Systems & Control Letters 97 (2016) 184–192

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Stability and stabilization of switched stochastic systems under asynchronous switching☆

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ARTICLE INFO

Article history:
Received 22 December 2015
Received in revised form
26 June 2016
Accepted 9 September 2016

Keywords:
Asynchronous switching
Average dwell time
Switched system
Stochastic stability

ABSTRACT

This paper studies the stability and stabilization problems for a class of switched stochastic systems under asynchronous switching. The asynchronous switching refers to that the switching of the candidate controllers does not coincide with the switching of system modes. Two situations are considered: (1) time-delayed switching situation, that is, the switching of the candidate controllers has a lag to the switching of the system modes; (2) mismatched switching situation, the switching of the candidate controllers does not match the switching of the system modes. Using average dwell time and Lyapunov-like function, sufficient conditions are established for stochastic input-to-state stability of the whole system. Also, the stabilizing controller design approach is proposed for switched stochastic linear systems. The minimal average dwell time and the controller gain are achieved. Finally, a numerical example is used to demonstrate the validity of the developed results.

1. Introduction

Switched systems are a special class of hybrid systems, and consist of a family of subsystems (also called system modes) and a switching law that orchestrates the switching among the system modes; see [1,2]. In practice, there are numerous physical systems that could be modeled as the switched systems, such as fermentation processes [1], networked control systems [3,4] and scalable video coding systems [2]. Because of practical application and theoretical development, switched systems have been given considerable attention in the last few decades. The readers are referred to [5–9] for a general introduction and the recent progresses in the field of switched systems.

In the practical systems, disturbances are inevitable and have impacts on the stability and the performances of the dynamical systems including switched systems. Furthermore, the stochastic disturbances lead to stochastic modeling and control for the control systems, which leads to switched stochastic systems. In the literature, there are some salient results on switched stochastic systems, such as stability [10–12], fault detection filtering [13], passivity and passification [14], H_∞ control [15], sliding mode control [16]. In the established methods, there are two widely applied approaches to study switched systems, i.e., average dwell time (ADT) approach [8,17] and Lyapunov function approach [18–20]. Average dwell time characterizes the switching rate that guarantees stability of the closed-loop system. In Lyapunov function approach, multiple Lyapunov function is an essential Lyapunov function. Combining ADT and multiple Lyapunov function, stability analyses and control syntheses of switched systems have been investigated; see [12,14,16,19,21].

In the previous works [8,10,11], there is a general assumption: the switching of the candidate controllers and the system modes is coincident, which is called synchronous switching. However, asynchronous switching, which is opposed to the synchronous switching, is more practical. Asynchronous phenomena like time delays can be found in many fields, such as networked control systems [4,22], chemical systems [23], Markovian jump systems [24] and neural systems [25]. For the switched systems, asynchronous switching may be caused by disturbances, identification of the system modes, implementation of the matched controller, time delays in information transmission and even the requirements of the switching law. Because the switched systems do not necessarily inherit the stability properties of the subsystems, asynchronous switching may further deteriorate the performances of switched systems. Some studies have been reported in the literature. For instance, asynchronous control problem of switched linear systems was addressed in [21]. The stability conditions were established in terms of ADT and Lyapunov-like conditions. Stability of

☆ The work in this paper was financially supported by National Natural Science Foundation of China under Grant 61374026, Program for New Century Excellent Talents in University 11-0880.

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http://dx.doi.org/10.1016/j.sysconle.2016.09.005
0167-6911/

switched nonlinear systems was considered in [26] by analyzing the Lie derivative of Lyapunov function. If time delays and asynchronous switching were considered, then Lyapunov–Krasovskii functional method was used in [25] to derive the stability conditions for switched nonlinear systems.

In this paper, we study the stability and stabilization problems for switched stochastic systems under asynchronous switching. Sufficient conditions are established for stochastic stability and controller design. Based on the different causes of asynchronous switching, two cases are considered. The first case is time-delayed switching, i.e., there are time delays between the switches of the candidate controllers and the system modes. The second one is mismatched switching, that is, there are no time delays but switching mismatches at the switching times. Under these two cases, stochastic stability of switched stochastic systems is studied in continuous-time context and discrete-time context. Using ADT and Lyapunov function approach, sufficient conditions are established to guarantee stochastic input-to-state stability (SISS). Furthermore, for switched stochastic linear systems, the stabilizing controllers design approach is proposed. Finally, a numerical example is used to demonstrate the effectiveness of the designed controllers. Compared with the previous works in [19,24–26,12], the contributions of this paper are three-fold. First, two asynchronous switching cases are studied, whereas only the time-delayed switching case was considered in the previous works [19,25,26,21]. Especially, the mismatched switching case is first studied in this paper. Second, for above two asynchronous switching cases, the stability conditions are established, which extends the previous results for the deterministic/linear/synchronous switched systems [12,19,21]. Moreover, both the continuous-time systems and the discrete-time systems are considered. Third, for switched stochastic linear systems with asynchronous switching, the stabilizing switched controller is designed, which recovers many previous works [21,27] as the special cases.

This paper is organized as follows. In Section 2, the considered problem is formulated and some preliminaries are given. Using average dwell-time and multiple Lyapunov-like function, sufficient conditions for SISS of switched stochastic systems are derived in Section 3. Both the time-delayed switching case and the mismatched switching case are considered. For these two cases, the stabilizing switched controllers are designed for switched stochastic linear systems in Section 4. In Section 5, a numerical example is used to illustrate the obtained results. Conclusions and future works are stated in Section 6.

Notation: The notation used in this paper is fairly standard. $\mathbb{N}^+$ stands for the set of nonnegative integers; $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $|\cdot|$ represents the Euclidean vector norm. $\mathbb{P}\{\cdot\}$ denotes the probability measure; $\mathbb{E}[\cdot]$ denotes the mathematical expectation. $\mathcal{C}^{1,2}$ stands for the space of the functions that are continuously differentiable on the first augment and continuously twice differentiable on the second augment. A function $\alpha(t):\mathbb{R}^+\to\mathbb{R}^+$ is of class $\mathcal{K}$ if it is continuous, zero at zero, and strictly increasing; $\alpha(t)$ is of class $\mathcal{K}_\infty$ if it is of class $\mathcal{K}$ and unbounded. A function $\beta(s,t):\mathbb{R}^+\times\mathbb{R}^+\to\mathbb{R}^+$ is of class $\mathcal{KL}$ if $\beta(s,t)$ is of class $\mathcal{K}$ for each fixed $t\geq 0$ and $\beta(s,t)$ decreases to zero as $t\to 0$ for each fixed $s\geq 0$. $\mathcal{L}_\infty^n$ denotes the set of all the measurable and locally essentially bounded signal $x\in\mathbb{R}^n$ on $\mathbb{R}^+$ with norm $\|x\| := \sup_{t\geq t_0}\inf_{\{\mathcal{A}\subset\Omega,\mathbb{P}\{\mathcal{A}\}=0\}}\sup\{|x(t,w)||w\in\Omega\setminus\mathcal{A}\}$. In addition, the symbols $tr[\cdot]$ and $diag\{\cdot\}$ denote trace operator and block diagonal matrix operator, respectively. The superscript "T" denotes the transpose, and the symmetric term in a matrix is denoted by $*$. $A>0$ ($A\geq 0$) means that the matrix A is positive definite (positive semidefinite). For simplicity, denote $\alpha_1\circ\alpha_2(s) := \alpha_1(\alpha_2(s))$ for all $\alpha_1,\alpha_2:\mathbb{R}\to\mathbb{R}$ and $s\geq 0$.

2. Problem formulation

Consider the switched stochastic nonlinear control system of the form

$$dx(t) = f_{\sigma(t)}(t,x,u,v)dt + g_{\sigma(t)}(t,x,u,v)dw(t) \tag{1}$$

for the continuous-time domain or

$$x(l+1) = f_{\sigma(l)}(l,x,u,v) + g_{\sigma(l)}(l,x,u,v)w(l) \tag{2}$$

for the discrete-time domain, where $x\in\mathbb{R}^{n_x}$ is the system state initializing at $x(t_0)=x_0$ and $t_0\geq 0$, $u\in\mathbb{R}^{n_u}$ is the control input which is assumed to be measurable and locally bounded, and $v\in\mathcal{L}_\infty^{n_v}$ is the exogenous disturbance. A piecewise constant and right continuous function $\sigma:\mathbb{R}^+\to\mathcal{M}$ is a switching signal specifying the index of the active subsystem, where $\mathcal{M}=\{1,\ldots,M\}$ is an index set. For the continuous-time version (1), $w(t)$ is an n_w-dimensional independent standard Wiener process (or Brownian motion) defined on a complete probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq t_0},\mathbb{P})$; for the discrete-time version (2), $w(l)$ is a scalar Gaussian white noise with $\mathbb{E}[w(l)]=0$ and $\mathbb{E}[w^2(l)]=\theta$. For each $i\in\mathcal{M}$, both $f_i:[t_0,\infty)\times\mathbb{R}^{n_x}\times\mathbb{R}^{n_u}\times\mathcal{L}_\infty^{n_v}\to\mathbb{R}^{n_x}$ and $g_i:[t_0,\infty)\times\mathbb{R}^{n_x}\times\mathbb{R}^{n_u}\times\mathcal{L}_\infty^{n_v}\to\mathbb{R}^{n_x\times n_w}$ are continuous with respect to t, x, u and v, and uniformly locally Lipschitz with respect to x and v; $f_i(\cdot,0,0,0)\equiv 0$ and $g_i(\cdot,0,0,0)\equiv 0$. For simplicity of notation, the solution process of the switched stochastic system (1) or (2) is assumed to be existent and unique for all the time; see [9,19]. Otherwise, the solution process is only defined on certain finite interval $[t_0,t_{\max})$ and $t_{\max}>t_0$. However, all the subsequent results are still valid for this case.

Definition 1 (*[8]*). For a switching signal σ and any $t_2>t_1\geq t_0$, let $N_\sigma(t_2,t_1)$ be the switching number of σ over the interval $[t_1,t_2)$. If there exist constants $N_0\geq 1$ and $\tau_a>0$ such that

$$N_\sigma(t_2,t_1)\leq N_0+\frac{t_2-t_1}{\tau_a}, \tag{3}$$

then N_0 and τ_a are called the chatter bound and the average dwell time, respectively.

In the following, the stability definitions are introduced for the continuous-time system (1). For the discrete-time version (2), the stability definitions are obtained similarly.

Definition 2 (*[12]*). The switched stochastic nonlinear system (1) is stochastically input-to-state stable (SISS), if for any $\varepsilon>0$, there exist $\beta\in\mathcal{KL}$ and $\gamma\in\mathcal{K}_\infty$ such that for all $x_0\in\mathbb{R}^{n_x}$, $u\in\mathbb{R}^{n_u}$ and $v\in\mathcal{L}_\infty^{n_v}$,

$$\mathbb{P}\{|x(t)|\leq\beta(|x(t_0)|,t-t_0)+\gamma(\|v\|)\}\geq 1-\varepsilon,\quad t\geq t_0. \tag{4}$$

If the inequality (4) holds for $v\equiv 0$, then the system (1) with $v\equiv 0$ is stochastically globally asymptotically stable (SGAS).

To stabilize the switched stochastic nonlinear system (1) and (2), the candidate mode-dependent controllers are designed as $u(t)=\kappa_{\sigma(t)}(x(t))$ for the continuous-time version (1) or $u(l)=\kappa_{\sigma(l)}(x(l))$ for the discrete-time version (2). In the literature, there is a common assumption for the candidate controllers: the switching of the candidate controllers is coincident with the switching of the system modes. In practice, this assumption is hard to be satisfied, whereas the asynchronous switching exists extensively in the physical systems [22–25]. However, the asynchronous switching deteriorates the stability and the performances of the switched stochastic control systems.

Therefore, the objectives of this paper are to establish the sufficient conditions to guarantee stochastic input-to-state stability of switched stochastic systems and to design the mode-dependent controllers under asynchronous switching. Based on