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Single-experiment observability decomposition of discrete-time analytic systems

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ABSTRACT

This paper addresses the single-experiment observability decomposition of discrete-time analytic systems. Unlike the continuous-time case, there exist systems which cannot be decomposed into observable and unobservable subsystems due to the fact that the observable space is not integrable. In this paper, a necessary and sufficient condition for integrability of observable space is given. As a corollary of this condition it is proven that if the system is reversible, the observability decomposition can be always achieved. Moreover, integrability of observable space is also addressed for delta-domain models of non-uniformly sampled systems.

1. Introduction

In the linear control theory, the Kalman decomposition plays a central role and has a close relationship with the realization theory of the transfer function matrix because the accessible and observable part corresponds to the transfer function matrix. The Kalman decomposition has been extended to the case of continuous-time nonlinear systems by differential geometric methods [1,2]. However, such decomposition is still missing in the discrete-time domain. This is due to the fact that not all single-experiment unobservable discrete-time nonlinear systems can be decomposed into single-experiment observable and unobservable subsystems [3], where single-experiment observability means that arbitrary initial state is uniquely determined by a single sequence of inputs and the corresponding sequence of outputs [4].

For continuous-time nonlinear systems, the single-experiment observability decomposition is carried out both via differential geometric [1,2] and algebraic methods [5]. In [5] decomposition is carried out first for globally linearized system equations, that is, into observable and unobservable subspaces of differential one-forms. This is always doable both in continuous- and discrete-time cases (for the latter, see [3]). Since in the continuous-time case the observable subspace of one-forms is proved to be always generically integrable, it can be locally spanned by exact one-forms whose integrals define the observable state coordinates. However, it was shown in [3] on the bases of simple bilinear examples that the observable space of the discrete-time system is not always integrable. The paper [6] presents a subclass of systems with non-integrable observable space.

For discrete-time nonlinear systems, the papers [3,7–10] addressed the state space decomposition into observable and unobservable subsystems. The paper [7] considers only autonomous systems without control whereas the decomposition in [8] is based on multiple-experiment observability, which is a weaker property than single-experiment observability; see [4] for more details about comparisons of observability notions. The paper [3] stated a conjecture that the observable space with respect to single-experiment observability is integrable for reversible analytic systems. This conjecture is based on two facts: (i) in the continuous-time case the observable subspace is integrable, and (ii) exact sampled-date model of the continuous-time system is reversible [11]. Because of the latter fact, reversibility assumption is not very restrictive. The paper [9] proves the conjecture only for polynomial systems. However, the proof carried out in [9], cannot be directly extended even for rational systems because it relies on the specific properties of polynomials; for instance the output at each time instant can be described as the sum of monomials of input variables with coefficients being polynomials of the state variables. Of course, in case one drops the requirement that the decomposed equations have to be in the form of the classical state equations and allows the generalized state equations (and the generalized state transformation) that, besides inputs, may depend also on forward shifts of inputs, it becomes always possible to carry out the decomposition [10]. However, as demonstrated in this paper, not all observability properties, for instance multiple-experiment observability, are preserved under the generalized state transformation.

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The goal of this paper is to prove that in case of reversible analytic systems, single-experiment unobservable systems can always be decomposed into single-experiment observable and unobservable subsystems in the form of classical state equations by classical state transformations, exactly like in the continuous-time counterpart [1,2,5], i.e., the conjecture from [3] is proven to be true. The main idea of the proof is to show that the multiple-experiment observability decomposition in [8] is nothing else but the single-experiment observability decomposition if the system is analytic and reversible. As an application of the result, one can extend the Kalman decomposition for this subclass of systems as demonstrated in the conference version of this paper [12].

A description of a dynamical system, based on difference operator is often referred to as delta-domain description. Delta-domain models are closely linked to the continuous-time systems. When signals are sampled at high rate such models are less sensitive to round-off errors and do not yield ill-conditioned models, as often happens with models based on the shift operators [13]. As an application of our main result, we will prove that the observable space of analytic delta-domain model is integrable (for almost all sampling times). This fact demonstrates once again that the properties of such models are closer to those of the continuous-time systems. Note that unlike [13], we do not assume constant sampling rate but address also non-uniformly sampled systems, the literature of which is not very large though such models are important in the nontraditional application areas such as, for instance, control over networks or biology and medicine.

Preliminary results of this paper were partly presented in conference paper [12]. The present improved version differs from it in a number of aspects. First, a necessary and sufficient condition for integrability of single-experiment observable space of not necessary reversible system is given. Second, a simpler proof of integrability of single-experiment observable space for reversible analytic systems is presented. Third, a vehicle model with zero slip angle [14] is added to demonstrate one of the main results. Finally, the analytic delta-domain models were studied.

The paper is organized as follows. Preliminary information about algebraic framework and notions of observability are given in Section 2. Integrability of observability space of discrete-time systems and delta-domain models are studied in Sections 3 and 4, respectively. Section 5 provides brief conclusions.

2. Preliminaries

2.1. Algebraic framework

In what follows we use the notation ξ for any variable $\xi(t)$, and $\xi^{\langle\ell\rangle}(t)$ for its time shift $\xi(t+\ell)$, $\ell \in \mathbb{Z}$. From this definition, $\xi(t) = \xi^{\langle 0\rangle}(t)$. Consider the discrete-time system

$$\begin{aligned} x^{\langle 1\rangle}(t) &= f(x(t), u(t)), \\ y(t) &= h(x(t)), \end{aligned} \tag{1}$$

where $x(t) \in X \subset \mathbb{R}^n$, $u(t) \in U \subset \mathbb{R}^m$, and $y(t) \in Y \subset \mathbb{R}^p$; X, U, and Y are open subsets; $f : X \times U \to X$ and $h : X \times U \to Y$ are assumed to be analytic vector functions. We denote the composition of function f as

$$(f_{u^{\langle 1\rangle}} \circ f_u)(x) = f(f(x, u), u^{\langle 1\rangle}).$$

For the sake of simplicity, $f_{U_\ell}(x)$ denotes $(f_{u^{\langle \ell-1\rangle}} \circ \cdots \circ f_{u^{\langle 1\rangle}} \circ f_u)(x)$, where U_ℓ $(\ell \geq 1)$ denotes the input sequence $(u, u^{\langle 1\rangle}, \ldots, u^{\langle \ell-1\rangle}) \in \mathbb{R}^{m\times\ell}$.

In this paper, reversibility is an important concept.

Definition 1. The system (1) is said to be generically reversible if

$$\operatorname{rank} \frac{\partial f(x, u)}{\partial x} = n \tag{2}$$

holds generically, i.e., this property holds on an open and dense subset of $X \times U$, provided it holds at some point of this domain.

The generic reversibility property is independent of the chosen coordinates. The coordinate transformation $z = \varphi(x)$ ($\operatorname{rank} \frac{\partial \varphi(x)}{\partial x} = n$) results in $z^{\langle 1\rangle}(t) = \varphi(f(\varphi^{-1}(z(t)), u(t)))$. For this system, we have

$$\frac{\partial \varphi(f(\varphi^{-1}(z), u))}{\partial z} = \left.\frac{\partial \varphi(x)}{\partial x}\right|_{x=f(\varphi^{-1}(z),u)} \left.\frac{\partial f(x, u)}{\partial x}\right|_{x=\varphi^{-1}(z)} \frac{\partial \varphi^{-1}(z)}{\partial z}.$$

and so, the reversibility is invariant.

The reversibility assumption is satisfied by a sampled model of continuous-time system for almost all sampling time [11] and is a bit stronger property than submersivity, typically made in the discrete-time context, when one assumes

$$\operatorname{rank} \frac{\partial f(x, u)}{\partial (x, u)} = n. \tag{3}$$

Let $\mathcal{K}$ be the field of meromorphic functions in a finite number of independent system variables from the infinite set $\mathcal{C} = \{x_1, \ldots, x_n, u_1^{\langle\ell\rangle}, \ldots, u_m^{\langle\ell\rangle}, \ell \geq 0\}$, where x_i and $u_j^{\langle\ell\rangle}$ are respectively the ith component of state vector x and jth component of input vector $u^{\langle\ell\rangle}$ [6]. The forward-shift operator $\sigma_f : \mathcal{K} \to \mathcal{K}$ is defined by

$$\sigma_f(\phi)(x, u, u^{\langle 1\rangle}, \ldots, u^{\langle\ell\rangle}) := \phi(f(x, u), u^{\langle 1\rangle}, u^{\langle 2\rangle}, \ldots, u^{\langle\ell+1\rangle}).$$

Under the assumption (2) (or (3)), $\mathcal{K}$ is a difference field.

Consider the infinite set of symbols $\mathrm{d}\mathcal{C} = \{\mathrm{d}x_1, \ldots, \mathrm{d}x_n, \mathrm{d}u_1^{\langle\ell\rangle}, \ldots, \mathrm{d}u_m^{\langle\ell\rangle}, \ell \geq 0\}$, and denote by $\mathcal{E}$ the vector space over the field $\mathcal{K}$ spanned by the elements of $\mathrm{d}\mathcal{C}$, namely, $\mathcal{E} = \operatorname{span}_{\mathcal{K}} \mathrm{d}\mathcal{C}$. Any element of $\mathcal{E}$ has the form

$$\sum_{i=1}^{n} a_i \mathrm{d}x_i + \sum_{\ell\geq 0} \sum_{j=1}^{m} b_{j,\ell} \mathrm{d}u_j^{\langle\ell\rangle},$$

where only a finite number of coefficients $b_{j,\ell}$, $j = 1, \ldots, m$, $\ell \geq 0$ are non-zero elements of $\mathcal{K}$. The elements of $\mathcal{E}$ are called differential one-forms. The field $\mathcal{K}$ and the vector space $\mathcal{E}$ is connected by the operator $\mathrm{d} : \mathcal{K} \to \mathcal{E}$:

$$\mathrm{d}(\phi)(x, u, u^{\langle 1\rangle}, \ldots, u^{\langle\ell\rangle}) = \sum_{i=1}^{n} \frac{\partial \phi}{\partial x_i} \mathrm{d}x_i + \sum_{\ell\geq 0} \sum_{j=1}^{m} \frac{\partial \phi}{\partial u_j^{\langle\ell\rangle}} \mathrm{d}u_j^{\langle\ell\rangle}.$$

The operator $\sigma_f : \mathcal{K} \to \mathcal{K}$ induces a forward-shift operator $\sigma_f : \mathcal{E} \to \mathcal{E}$ by

$$\sigma_f : \sum_i a_i \mathrm{d}\phi_i \to \sum_i \sigma_f(a_i) \mathrm{d}(\sigma_f(\phi_i)), \; a_i \in \mathcal{K}, \phi_i \in \mathcal{C}.$$

A differential one-form $\omega \in \mathcal{E}$ is said to be exact if $\omega = \mathrm{d}F$ for some function $F \in \mathcal{K}$. It is said to be integrable if $\omega = G\mathrm{d}F$ for some functions $F, G \in \mathcal{K}$. The notion of integrability is extended to the vector space and can be checked by Frobenius theorem [15].

Definition 2. Let $V := \operatorname{span}_{\mathcal{K}}\{\omega_1, \ldots, \omega_r\} \subset \mathcal{E}$ be the r-dimensional subspace. The subspace V is said to be integrable if $V = \operatorname{span}_{\mathcal{K}}\{\mathrm{d}F_1, \ldots, \mathrm{d}F_r\}$ for some functions $F_i \in \mathcal{K}$, $i = 1, \ldots, r$.

2.2. Observability

Two types of observability are addressed in this paper: single- and multiple-experiment observability. In the next section, the single-experiment observability decomposition is studied via the multiple-experiment observability decomposition.

First, we recall the definition of single-experiment observability. Introduce the subspaces $\mathcal{X}$, $\mathcal{U}$, and $\mathcal{Y}$ of $\mathcal{E}$ as follows (see more in [3]): $\mathcal{X} = \operatorname{span}_{\mathcal{K}}\{\mathrm{d}x_1, \ldots, \mathrm{d}x_n\}$, $\mathcal{U} = \operatorname{span}_{\mathcal{K}}\{\mathrm{d}u_1^{\langle\ell\rangle}, \ldots, \mathrm{d}u_m^{\langle\ell\rangle}, \ell \geq$