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Robust time-domain output error method for identifying continuous-time systems with time delay

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ABSTRACT

In this paper, an output error method is proposed for the identification of continuous-time systems with time delay from sampled data. The challenge of time delay system identification lies in the presence of nonlinear time delays in models, then starting value-based optimization methods may be trapped easily by local minima. In order to improve the convergence performance to the choice of initial parameters, several approaches to smooth the loss function are presented. It is shown that the loss function may possess many local minima when data are regularly sampled with the inter-sample behavior of zero-order hold. Interestingly, irregular sampling can be an efficient approach to overcome these local minima. To achieve superior convergence performance, an over-parametrization approach incorporating a low-pass filtering technique is proposed to enlarge the convergence region. Theoretical and simulated results are presented to demonstrate the effectiveness of the proposed method.

1. Introduction

Control-oriented system identification, which aims to build mathematical models for characterizing system behaviors between the manipulated and controlled variables from experimental data, has been increasingly appealing for obtaining good control system performance. With the fast development of digital computers, system identification methods have been developed based on discrete-time (DT) models in terms of shift operators to facilitate the implementation. Recently, there has been a resurgence of interest in the study of continuous-time (CT) model identification, which is motivated by the advantages of CT models, for example: physical insights provided by CT model parameters; flexibility in dealing with fractional time delays; invariance of model parameters in handling irregularly sampled data [1,2], note that in such case DT model parameters are usually time-varying even if the original systems are stationary and invariant. A common feature of industrial processes is the presence of a time delay between the input command and output response. Modeling of these systems, typically by low order linear models plus time delay, has been widely recognized for obtaining good approximations of the system input–output behaviors. Identification of process models from sampled data is the problem that will be studied in this paper.

The main difficulty of process model identification arises from the presence of a nonlinear parameter, *i.e.* the time delay, in a linear model. Here a nonlinear parameter means that the differentiation with respect to this parameter does not yield a constant. Then some traditional methods for linear system identification cannot be applied directly. The study of time delay system identification has led in the literature to several methods, recent reviews can be found in [3–5]. The existing methods fall into three categories roughly: (1) identification using different experiment tests, *e.g.* step or relay feedback tests [5–8], persistent excitation tests [9,10]; (2) identification using different strategies to estimate the whole model parameters, *e.g.* one-step approaches that estimate all the parameters simultaneously [11–14], two-step approaches that estimate the rational model parameters and time delay separately [9,10]; (3) identification using different criteria for optimal model fitting, *e.g.* cross-correlation maximization [15,16], output or prediction error minimization [10,11].

The simplified refined instrumental variable method for CT systems (SRIVC) has been popular for direct CT modeling [17–19]. This method was first proposed in [20] and later developed in *e.g.* [21–23]. By extension, a SRIVC-based method for time delay systems (TDSRIVC) was proposed in [10] to identify simple process models from irregularly sampled data, in which a numerical search was incorporated to solve for the optimal time delay. Since the numerical search is quite sensitive to the choice of initial time

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delays, a low-pass filtering technique was suggested in [10,24] in order to widen the attraction region, therefore improving the convergence performance. However, the study in [10] is still far from being enough. In this paper, we present an output error (OE) method for the considered identification problem, in which the rational model parameters and time delay are simultaneously estimated. As the contributions of this paper, three issues related to this identification problem are studied, *i.e.* the convergence of parameter estimates, the merit of irregular sampling, and possible strategies to widen the convergence region. It is shown that irregular sampling may smooth the local minima originating from the inter-sample behavior of zero-order hold (ZOH). Note that irregular sampling only gives local refinements to the loss function, while the overall function shape still remains the same. More powerful approaches are based on low-pass filtering to widen the convergence region [10,24]. Though this technique has been proposed some time ago, strategies for optimal implementation have seldom been investigated, and this is an issue that will be addressed in this paper.

1.1. Problem formulation

Consider a linear time-invariant, single-input single-output, CT system with input $u(t)$ and output $y(t)$ related by

$$\mathcal{S}:\begin{cases} x(t)=G_o(p)u(t-\tau_o)=\dfrac{B_o(p)}{A_o(p)}u(t-\tau_o)\\ y(t_k)=x(t_k)+e(t_k)\end{cases}\tag{1}$$

where $0\le\tau_o<\infty$ is the pure time delay, $x(t)$ the state response of the system, and $y(t_k)$ the observed output at time-instant t_k. $\{e(t_k)\}$ is a DT white noise process of any possible distribution uncorrelated to the excitation signal. Moreover, as required by Lemma 4, $u(t)$ is locally integrable and persistently exciting of order no less than n_a+n_b+2. $B_o(p)$ and $A_o(p)$ are the system polynomials in the differential operator p, *i.e.* $p=\frac{d}{dt}$

$$B_o(p)=b_0^o p^{n_b}+\cdots+b_{n_b-1}^o p+b_{n_b}^o\tag{2}$$
$$A_o(p)=a_0^o p^{n_a}+\cdots+a_{n_a-1}^o p+1,\quad n_a\ge n_b+1.\tag{3}$$

It is assumed that the system is stable, or equivalently, $A_o(p)$ has all roots located on the left-half plane. $A_o(p)$ and $B_o(p)$ are co-prime. The polynomial degrees satisfy $n_a\ge n_b+1$, this assumption will become clear once Eq. (18) is presented. The unknown parameters are stacked in

$$\boldsymbol{\rho}_o^\top=\left[\boldsymbol{\theta}_o^\top,\tau_o\right]\tag{4}$$
$$\boldsymbol{\theta}_o^\top=\left[a_0^o,\ldots,a_{n_a-1}^o,b_0^o,\ldots,b_{n_b}^o\right]\in\mathbb{R}^{n_a+n_b+1}.\tag{5}$$

Assume that the input–output signals are observed at regular or irregular time-instants $\{t_1,\ldots,t_N\}$ with $t_1\ge 0$, the resulting sampled data are denoted by $Z^N=\{u(t_k),y(t_k)\}_{k=1}^N$. The (time-varying) sampling interval is denoted by

$$h_k=t_{k+1}-t_k,\quad k=1,2,\ldots,N.\tag{6}$$

The initial conditions of both input/output signals are assumed to be zero. It is also necessary to constrain the final observation time-instant $t_N>\tau_o$ in order to ensure that the system is sufficiently excited during the interval (t_1,t_N). In direct CT identification, it is necessary to know, or at least to make some assumptions on, inter-sample behaviors of signals in order to reconstruct their CT counterparts from DT data. In this paper we assume that $u(t)$ is piecewise constant (namely ZOH) between contiguous sampling instants, this assumption is feasible since it has been widely used in applications, *e.g.* computer control systems. On the other hand, the inter-sample behavior of $y(t)$ is usually unknown. A first-order hold (FOH) assumption typically gives rise to a satisfactory approximation if the sampling interval is small.

Finally, provided that the model structure, and the polynomial degrees $\{n_a,n_b\}$ are known, the identification objective is to estimate the parameter vector $\boldsymbol{\theta}_o$ and time delay τ_o from the sampled data Z^N.

1.2. CT OE optimization problem

When the model structure, the polynomial degrees $\{n_a,n_b\}$ are known, System (1) can be represented by the following parametrized CT OE (COE) process model, in which $B(p,\boldsymbol{\theta})$ and $A(p,\boldsymbol{\theta})$ have the same structures as $B_o(p)$ and $A_o(p)$, but the dependence on the parameter vector $\boldsymbol{\theta}$ should be emphasized

$$\mathcal{M}:\begin{cases}\hat{x}(t)=G(p,\boldsymbol{\theta})u(t-\tau)=\dfrac{B(p,\boldsymbol{\theta})}{A(p,\boldsymbol{\theta})}u(t-\tau)\\ y(t_k)=\hat{x}(t_k)+\epsilon(t_k)\end{cases}\tag{7}$$

where $\epsilon(t_k)$ is the output error. The unknown parameters in the above model can be obtained by minimizing the OE loss function

$$\hat{\boldsymbol{\rho}}=\arg\min_{\boldsymbol{\rho}}V(\boldsymbol{\rho})\tag{8}$$

$$V(\boldsymbol{\rho})=\frac{1}{2(N-r)}\sum_{k=r+1}^{N}\epsilon^2(t_k)\tag{9}$$

where r is chosen to ensure $t_r<\tau\le t_{r+1}$. This choice is because $u(t_k-\tau)=0$ and $y(t_k)=e(t_k)$ for $t_k<\tau$, only the data $\{u(t_k),y(t_k)\}_{k=r+1}^N$ are informative for identification.

The remainder of this paper is arranged as follows: the COE method for time delay systems (TDCOE) is presented in Section 2, with the convergence analysis demonstrated in Section 3; subsequently, the merit of irregular sampling is investigated in Section 4; the over-parametrization approach to widen the convergence region is described in Section 5; then, numerical examples are given in Section 6 to illustrate the effectiveness of the proposed approaches; finally, conclusions are drawn in Section 7.

2. TDCOE method

A well-known method to solve (8) is the nonlinear least-squares method, which searches iteratively the optimal value of $\boldsymbol{\rho}$ starting from an initial guess $\hat{\boldsymbol{\rho}}^0$ as follows

$$\hat{\boldsymbol{\rho}}^{j+1}=\hat{\boldsymbol{\rho}}^j+\mu^j\left[\nabla^2V(\hat{\boldsymbol{\rho}}^j)\right]^{-1}\nabla V(\hat{\boldsymbol{\rho}}^j)\tag{10}$$

where μ^j is the step length, $\nabla V(\hat{\boldsymbol{\rho}}^j)$ and $\nabla^2V(\hat{\boldsymbol{\rho}}^j)$ are the gradient vector and Hessian matrix of $V(\hat{\boldsymbol{\rho}}^j)$, respectively. In view of (9), $\nabla V(\hat{\boldsymbol{\rho}}^j)$ can be readily computed by

$$\nabla V(\hat{\boldsymbol{\rho}}^j)=\frac{1}{N-r}J(\hat{\boldsymbol{\rho}}^j)\boldsymbol{\epsilon}(\hat{\boldsymbol{\rho}}^j)\tag{11}$$

where

$$\boldsymbol{\epsilon}(\hat{\boldsymbol{\rho}}^j)=\left[\epsilon(t_{r+1})\ \ \cdots\ \ \epsilon(t_N)\right]^\top\tag{12}$$

$$J(\hat{\boldsymbol{\rho}}^j)=-\begin{bmatrix}\dfrac{\partial\epsilon(t_{r+1})}{\partial\hat{\rho}_1^j} & \cdots & \dfrac{\partial\epsilon(t_N)}{\partial\hat{\rho}_1^j}\\ \vdots & & \vdots\\ \dfrac{\partial\epsilon(t_{r+1})}{\partial\hat{\rho}_\ell^j} & \cdots & \dfrac{\partial\epsilon(t_N)}{\partial\hat{\rho}_\ell^j}\end{bmatrix},\ \ell=n_a+n_b+2\tag{13}$$

with $\hat{\rho}_i^j$ denotes the ith element of the vector $\hat{\boldsymbol{\rho}}^j$. In order to avoid computing second order derivatives, the Gauss–Newton method is used to approximate $\nabla^2V(\hat{\boldsymbol{\rho}}^j)$

$$\nabla^2V(\hat{\boldsymbol{\rho}}^j)=\frac{1}{N-r}J(\hat{\boldsymbol{\rho}}^j)J^\top(\hat{\boldsymbol{\rho}}^j).\tag{14}$$