 1], \\ 0, & \text{if } \eta \in (1, \infty), \end{cases} \tag{5}$$

which decreases from 1 to 0 as $\eta$ increases from 0 to $\infty$, and the rate of change of $\rho_h$ depends on $h$. Let $b \geq a > 0$, define $c \triangleq (b - a)/\sqrt{4ab}$, and consider $\phi : \mathbb{R} \to (-b, a)$ defined by

$$\phi(\eta) \triangleq \tfrac{1}{2}\left[\frac{(a+b)(\eta+c)}{\sqrt{1+(\eta+c)^2}} + (a - b)\right], \tag{6}$$

which is a sigmoidal function.

Next, let $r_c > 0$ be the *communication radius*, which is the maximum distance at which an agent can sense another agent's relative position and relative velocity. For all $t \geq 0$, define the *neighbor set* $\mathcal{N}_i(t) \triangleq \{j \in \mathcal{I} \setminus \{i\} : \|q_j(t) - q_i(t)\| < r_c\}$, which is the set of agents whose distance to the $i$th agent is no greater than the communication radius $r_c$ at time $t$. Let $d \in (0, r_c]$ be the *flock radius*, which is the desired distance between agents in the flock.

For all $t \geq 0$, define $q(t) \triangleq [q_1^{\mathrm{T}}(t) \ \cdots \ q_n^{\mathrm{T}}(t)]^{\mathrm{T}}$ and $p(t) \triangleq [p_1^{\mathrm{T}}(t) \ \cdots \ p_n^{\mathrm{T}}(t)]^{\mathrm{T}}$. Then, [10, Algorithm 1] considers the control $u_i(t) = v_i(q(t), p(t))$, where

$$v_i(q, p) \triangleq \underbrace{\sum_{j\in\mathcal{N}_i} \rho_h\left(\frac{\|q_j - q_i\|_\epsilon}{\|r_c\|_\epsilon}\right)\Phi(q_j - q_i)}_{\text{Flock attraction and repulsion}} + \underbrace{\sum_{j\in\mathcal{N}_i} \rho_h\left(\frac{\|q_j - q_i\|_\epsilon}{\|r_c\|_\epsilon}\right)[p_j - p_i]}_{\text{Velocity consensus}}, \tag{7}$$

and $\Phi : \mathbb{R}^m \to \mathbb{R}^m$ is defined by $\Phi(x) \triangleq \phi(\|x\|_\epsilon - \|d\|_\epsilon)\sigma_\epsilon(x)$.

For each $t \geq 0$ and each $j \in \mathcal{N}_i(t)$, the flock-attraction-and-repulsion term in (7) is such that the $i$th agent is attracted to the $j$th agent if $\|q_j(t) - q_i(t)\| > d$, and repelled from the $j$th agent if $\|q_j(t) - q_i(t)\| < d$. The velocity-consensus term in (7) attempts to match the $i$th agent's velocity with a weighted average of the velocities of all agents in the neighbor set.

The parameters in the control (7) are $\epsilon$, $h$, $a$, and $b$. Increasing $\epsilon$ decreases the strength of the flock-attraction-and-repulsion term relative to the strength of the velocity-consensus term. Increasing $h$ increases the rate of change of $\rho_h$. Increasing $a$ increases the strength of attraction relative to the strength of repulsion and velocity consensus. Increasing $b$ increases the strength of repulsion relative to the strength of attraction and velocity consensus.

Theorem 1 of [10] provides conditions such that the agents with dynamics (1), (2) and control $u_i = v_i$ form at least one