Systems & Control Letters 97 (2016) 27–35

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Stabilization of exponentially unstable discrete-time linear systems by truncated predictor feedback☆

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ARTICLE INFO

Article history:
Received 8 February 2016
Received in revised form
30 August 2016
Accepted 5 September 2016

Keywords:
Discrete-time
Exponentially unstable systems
Stabilization
Time delay
Truncated predictor feedback

ABSTRACT

Predictor state feedback solves the problem of stabilizing a discrete-time linear system with input delay by predicting the future state with the solution of the state equation and thus rendering the closed-loop system free of delay. The solution of the state equation contains a term that is the convolution of the past control input with the state transition matrix. Thus, the implementation of the resulting predictor state feedback law involves iterative calculation of the control signal. A truncated predictor feedback law results when the convolution term in the state prediction is discarded. When the feedback gain is constructed from the solution of a certain parameterized Lyapunov equation, the truncated predictor feedback law has been shown to achieve asymptotic stabilization of a system that is not exponentially unstable in the presence of an arbitrarily large delay by tuning the value of the parameter small enough. In this paper, we extend this result to exponentially unstable systems. Stability analysis leads to a bound on the delay and a range of the values of the parameter for which the closed-loop system is asymptotically stable as long as the delay is within the bound. The corresponding output feedback result is also derived.

1. Introduction

Time delay in the control input, as a cause of performance deterioration, is ubiquitous in control engineering. A basic control problem for systems with time delay is the problem of stabilization. Thanks to intensive research during the past few decades, various control design methods have been developed and numerous stability conditions established for linear systems with input delay (see, for example, [1–15]). Both continuous-time and discrete-time systems have been studied.

Among the various methods that achieve asymptotic stabilization for a linear system with input delay, predictor state feedback [16] is particularly appealing. It feeds the prediction of the future state into input of the system and results in a closed-loop system free of delay. The state prediction is simply the solution of the state equation of the system. As a result, the predictor feedback consists of two terms, one corresponding to the zero input solution, which is the product of the transition matrix from the current state to the future state in the time equal to the amount of delay, and the other the zero state solution that involves the convolution of the state transition matrix and the past input. The convolution with the past input increases the complexity in the implementation of the feedback law. Discarding the term associated with the zero state solution in the predictor feedback law leads to the so-called truncated predictor feedback (TPF), which leads to remarkable simplification.

Beginning from [17], the TPF has been utilized to stabilize linear systems with input delay. In particular, in [17], it was established that, a continuous-time linear system that is not exponentially unstable can be asymptotically stabilized by the TPF for an arbitrarily large delay if the feedback gain is constructed by the low gain feedback design technique [18] and the value of the low gain parameter is sufficiently small. Following [17,19] reached a parallel conclusion in the discrete-time setting, that is, a discrete-time linear system that is not exponentially unstable can be stabilized by the low gain based TPF in the presence of an arbitrarily large delay as long as the low gain parameter is tuned sufficiently small. The low gain feedback designs adopted in [17,19] are based on eigenstructure assignment [18]. Alternative TPF designs that adopt a parametric Lyapunov equation based low gain feedback design [20,21] were later proposed in [22,23] for continuous-time and discrete-time systems, respectively.

As an extension of the results in [22], [24] develops the truncated predictor feedback for exponentially unstable systems. The feedback law in the TPF law was designed based on the same

☆ This work was supported in part by the National Science Foundation under grant CMMI-1462171.

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http://dx.doi.org/10.1016/j.sysconle.2016.09.001
0167-6911/

parametric Lyapunov equation. A bound on the delay that can be tolerated by the TPF law is derived which is inversely proportional to the sum of the unstable poles of the open loop system. The range of the values of the parameter is determined for which the closed-loop system is asymptotically stable as long as the delay is within the bound.

The aim of this paper is to extend the results in [24] to the discrete-time setting. We consider a general discrete-time linear system, which could be exponentially unstable, subject to time-varying delay in the input. A discrete-time parametric Lyapunov equation based approach is adopted to construct feedback gain in the TPF law. Properties of the solution, some of which do not have a continuous-time counterparts, of the parametric Lyapunov equation are established. With the help of these properties, a delay bound and explicit conditions on the feedback gain parameter are then determined under which the asymptotic stability of the system is guaranteed. It is also observed that, as all the exponentially unstable poles of the system approach the unit circle, the delay bound goes to infinity. This observation is consistent with the results in [19,23], which showed that for a system whose poles are inside or on the unit circle, asymptotic stabilization can be achieved for an arbitrarily large delay by the TPF. We will also develop output feedback results.

The remainder of this paper is organized as follows. Section 2 states the problem and establishes properties of the solution to the discrete-time parametric Lyapunov equation for exponentially unstable systems. In Section 3, a delay bound and explicit conditions on the parameter are established under which the asymptotic stability of the closed-loop system is guaranteed. Output feedback results are then developed in Section 4. Numerical examples are provided in Section 5. Section 6 concludes the paper.

Notation. Throughout the paper, we use standard notation. In particular, we use $\mathbb{R}$, $\mathbb{N}$ and $\mathbb{Z}$ to denote the sets of all real numbers, all nonnegative integers and all integers, respectively. Also, for $a, b \in \mathbb{Z}$, $a \leq b$, $\mathbf{I}[a, b]$ is the set of all integers in $[a, b]$. For a real matrix M, $\|M\|$ is its induced norm. Finally, N_p denotes a neighborhood of a point $p \in \mathbb{R}$.

2. Problem statement and preliminaries

In this paper, we consider a discrete-time linear system with time-varying input delay,

$$\begin{cases} x(k+1) = Ax(k) + Bu(\phi(k)), & k \geq 0, \\ y(k) = Cx(k), \\ x(k) = \psi(k), & k \in \mathbf{I}[-K, 0], \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $u(k) \in \mathbb{R}^m$ and $y(k) \in \mathbb{R}^q$ are the state vector, input vector and output vector, respectively. The time-varying delay function $\phi(k) : \mathbb{N} \to \mathbb{Z}$ is assumed to have the standard form of $\phi(k) = k - \kappa(k)$, whose inverse function $\phi^{-1}(k) : \mathbb{Z} \to \mathbb{N}$ exists and is known. Also, $\kappa(k) \in \mathbb{N} \to \mathbb{N}$ denotes time-varying delay that satisfies $\kappa(k) \in \mathbf{I}[0, K]$, where $K \in \mathbb{N} \setminus \{0\}$ is the maximal value of the delay. We note that $\phi^{-1}(k)$ exists as long as $\phi(k)$ is a strictly increasing function of k. The initial condition is given by $\psi(k)$ for $k \in \mathbf{I}[-K, 0]$.

It is also assumed that (A, B) is stabilizable and (A, C) is detectable. Without loss of generality, we further assume that the pair (A, B) is in the form of

$$A = \begin{pmatrix} A_{\mathrm{I}} & 0 \\ 0 & A_{\mathrm{O}} \end{pmatrix}, \qquad B = \begin{pmatrix} B_{\mathrm{I}} \\ B_{\mathrm{O}} \end{pmatrix}, \tag{2}$$

where all eigenvalues of A_{I} are inside the unit circle and all eigenvalues of A_{O} are on or outside the unit circle.

Consider the following feedback law for system (1),

$$u(\phi(k)) = F(\gamma)x(k), \tag{3}$$

where $F(\gamma)$, $\gamma > 0$, is a parametric feedback gain matrix which renders $A + BF(\gamma)$ Schur stable. Under the feedback law (3), the closed-loop system is given by

$$x(k+1) = (A + BF(\gamma))x(k), \tag{4}$$

which is asymptotically stable because the matrix $A + BF(\gamma)$ is Schur stable.

Since $\phi^{-1}(k)$ exists and is known, we obtain from (3) that

$$u(k) = F(\gamma)x(\phi^{-1}(k)). \tag{5}$$

Recall that $\phi(k) = k - \kappa(k)$ and $\kappa(k) \geq 0$ for $k \in \mathbb{N}$, we have $k = \phi(\phi^{-1}(k)) = \phi^{-1}(k) - \kappa(\phi^{-1}(k)) \leq \phi^{-1}(k)$. Thus, the right side of (5) contains future state of $x(k)$, namely, $x(\phi^{-1}(k))$. This future state can be predicted with the solution of the closed-loop state equation as,

$$x(\phi^{-1}(k)) = A^{\phi^{-1}(k)-k}x(k) + \sum_{s=1}^{\phi^{-1}(k)-k} A^{s-1}Bu\big(\phi^{-1}(k) - s - \kappa(\phi^{-1}(k) - s)\big). \tag{6}$$

Substitution of (6) in (5) yields the classical predictor state feedback law,

$$u(k) = F(\gamma)A^{\phi^{-1}(k)-k}x(k) + F(\gamma)\sum_{s=1}^{\phi^{-1}(k)-k} A^{s-1}Bu\big(\phi^{-1}(k) - s - \kappa(\phi^{-1}(k) - s)\big). \tag{7}$$

Discarding the term containing the summation sign that involves the past values of the control input results in the truncated predictor feedback law,

$$u(k) = F(\gamma)A^{\phi^{-1}(k)-k}x(k). \tag{8}$$

Following [21], for a controllable pair (A, B) with A being nonsingular, $F(\gamma)$ can be constructed as

$$F(\gamma) = -(I + B^{\mathrm{T}}P(\gamma)B)^{-1}B^{\mathrm{T}}P(\gamma)A, \tag{9}$$

where $P(\gamma)$ is the unique positive definite solution to the discrete-time parametric algebraic Riccati equation

$$A^{\mathrm{T}}PA - P - A^{\mathrm{T}}PB(I + B^{\mathrm{T}}PB)^{-1}B^{\mathrm{T}}PA = -\gamma P. \tag{10}$$

A necessary and sufficient condition for the existence and uniqueness of such a solution $P(\gamma)$ is that

$$\gamma \in (1 - |\lambda(A)|^2_{\min}, 1), \tag{11}$$

where $|\lambda(A)|_{\min}$ denotes the minimal modulus of all eigenvalues of A. We note that $P(\gamma) = W^{-1}(\gamma)$, where $W(\gamma)$ is the unique positive definite solution to the discrete-time Lyapunov equation

$$W(\gamma) - \frac{1}{1-\gamma}AW(\gamma)A^{\mathrm{T}} = -BB^{\mathrm{T}}, \tag{12}$$

which is equivalent to (10).

The objective of this paper is to establish a bound on the delay and a range of the values of the parameter for which the closed-loop system consisting the system (1) and the truncated predictor feedback law (8) is asymptotically stable as long as the delay is within the bound. The corresponding output feedback result will also be derived. To this end, we need to establish some properties of the solution of the algebraic Riccati equation (10) as well as to generalize an existing version of Jensen's inequality.

The following two lemmas establish properties of $P(\gamma)$ for a general system (1) that may be exponentially unstable. The proofs follow similar steps as in the proof of Proposition 2, Theorem 1 and Corollary 2 in [23], where the systems considered are not exponentially unstable.

Lemma 1. *For a controllable pair (A, B) with A being nonsingular, and any nonnegative integers a and b, the following inequality holds*