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Event-triggered control for synchronization of coupled harmonic oscillators ☆

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ABSTRACT

This paper addresses the synchronization problem of coupled harmonic oscillators by event-triggered control. A centralized event-triggered control strategy is first developed and is further extended to a decentralized counterpart, in which the control protocol and event-triggering conditions only require local information. With the event-triggered control strategies, controllers update at the discrete instants when the related measurement errors exceed some proper state-dependent thresholds, which can reduce the computation and transmission costs. By the tools from nonsmooth analysis, it is shown that the proposed event-triggered control strategies synchronize asymptotically all oscillator states. Furthermore, a decentralized event-triggered strategy with a fixed threshold is proposed for the sake of excluding the Zeno behavior. The effectiveness of the proposed strategies is illustrated by numerical simulations.

1. Introduction

Many complex systems in nature exhibit synchronized behavior, such as fireflies' flashing or crickets' chirping. The interaction mechanisms of synchronization have been extensively studied in different disciplines [1,2]. The synchronization of coupled harmonic oscillators is an important subject in the study of synchronization phenomena (see e.g., [3–7]).

Recently, researchers from control community have developed several control algorithms to drive groups of coupled harmonic oscillators into state synchronization. In general, these control algorithms can be categorized into continuous algorithms [3,7,8] and discontinuous algorithms [9–11]. Note that these existing algorithms depend on that the agents should have continuous or periodic access to information from neighboring agents which may lead to inefficient utilization of energy and communication bandwidth. Most recently, event-triggered control has become a promising alternative to reduce the usage of system resources [12–16]. With the event-triggered control, the sampled data are sent to controllers only when the measurement errors exceed some predefined thresholds, which often depend on system states.

In this paper, the event-triggered control strategies are proposed to implement the synchronization of coupled harmonic oscillators with low resource costs. The states of coupled harmonic oscillators are time-varying and periodically oscillating even when the synchronization is achieved. The measurement errors do not converge to zero while the state-dependent thresholds tend to zero. In such a case, the Zeno triggering may exist under the event-triggered control. Thus, under the event-triggered control, we shall answer the following questions: How to define the solution of the event-triggered system; how to analyze the synchronization behavior of oscillators. This paper mainly concentrates on the two problems. Furthermore, an event-triggered strategy with a fixed triggering threshold is proposed to exclude the Zeno behavior.

The main contributions of this paper are summarized as follows: (i) The event-triggered algorithms are proposed for the first time to synchronize a group of coupled harmonic oscillators. (ii) The tools of nonsmooth analysis are used in the mathematical treatments. Since the investigated closed-loop system is discontinuous on the right-hand side, the classical solutions may not exist in such a case and the closed-loop system may also exhibit the Zeno behavior. However, one can still define suitable solutions for these systems in the framework of nonsmooth analysis [17,18]. The literature [17–20], using the theory of nonsmooth analysis, has provided new approaches to tackle the discontinuous systems and has shown that the mathematical analysis is effective.

Notation. $\mathbb{R}^n$ is the Euclidean space with n-dimensions. $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrices and I_n denotes the

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n-dimensional identity matrix. For a matrix A, $\|A\|$ is the induced 2-norm. Let $\mathbf{1}_n = [1, 1, \ldots, 1]^T$. Let $B(x, \delta)$ be an open ball centered at x with radius δ, $\overline{co}$ be the convex closure and $\mu(\cdot)$ be the Lebesgue measure. A continuous function $f : [0, a) \mapsto \mathbb{R}$, $a > 0$, belongs to the class of K function if it is strictly increasing and $f(0) = 0$.

2. Preliminaries and problem statement

The interaction topology of oscillators can be conveniently modeled by a graph. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be an undirected connected graph, where $\mathcal{V} = \{1, 2, \ldots, n\}$ represents the set of oscillators and $\mathcal{E} = \{(i, j) : i, j \in \mathcal{V}\} \subseteq \mathcal{V} \times \mathcal{V}$ stands for the set of edges. An edge $(i, j) \in \mathcal{E}$ implies that there exists a communication channel between nodes i to j, and i is called a neighbor of j. Let $\mathcal{N}_i$ index the set of the neighbors of i, and $|\mathcal{N}_i|$ denote its cardinality. The adjacency matrix $A = [a_{ij}]$ of an undirected graph is a symmetric matrix with $a_{ij} = 1$ if i and j are neighbors, and $a_{ij} = 0$ otherwise. Denote the degree matrix of $\mathcal{G}$ by D and D is a diagonal matrix with the ith diagonal entry $d_{ii} = \sum_{j\in\mathcal{N}_i} a_{ij}$. The graph Laplacian L is given by $L = D - A$.

Consider a group of coupled harmonic oscillators with the following dynamics:

$$\begin{cases} \dot{r}_i(t) = v_i(t) \\ \dot{v}_i(t) = -\alpha r_i(t) + u_i(t) \end{cases} \quad i = 1, 2, \ldots, n, \tag{1}$$

where $r_i(t), v_i(t) \in \mathbb{R}$ denote the position and velocity of the ith oscillator respectively, $u_i(t)$ is the control protocol to be designed, and $\sqrt{\alpha}$ is the frequency of the oscillators.

In [3,7], a continuous protocol was proposed. The coupled harmonic oscillators based on periodic sampled-data control ware considered in [10] and [11] respectively. In this paper, we aim to address the synchronization problem of coupled harmonic oscillators by event-triggered control. The controllers update necessarily at some discrete time instants, which are completely decided by the event-triggered detectors.

As the considered system has discontinuous right-hand side under eventtriggered controllers, the solutions of the system are considered in the Filippov sense.

Definition 1 (*[21]*). Consider a system given by

$$\dot{x}(t) = f(t, x), \tag{2}$$

where $f(t, x) : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ is a measurable and essentially locally bounded function. A vector function $x(t)$ is called a Filippov solution of (2) on $[t_0, t_1)$ if $x(t)$ is absolutely continuous on $[t_0, t_1)$ and for almost all $t \in [t_0, t_1)$, satisfies

$$\dot{x}(t) \in \mathcal{K}[f(t, x)], \tag{3}$$

where $\mathcal{K}[f(t, x)] = \cap_{\delta>0} \cap_{\mu(S)=0} \overline{co}\{f(t, B(x, \delta))\backslash S\}$ is a set-valued map.

A solution $x(t)$ is complete if $x(t)$ satisfies (3) for almost all $t \in [t_0, \infty)$.

Definition 2 (*[22]*). The set-valued Lie derivative of $V(t, x)$ with respect to $x(t)$, the trajectory of (2), is defined by

$$\dot{\tilde{V}} \triangleq \bigcap_{\xi\in\partial V(t,x)} \xi^T \begin{bmatrix} \mathcal{K}[f(t, x)] \\ 1 \end{bmatrix},$$

where $\partial V(t, x)$ is the generalized gradient of V at (t, x).

In particular, if the function $V(t, x)$ does not explicit depend on t, the set valued Lie derivative of $V(x)$ with respect to $x(t)$ becomes $\dot{\tilde{V}} \triangleq \cap_{\xi\in\partial V(x)} \xi^T \mathcal{K}[f(t, x)]$. Based on the fact that if let $V(x)$ be a Lipschitz and regular function, then $V(x)$ is absolutely continuous and $\frac{d}{dt}V(x) \in \dot{\tilde{V}}$ for almost everywhere $t > t_0$.

3. Event-triggered synchronization protocols

Denote the position and velocity state averages of oscillators as $\bar{r}(t) = \frac{1}{n}\sum_1^n r_i(t)$ and $\bar{v}(t) = \frac{1}{n}\sum_1^n v_i(t)$, respectively. Define synchronization position error and velocity error as $\xi_i(t) = r_i(t) - \bar{r}(t)$ and $\eta_i(t) = v_i(t) - \bar{v}(t)$, respectively. Obviously, the synchronization is achieved asymptotically if $\xi_i(t) \to 0$ and $\eta_i(t) \to 0$ as $t \to \infty$. For simplicity, the time variable t is omitted if no confusion. Let $\xi = [\xi_1, \xi_2, \ldots, \xi_n]^T$ and $\eta = [\eta_1, \eta_2, \ldots, \eta_n]^T$.

Lemma 1 (*[23]*). *If $\mathcal{G}$ is a connected undirected graph of order n, then the eigenvalues of L can be arranged in the increasing order $0 = \lambda_1 < \lambda_2 \leq \cdots \leq \lambda_n$. Furthermore, for any $\zeta \in \mathbb{R}^n$ such that $\zeta^T\mathbf{1}_n = 0$, then*

$$\lambda_2\zeta^T\zeta \leq \zeta^T L\zeta \leq \lambda_n\zeta^T\zeta.$$

It is obvious that $\xi^T\mathbf{1}_n = \eta^T\mathbf{1}_n = 0$.

Lemma 2. *For any $a > 0$, $x, y \in \mathbb{R}^n$ and positive semi-definite matrix L, one has*

$$x^T Ly \leq \frac{a}{2}x^T L^2 x + \frac{1}{2a}y^T y.$$

Lemma 3 (*[24]*).

(1) *Assume that $f(x), g(x) : \mathbb{R}^m \to \mathbb{R}^n$ are locally bounded, then*

$$\mathcal{K}[f(x) + g(x)] \subseteq \mathcal{K}[f(x)] + \mathcal{K}[g(x)].$$

(2) *Let $g(x) : \mathbb{R}^n \to \mathbb{R}^{p\times n}$ (i.e., matrix valued) be continuous and $f(x) : \mathbb{R}^m \to \mathbb{R}^n$ be locally bounded, then*

$$\mathcal{K}[g(x)f(x)] = g(x)\mathcal{K}[f(x)].$$

3.1. Centralized event-triggered control protocol

In this subsection, the centralized event-triggered synchronized protocol is proposed, where all the oscillators share a common event-triggering condition. The event instants of all oscillators are denoted by $t_0, t_1, \ldots, t_k, \ldots$, with $t_k < t_{k+1}$. Between any two consecutive event instants, all controllers are held constant in a zero-order hold fashion. The proposed centralized event-triggered control protocol is given by

$$u_i(t) = -\sum_{j\in\mathcal{N}_i}(v_i(t_k) - v_j(t_k)), \quad t \in [t_k, t_{k+1}). \tag{4}$$

By (1), (4) and the symmetry of the communication topology, it gives

$$\begin{cases} \dot{\bar{r}}(t) = \bar{v}(t) \\ \dot{\bar{v}}(t) = -\alpha\bar{r}(t), \end{cases} \tag{5}$$

which means that the dynamic average of the system (1) is preserved. For oscillator i, introduce the measurement error $e_i(t) = \eta_i(t_k) - \eta_i(t)$, $t \in [t_k, t_{k+1})$. Let $e = [e_1, e_2, \ldots, e_n]^T$. By protocol (4), the dynamics of error system is given by

$$\begin{cases} \dot{\xi}(t) = \eta(t), \\ \dot{\eta}(t) = -\alpha\xi(t) - L(\eta(t) + e(t)). \end{cases} \tag{6}$$

Note that an event occurs whenever the measurement error increases to a proper defined threshold and then the measurement error resets to zero immediately. This leads to discontinuous right-hand side of system (6). Thus, it is more reasonable to analyze the dynamic behavior within the framework of nonsmooth analysis.