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Hierarchical analysis of large-scale control systems via vector simulation function ☆

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ABSTRACT

In this paper, we study the hierarchical problem of large-scale control systems. As a natural extension of traditional simulation function, the new notion of vector simulation function is introduced for investigating the hierarchies between abstract systems and concrete systems. By constructing a comparison system, we present a generalized result on this problem which, in the case of a scalar simulation function, specializes to the classical result. When interconnected nonlinear systems are considered, an easily checkable sufficient condition is obtained to facilitate the compositional construction of abstractions. Based on the condition, we propose a particular construction of abstractions for interconnected linear systems. Finally, two examples are included.

1. Introduction

Addressing the inherent difficulty in analyzing and controlling large-scale interconnected system is an important relevant problem in both modern engineering and control theory. One way is to place a hierarchical structure on this system such that the high-level abstract system could act as a substitute in the performance analysis and controller design processes. In the area of formal verification of dynamic system, a quotient system is used to verify whether a given dynamic system satisfies certain properties [1]. In this context, the abstract system is defined as the quotient system dynamics [2], which is constructed by a so-called quotient map. Another closely related notion is simulation relation [3]. In fact, there always exists a simulation relation from an original control system to the associated quotient system [4].

For the controller design processes, the simulation function [5,6] allows us to synthesize controller for a large-scale system from a simple system. In our context of this paper, we will call this simple system as abstract system, or simply, abstraction if there exists a simulation function from it to the original complex system whereas it was referred to as approximate abstraction instead of abstraction in [7]. A simulation function relates the concrete system and its abstraction by establishing a quantitative bound of the distance between the output trajectories of both systems. In order to employ the advanced nonlinear control techniques, the simulation function in this paper is defined in the similar manner as Lyapunov function with some small modifications [7]. In addition, model reduction is a notion related to model abstraction. For an overview of model reduction techniques, we refer the reader to [8] and the references therein. The relations between them have been clarified in [2,9].

In this paper, we study the hierarchical problem which aims to verify whether a simple system can serve as an abstraction and construct it if possible. To simplify the verification of an abstraction by the simulation function, as a natural extension, the new notion of vector simulation function (VSF) is introduced. Just as vector Lyapunov function compared to scalar Lyapunov function [10], VSF offers more flexibilities for analyzing and controlling large-scale systems as compared to (scalar) simulation function. The time derivative of the VSF satisfies an element-by-element inequality involving a vector field of a certain comparison system. By the input-to-state stable (ISS) property of this comparison system, we present a generalized result which, in the case of a simulation function, specializes to the classical hierarchical result. Then for a class of interconnected control systems, a VSF is obtained by augmenting some simulation functions for subsystems and an easily checkable sufficient condition is derived to facilitate the compositional construction of an abstraction. Moreover, we show the fact that the interconnection of abstractions of some subsystems is, under this condition, an abstraction of the interconnection of these subsystems. Finally, we propose a particular scheme for the compositional construction of abstractions for interconnected linear systems.

The remainder of this paper is organized as follows. In Section 2, we recall some necessary notions and definitions, in particular the hierarchical framework for general nonlinear systems. Section 3 contains the main results of this paper. First of all, Section 3.1

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defines the VSF by the ISS property of a comparison system and proves the more general result than what introduced in Section 2.1. This is followed by the application of this new result to a class of interconnected control systems in Section 3.2. This naturally leads to a particular construction of abstractions for interconnected linear systems in Section 3.3. Section 4 presents two examples including a linear case and a nonlinear case.

2. Preliminaries

Let $\mathbb{R}^n$ denote the set of $n \times 1$ column vectors. $\mathbb{R}^n_+$ denotes the nonnegative orthant of $\mathbb{R}^n$, that is, the set of vectors with all the components being nonnegative real numbers. In particular, $\mathbb{R}_+$ is the set of nonnegative real numbers. For a vector x in $\mathbb{R}^n$, $\|x\|$ is the Euclidean norm while $\|x\|_1$ denotes the 1-norm. For an essentially bounded and measurable signal x, defined from $\mathbb{R}_+$ to $\mathbb{R}^n$, the $\mathcal{L}_\infty$ signal norm is denoted by $\|x\|_\infty := ess\ sup_{\tau\geq 0}\|x(\tau)\|$. For brevity, the set of measurable functions for which this norm is finite is denoted by $\mathcal{L}_\infty[\mathbb{R}_+, \mathbb{R}^n]$. For a pair of vectors x, y in $\mathbb{R}^n$, we write $x \prec y$ (respectively, $x \preceq y$) to indicate that every component of $x-y$ is negative (respectively, nonpositive), i.e., $x_i < y_i$ (respectively, $x_i \leq y_i$) for all $i = 1, \ldots, n$, where x_i denotes the ith component of x. $\succ$ and $\succeq$ are defined in an obvious way. A continuous function $\gamma : [0, a) \to \mathbb{R}_+$ is a $\mathcal{K}$ function if it is strictly increasing and satisfies $\gamma(0) = 0$. It is said to belong to class $\mathcal{K}_\infty$ if $a = \infty$ and $\gamma(r) \to \infty$ as $r \to \infty$. A continuous function $\beta : \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R}_+$ is a $\mathcal{KL}$ function if for each fixed s, the mapping $\beta(r, s)$ belongs to $\mathcal{K}$ function with respect to r, for each fixed r, the mapping $\beta(r, s)$ is decreasing with respect to s and $\beta(r, s) \to 0$ as $s \to \infty$. A function $\Lambda = [\Lambda_1, \ldots, \Lambda_n] : \mathbb{R}^n \to \mathbb{R}^n$ is quasimonotone nondecreasing if $\Lambda_i(x) \leq \Lambda_i(y)$, $i = 1, \ldots, n$ for any two points x, y in $\mathbb{R}^n$ satisfying $x_i = y_i$ and $x_j \leq y_j$, $j = 1, \ldots, n, j \neq i$. Λ is essentially nonnegative if $\Lambda_i(x) \geq 0$ for all $i = 1, \ldots, n$, and $x \in \mathbb{R}^n_+$ such that $x_i = 0$. A real square matrix A is *Metzler* if its off-diagonal entries are nonnegative. In addition, $\circ$ denotes the composition of two functions, and $col(x_1, \ldots, x_r)$ denotes $[x_1^T, \ldots, x_r^T]^T$ for any column vectors $x_1, \ldots, x_r$. Recall some concepts in linear systems theory. We use the usual symbols imB and $kerB$ to denote image and kernel of $B \in \mathbb{R}^{n\times m}$. (A, B) with $A \in \mathbb{R}^{n\times n}$ is stabilizable if there exists $K \in \mathbb{R}^{m\times n}$ such that $A + BK$ is Hurwitz.

2.1. Hierarchical framework of nonlinear systems

Let us consider the following nonlinear system

$$\Sigma : \begin{cases} \dot{x}(t) = f(x(t), u(t)) \\ y(t) = g(x(t)) \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^p$, $y(t) \in \mathbb{R}^k$. Assume that the vector field $f : \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ satisfies usual regularity assumptions so that for any essentially bounded measurable input function $u : \mathbb{R}_+ \to \mathbb{R}^p$ and initial state $x(0) \in \mathbb{R}^n$, this differential equation has a unique state trajectory on the whole non-negative time axis.

Now, we assume a much simpler system in the similar form:

$$\hat{\Sigma} : \begin{cases} \dot{\hat{x}}(t) = \hat{f}(\hat{x}(t), \hat{u}(t)) \\ \hat{y}(t) = \hat{g}(\hat{x}(t)) \end{cases} \tag{2}$$

where $\hat{x}(t) \in \mathbb{R}^{\hat{n}}$, $\hat{u}(t) \in \mathbb{R}^{\hat{p}}$, $\hat{y}(t) \in \mathbb{R}^k$. Similarly, we impose the same assumption on uniqueness and existence of solutions. Noting that both systems must have the same dimension of output space while $\hat{\Sigma}$ is, in general, smaller in state space dimension as compared to Σ, i.e., $\hat{n} < n$.

The system Σ is the practical model we actually want to control. However, the complexities in analyzing and controlling large-scale, interconnected systems lead to inherent difficulties and untractable problems. In our approach, control will be synthesized hierarchically in $\hat{\Sigma}$, which acts as a substitute in the analyses and controller design processes. Both systems are linked by the following definition:

Definition 1. For both systems Σ and $\hat{\Sigma}$, if there exist a $\mathcal{KL}$ function β and a $\mathcal{K}$ function γ and a continuous function $\varrho : \mathbb{R}^n \times \mathbb{R}^{\hat{n}} \to \mathbb{R}_+$ such that for any input $\hat{u}(t) \in \mathcal{L}_\infty[\mathbb{R}_+, \mathbb{R}^{\hat{p}}]$ of $\hat{\Sigma}$ and $x(0) \in \mathbb{R}^n$, $\hat{x}(0) \in \mathbb{R}^{\hat{n}}$, there exists an input $u(t) \in \mathcal{L}_\infty[\mathbb{R}_+, \mathbb{R}^p]$ of Σ so that we have

$$\|y(t) - \hat{y}(t)\| \leq \beta(\varrho(x(0), \hat{x}(0)), t) + \gamma(\|\hat{u}\|_\infty) \tag{3}$$

where $y(t)$ and $\hat{y}(t)$ are output trajectories of Σ associated with the input $u(t)$, the initial state $x(0)$ and $\hat{\Sigma}$ associated with the input $\hat{u}(t)$, the initial state $\hat{x}(0)$, respectively, then $\hat{\Sigma}$ is called an abstract system or simply abstraction, and Σ is called a concrete system.

If we think of (1)–(2) as a whole with the state space in $\mathbb{R}^{n+\hat{n}}$, (3) is similar to a special case of Multiple-Measure Input Stable (MMIS) [11]. However, in our notion two inputs u and $\hat{u}$ are introduced with an existential quantifier as opposed to a universal quantifier while only one input exists in MMIS, which is the main difference between them. This will facilitate hierarchical control design.

One way to verify (3) is to use a so-called simulation function, which is a Lyapunov-like function introduced in [5,12]. We recall this notion by the following modified definition.

Definition 2. A continuous function $V_s : \mathbb{R}^n \times \mathbb{R}^{\hat{n}} \to \mathbb{R}_+$ is called a simulation function from $\hat{\Sigma}$ to Σ if for any $x \in \mathbb{R}^n$, $\hat{x} \in \mathbb{R}^{\hat{n}}$, $\hat{u} \in \mathbb{R}^{\hat{p}}$, there exists $u \in \mathbb{R}^p$ so that the following inequalities hold:

$$V_s(x, \hat{x}) \geq \alpha_s(\|g(x) - \hat{g}(\hat{x})\|) \tag{4}$$

$$\begin{bmatrix} \dfrac{\partial V_s}{\partial x}, & \dfrac{\partial V_s}{\partial \hat{x}} \end{bmatrix} \begin{bmatrix} f(x, u) \\ \hat{f}(\hat{x}, \hat{u}) \end{bmatrix} \leq -\rho_s(V_s(x, \hat{x})) + \sigma_s(\|\hat{u}\|) \tag{5}$$

where α_s and ρ_s are $\mathcal{K}_\infty$ functions, σ_s is $\mathcal{K}$ function.

The definition of simulation function is similar to that of Input-to-State Stability-Control Lyapunov Function (ISS-CLF) [13,14]. Despite this, some differences between them still exist. At first, the simulation function is employed to bound the distance between the output trajectories of both systems. Hence, V_s is linked to the output error by (4), which implies V_s is only positive semi-definite. Also, the argument of ρ_s in (5) is set to be V_s instead of $\|(x^T, \hat{x}^T)\|$ since the comparison in the output sense between concrete system and abstract system is our main focus.

Theorem 1. *Suppose there exists a simulation function V_s from $\hat{\Sigma}$ to Σ. Then $\hat{\Sigma}$ is an abstraction of the concrete system Σ.*

Proof. The proof is similar to that in [7] hence omitted. And $\varrho = V_s$. □

If a continuous function $u = u(x, \hat{x}, \hat{u})$ exists so that (5) is satisfied, then this function will be called an interface function or simply interface [5]. An interface shows its usefulness by transferring or refining the controller that we design for the abstraction $\hat{\Sigma}$ to a controller for the concrete system Σ.

3. Main results

For large-scale system, especially the interconnected system, the construction of simulation function is usually complicated. VSF may be a good alternative to simulation function as a more general concept.

3.1. The introduction of VSF

Now, we will introduce the notion of VSF upon which our main results build.