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Model matching with strong stability in switched linear systems

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ABSTRACT

This work deals with model matching by output dynamic feedback in switched linear systems. The plant and the model are assumed to be subject to different switching signals. A necessary and sufficient condition for achieving model matching with asymptotic stability of both the closed-loop compensated system and the compensator, for a sufficiently large dwell time, is proven. Such condition is obtained by specializing the structural condition with a suitable redefinition of the robust controlled invariant subspace involved, capable of capturing not only the structural aspect of the problem but also the stability aspects. The effect of the combined action of nonzero initial states and nonzero inputs is dealt with. The solution to a more demanding problem formulation, where the dwell time is assumed to be fixed and given is also discussed.

1. Introduction

Switched dynamical systems have recently attracted considerable interest, mainly because of the significant number of fields where real systems featuring a multiplicity of operation modes are involved. State-of-the-art methods for studying stability and stabilizability of switched systems have been presented in the survey paper by Lin and Antsaklis [1] as well as in the later articles by Agrachev et al. [2], Chesi et al. [3], Mason et al. [4], Xiang [5]. Novel results in the design of switched control systems have been shown in different areas: LQR optimal control [6,7], $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control [8,9], output regulation [10,11], disturbance rejection [12–16].

In addition to the problems formerly mentioned, also model matching has been lately formulated for switched systems and solved through different control schemes [17–20]. Indeed, the problem of compensating a given plant so as it behaves like a suitable model is a classic topic of linear control theory [21–23]. Moreover, this problem has been extensively studied for many other classes of dynamical systems: e.g., nonlinear systems [24], time-delay and 2D systems [25], descriptor systems [26], periodic systems [27]. A main reason for these previous investigations – as well as a strong motivation for this work – is not only the intrinsic, theoretic interest of providing necessary and sufficient solvability conditions holding under less restrictive assumptions, but also the fact that these conditions generally provide new, powerful tools to deal with more complex control problems (see, e.g., Bao et al. [28], Seyboth and Allgöwer [29]).

As to the previous works on model matching for switched linear systems, first of all, it is worth noting that the necessary and sufficient conditions for the existence of a solution to the model matching problem proven in Conte et al. [17], Perdon et al. [20] are based on a switched state feedback control. However, a complete characterization of solvability of the model matching problem by switched output dynamic feedback – which was the subject of Zattoni et al. [18], Zattoni [19] – is fairly destined to have a stronger impact on applications. For this reason, also this work is focused on model matching by switched output dynamic feedback. Nonetheless, it neatly differs from Zattoni et al. [18], Zattoni [19] in several aspects.

In the first place, this article considers a more general setting than that outlined in Zattoni et al. [18]. While in the earlier paper the model was assumed to be linear and time-invariant, in this work both the plant and the model are assumed to be switched linear systems and, in particular, they are assumed to be subject to different switching signals: namely, the switching signals are defined over distinct indexed sets and the sequences of the switching times are allowed to be affected by mismatches. A further, relevant feature of the approach to the solution of the model matching problem developed in this work deals with the fact that model matching with stability (under restricted switching, in particular) of the closed loop is achieved by means of a compensator which is itself stable (in the same sense). Indeed, the

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general idea of achieving closed-loop stability by means of a stable compensator – also known in the literature as strong stability [30–32] – has drawn noticeable attention, due to the benefits of relying on a stable compensator, which may be even more relevant in the case of switched systems.

In particular, the strong stability requirement characterizes this work with respect to those, addressing linear time-invariant systems, that have inspired the way of dealing with output feedback model matching adopted herein [33,34] as well as with respect to their direct counterpart for discrete-time switched linear systems [19]. Indeed, the reduction of structural model matching by output dynamic feedback to structural feedforward disturbance decoupling was first presented in Marro and Zattoni [33,34]. However, the feedback connection considered in this work differs from the one studied in the previous papers for the presence of the output feedback gain matrix, which replaces the unitary feedback and which is itself part of the to-be-designed compensation scheme. In particular, it will be shown that, while solvability of the structural problem is independent of the choice of the switched output feedback gain matrix, the latter plays a key role in achieving strong stability. Namely, an appropriate choice of the output feedback gain matrix guarantees stability of the so-called auxiliary model and, consequently, of the switched compensator.

In this context, the present work first contributes a necessary and sufficient condition to solve model matching by output dynamic feedback in switched linear systems, with asymptotic stability of the closed-loop compensated system and of the compensator, for all switching signals with a sufficiently large dwell time. Such condition – which is a geometric one, expressed in coordinate free terms as an inclusion of subspaces – is derived by specializing the corresponding structural condition with the appropriate choice of the robust controlled invariant subspace involved. This subspace – which was introduced in Zattoni et al. [16] to characterize solvability of disturbance decoupling with stability in switched linear system – enjoys not only the property of being a controlled invariant subspace for all modes, but also that of being internally stabilizable with respect to all modes, thus allowing both the structural and the stability issues of the problem to be caught in one, sharp, coordinate-free statement.

As to the necessary and sufficient condition for the solution of structural model matching used, as starting point, in this paper, it is a generalization of the condition presented in Otsuka [12], Yurtseven et al. [14] for structural disturbance decoupling in switched linear systems. Actually, as a merely structural one, the condition by Otsuka [12] – or, by Yurtseven et al. [14] – coincides with the one first shown by Basile and Marro [35] for a set of linear time-invariant systems — the latter being regarded as the modes of the switched system. In more detail, the necessary and sufficient condition for structural model matching exploited in this work is an extension of the abovementioned condition to the case of disturbances accessible for measurement and it has been first used in Perdon et al. [20] to characterize solvability of model matching by (static or dynamic) state feedback.

This work also contributes to shed light on the solution of a more demanding formulation of the model matching problem in switched linear systems, where asymptotic stability of the compensated system and of the compensator is required for all switching signals with a fixed and given dwell time. In this framework, a further particularization of the main condition proven in this work, where stabilizability is intended over the class of the switching signals with the given dwell time, is shown to be necessary for the solution of model matching problem by output feedback, while it is shown to be still necessary and sufficient for the solution by dynamic feedforward.

The paper is organized as follows. The problem statement is introduced in Section 2. Section 3 is focused on the equivalence of structural model matching by output dynamic feedback to structural decoupling by dynamic feedforward for a modified layout. The structural solution to model matching is derived in Section 4. In Section 5, the characterization of solvability of model matching with asymptotic stability over the class of switching signals with a sufficiently large dwell time is provided. Solvability of model matching with asymptotic stability over the class of switching signals with a given dwell time is discussed in Section 6. Section 7 contains some concluding remarks.

Notation: $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{C}^-$ stand for the sets of real numbers, nonnegative real numbers, and complex numbers with negative real part, respectively. Matrices and linear maps are denoted by capital letters, like A. The image and the kernel of A are denoted by $\operatorname{Im} A$ and $\operatorname{Ker} A$, respectively. The spectrum of A is denoted by $\lambda(A)$. Vector spaces and subspaces are denoted by calligraphic letters, like $\mathcal{V}$. The restriction of a linear map A to an A-invariant subspace $\mathcal{J}$ is denoted by $A|_{\mathcal{J}}$. The symbol diag $\{M_1, \ldots, M_k\}$ denotes a block diagonal matrix whose blocks on the main diagonal are $M_1, \ldots, M_k$, respectively.

2. Problem statement

The plant $\Sigma_{P\,\sigma_P}$ is defined as the continuous-time switched linear system

$$\Sigma_{P\,\sigma_P} \equiv \begin{cases} \dot{x}_P(t) \;=\; A_{P\,\sigma_P(t)}\, x_P(t) + B_{P\,\sigma_P(t)}\, u(t), \\ \;y(t) \;=\; C_{P\,\sigma_P(t)}\, x_P(t), \end{cases}$$

where $t \in \mathbb{R}^+$ is the time variable, $x_P \in \mathcal{X}_P = \mathbb{R}^{n_P}$ is the state, $u \in \mathbb{R}^p$ is the control input, and $y \in \mathbb{R}^q$ is the output, with $p, q \leq n_P$. The modes of $\Sigma_{P\,\sigma_P}$ are the continuous-time linear time-invariant systems of the finite indexed set $\{\Sigma_{P\,i},\ i \in \mathcal{I}\}$, where $\mathcal{I} = \{1, 2, \ldots, N_P\}$ and

$$\Sigma_{P\,i} \equiv \begin{cases} \dot{x}_P(t) \;=\; A_{P\,i}\, x_P(t) + B_{P\,i}\, u(t), \\ \;y(t) \;=\; C_{P\,i}\, x_P(t), \end{cases}$$

with $A_{P\,i}$, $B_{P\,i}$, $C_{P\,i}$ constant real matrices of suitable dimensions and $B_{P\,i}$, $C_{P\,i}$ full-rank matrices, for all $i \in \mathcal{I}$. The control input function $u(t)$, with $t \in \mathbb{R}^+$, is assumed to belong to the set of the admissible control input functions $\mathscr{U}$, which is defined as the set of all piecewise-continuous time functions with finite values in $\mathbb{R}^p$. The switching signal σ_P is assumed to belong to the set of the admissible switching signals $\mathscr{S}_0(\mathcal{I})$, which is defined as the set of all piecewise-constant functions $\sigma_P : \mathbb{R}^+ \to \mathcal{I}$, having a finite number of discontinuity points in any finite time interval. Given $\tau_P \in \mathbb{R}^+$, $\mathscr{S}_{\tau_P}(\mathcal{I})$ denotes the set of the switching signals of $\mathscr{S}_0(\mathcal{I})$, whose interval between any two consecutive discontinuity points is greater than or equal to τ_P. The switching signals in $\mathscr{S}_{\tau_P}(\mathcal{I})$ are said to have dwell time τ_P. The plant $\Sigma_{P\,\sigma_P}$ is assumed to be asymptotically stable for all $\sigma_P \in \mathscr{S}_{\tau_P}(\mathcal{I})$, for a given positive real τ_P. This property will be briefly referred to as asymptotic stability *over* $\mathscr{S}_{\tau_P}(\mathcal{I})$. Moreover, the switching signal σ_P is assumed to be accessible for measurement.

The model $\Sigma_{M\,\sigma_M}$ is defined as the continuous-time switched linear system

$$\Sigma_{M\,\sigma_M} \equiv \begin{cases} \dot{x}_M(t) \;=\; A_{M\,\sigma_M(t)}\, x_M(t) + B_{M\,\sigma_M(t)}\, w(t), \\ y_M(t) \;=\; C_{M\,\sigma_M(t)}\, x_M(t), \end{cases}$$

where $x_M \in \mathcal{X}_M = \mathbb{R}^{n_M}$ is the state, $w \in \mathbb{R}^m$ is the input, and $y_M \in \mathbb{R}^q$ is the output. The modes of $\Sigma_{M\,\sigma_M}$ are the continuous-time linear time-invariant systems of the finite indexed set $\{\Sigma_{M\,j},\ j \in \mathcal{J}\}$, where $\mathcal{J} = \{1, 2, \ldots, N_M\}$ and

$$\Sigma_{M\,j} \equiv \begin{cases} \dot{x}_M(t) \;=\; A_{M\,j}\, x_M(t) + B_{M\,j}\, w(t), \\ y_M(t) \;=\; C_{M\,j}\, x_M(t). \end{cases}$$