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# Incorporating prior knowledge in observability-based path planning for ocean sampling

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ABSTRACT

Observability-based path planning of autonomous sampling platforms for flow estimation is a technique by which candidate trajectories are evaluated based on their ability to enhance the observability of underlying flow-field parameters. Until now, observability-based path planning has focused primarily on forward-in-time integration. We present a novel approach that makes use of the background error covariance at the current time to account properly for uncertainty of the underlying flow. The reduced Hessian of an optimal, linear data-assimilation strategy properly accounts for prior knowledge in the linear case and must be full rank to infer the initial state. The reduced Hessian represents an observability Gramian augmented with an inverse prior covariance. We extend this concept to the nonlinear case to yield a new criterion for scoring candidate trajectories: the empirical augmented unobservability index. Solving the differential covariance Riccati equation of the Kalman Filter for deterministic dynamics also properly accounts for prior knowledge in the linear case, but at a later time. The solution to this equation reveals the important distinctions between observability-based, augmented observability-based, and anticipated covariance-based path planning. Path planning based on this unobservability index in the presence of prior information yields the desired behavior in numerical experiments of a guided Lagrangian sensor in a two-vortex flow pertinent to ocean sampling.



## 1. Introduction

Ocean-observing systems provide essential information on the state of the ocean for use in oceanographic, atmospheric, and climatological modeling and forecasting. One such system is Argo, a continuously deployed, global array of drifting platforms [1]. These sensor systems will continue to increase in coverage and sampling capabilities as demand for ocean data increases. Incorporation of autonomous sampling platforms reduces oceanographic uncertainty through the determination of advantageous routes for measurement collection in response to uncertainty in estimates of a real-time environmental process [2]. The flow measurements of ocean sampling vehicles are often their Lagrangian data, i.e., measurements of the vehicle position under the influence of the flow. One such sampling platform is the underwater glider, which is a buoyancy-driven vehicle that alters its depth in a sinusoidal manner to induce flow over attached wings to make forward progress [3,4]. Sensor platforms like gliders are minimally actuated to extend endurance; planning efficient, feasible, and informative routes is therefore essential.

Observability is the property of being able to infer the initial state of a system or underlying model parameters by observing the system output over a fixed time interval. Many researchers have planned informative routes by considering the path's observability or empirical observability, which is an approximation to observability for nonlinear systems. Hinson et al. [5] analytically derive a trajectory that maximizes the observability of inertial position and heading for a self-propelled vehicle in a uniform flow. They pose an optimal-control problem to choose a path that minimizes the condition number of the observability Gramian for the linearized dynamics. Unfortunately, analytical solutions to the optimal-control problem only exist in specialized cases, due to the non-differentiability and non-convexity of the cost functional [5]. This problem may be addressed with grid-based optimization such as in the multi-vehicle sampling algorithm of DeVries et al. [6], if the application permits. Quenzer and Morgansen [7] also perform a finite-dimensional optimization over a discrete set of constant

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turning rates for an empirical observability-based controller in a multi-vehicle helming application.

Another method for performing finite-dimensional optimization of observability is to consider evaluation over a family of pre-determined candidate trajectories. The Adaptive Sampling and Prediction (ASAP) field experiment in Monterey Bay performed a similar optimization over a family of coordinated sampling patterns with respect to a sampling performance metric [8]. In addition to reducing computational cost, this method permits integration of observability optimization with other control policies that may have generated the candidate trajectories. In prior work, we generated candidate trajectories that steer a vehicle to separating boundaries of invariant regions in a two-vortex flow field [9,10].

Previous observability-based path-planning research has only been forward-looking. We propose to score trajectories using a new measure, the augmented unobservability index, to quantify how much each path increases the observability of estimated flow parameters given prior information in the form of a background error covariance matrix. Incorporating prior information in adaptive sampling has been accomplished by maximizing the anticipated reduction in error covariance. For example, Bishop et al. [11] consider an adaptive network design problem by optimizing the forecasted error covariance of an Ensemble Transform Kalman Filter over a finite set of possible network realizations. Davis et al. [12] also consider the forecasted covariance reduction in an objective analysis estimation technique to simulate routes for underwater gliders. Anticipated error covariance analysis is similar in the case of a linear deterministic model to augmented observability. However, we highlight the distinctions in Section 3. In the nonlinear case, the approaches differ because the anticipated reduction in covariance approach depends on the estimation scheme. We define empirical augmented observability independently of the estimator; it includes only the system dynamics, output equations, and the background error covariance.

Our technical approach first considers the variational data assimilation strategy 4D-Var with deterministic, linear dynamics and uncertain measurements. These dynamics correspond to a tangent-linear approximation of a nonlinear system, similar to the tangent-linear model used in the definition of empirical observability by Krener and Ide [13]. The optimal solution of this problem requires inversion of a matrix known as the reduced Hessian. Since we formulate the reduced Hessian in terms of linear observability with the addition of an inverse background error covariance, we refer to it as the augmented observability Gramian. The minimum-variance solution for a posterior filter covariance is given by the continuous-time Kalman Filter, which provides a differential Riccati equation describing its evolution. We derive the analytical solution to this differential Riccati equation by connecting the inverse covariance of a Kalman Filter to the augmented observability. We extend the concept of augmented observability to the nonlinear setting using an empirical observability Gramian and derive an upper bound on the associated empirical augmented unobservability index.

The contributions of this paper are: (1) an analytical solution to a continuous-time 4D-Var variational data assimilation problem in terms of the linear stochastic observability Gramian with an inverse background error covariance, which we refer to as augmented observability; (2) an analytical solution to the continuous-time Kalman Filter Riccati equation for a linear time-varying system with deterministic dynamics and uncertain measurements in terms of the stochastic observability Gramian; and (3) an extension of augmented observability to nonlinear systems based on the empirical observability Gramian, yielding a novel method for scoring candidate trajectories, the empirical augmented unobservability index. These contributions are important because they provide a quantitative evaluation criterion for automatic selection of the candidate path that maximizes the anticipated observability given existing state uncertainty. The strategy of path planning with empirical augmented observability is illustrated for a single vehicle in a two-vortex flow pertinent to ocean sampling. This example demonstrates that augmenting observability with prior information improves sampling by changing the optimal path in an intuitive manner.

Section 2 reviews empirical observability for nonlinear systems, the two-vortex system, and model-predictive path planning based on observability. Section 3 solves a linear 4D-Var variational data assimilation problem, defines the augmented observability Gramian, and derives the optimal inverse posterior covariance for a continuous-time Kalman Filter with deterministic dynamics. Section 4 extends augmented observability to the nonlinear setting and presents numerical experiments showing path planning using the empirical augmented unobservability index. Section 5 summarizes the paper and ongoing research.

## 2. Observability-based path planning in a two-vortex flow

This section reviews the empirical observability Gramian and observability-based path planning. It also presents background information on the dynamics of a Lagrangian sensor platform in a two-vortex flow.

### 2.1. Linear and empirical observability

Observability describes the ability to infer the initial state of a system by observing the output over a specified time interval. Consider the linear system

$$\dot{x}(t) = A(t)x(t), \qquad y(t) = C(t)x(t) \tag{1}$$

with $x(t) \in \mathbb{R}^n$, $y(t) \in \mathbb{R}^m$, $A(t) \in \mathbb{R}^{n\times n}$, and $C(t) \in \mathbb{R}^{m\times n}$. Observability can be assessed through inspection of the linear observability Gramian [14]

$$\mathcal{W}_o(t_0, t) = \int_{t_0}^{t} \Phi(\tau, t_0)^T C^T(\tau) C(\tau) \Phi(\tau, t_0) d\tau, \tag{2}$$

where $\Phi(\tau, t_0)$ is the state transition matrix for the dynamics from time $t_0$ to $\tau$. By uniqueness of the state solution to (1), the state transition matrix has the property that $\Phi(\tau, t_0)^{-1} = \Phi(t_0, \tau)$. Assessing the rank of $\mathcal{W}_o(t_0, t)$ is a boolean test to determine whether the system is observable on the time interval $[t_0, t]$: if $\mathcal{W}_o(t_0, t)$ is full rank, then the system state is observable.

Next, consider the nonlinear system

$$\dot{x}(t) = f(t, x(t)), \qquad y(t) = h(t, x(t)) + \upsilon(t), \tag{3}$$

where $f$ and $h$ are nonlinear functions and $\upsilon(t)$ is white Gaussian noise with covariance $R(t)$. The tangent-linear model for the dynamics (3) along a reference trajectory $x_r(t)$ with output $y_r(t)$ is given by the linear system

$$\frac{d}{dt}(\delta x(t)) = \left.\frac{\partial f}{\partial x}\right|_{x_r(t)} \delta x(t), \qquad \delta y(t) = \left.\frac{\partial h}{\partial x}\right|_{x_r(t)} \delta x(t). \tag{4}$$

For an initial condition $x_0$, the solution to (4) for $\delta x(t_0) = x(t_0) - x_r(t_0)$ yields the approximations $x_r(t) + \delta x(t) \approx x(t)$ and $y_r(t) + \delta y(t) \approx y(t)$. The local observability Gramian for the nonlinear system (3) is defined to be the linear observability Gramian (2) for the tangent-linear approximation (4) with $C(\tau) = \partial h/\partial x|_{x_r(\tau)}$ and $\Phi(\tau, t_0)$ as the state transition matrix for $\partial f/\partial x|_{x_r(\tau)}$ [13].

The empirical observability Gramian [13] is an approximation of the linear observability Gramian (2) for the nonlinear system (3). Let $\phi(\cdot, t_0, x(t_0))$ denote the state solution of (3) from $(t_0, x(t_0))$, $h^{\pm j}$ the system output corresponding to perturbed initial condition