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Sliding mode control of Schrödinger equation-ODE in the presence of unmatched disturbances[☆]

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ABSTRACT

In this paper, we consider boundary stabilization for a cascade of Schrödinger equation-ODE system with both, matched and unmatched disturbances. The backstepping method is first applied to transform the system into an equivalent target system where the target system is input-to-state stable. To reject the matched disturbance, the sliding mode control (SMC) law is designed for the target system. The well-posedness of the closed-loop system is proved, and the reachability of the sliding manifold in finite time is justified by infinite-dimensional system theory. It is shown that the resulting closed-loop system is input-to-state stable. A Numerical example illustrates the efficiency of the sliding mode design that reduces the ultimate bound of the closed-loop system by rejecting the matched disturbance.

1. Introduction

In the present paper, we consider stabilization of the Schrödinger equation-ODE cascade system with matched and unmatched disturbances. The main contribution of this paper is the design of a state feedback controller that practically stabilizes the coupled system in the presence of small unmatched disturbance by rejecting the matched disturbance. The control problems for unperturbed Schrödinger equations have been well studied and many nice results have been obtained. For instance, E. Machtyngier [1] discussed the exact controllability of Schrödinger equation in bounded domains with Dirichlet boundary condition. E. Machtyngier and E. Zuazua in [2] further considered the stabilization problem of the Schrödinger equation. By introducing multiplier techniques and constructing energy functionals, they have proved the exponential stabilization of the system. M. Krstic developed backstepping approach to deal with the problem of stabilization of Schrödinger equation in [3–5].

During the last decade, a considerable amount of attention has been paid to stability and control of systems described by partial differential equations (PDEs) subject to external disturbances. In [6,7], a stabilizing controller is designed for vibrating system with uncertainty by the Lyapunov functional approach. Input-to-state stability of the wave equation with a boundary disturbance is studied in [8]. Stabilization for a wave equation with distributed control and uncertainty by variable structure control is considered in [9]. Direct output feedback stabilization for a heat equation by the Lyapunov function method is discussed in [9]. More recently, the sliding mode boundary control is designed for a one-dimensional unstable heat equation in [10]. The sliding mode control is also applied to deal with stabilization for one dimensional wave equation, Euler–Bernoulli equation, Schrödinger equation, and cascaded heat partial differential equation system, where the control channel is subject to external disturbance, in [11,12], [13] and [14] respectively. SMC of finite-dimensional systems in the presence of unmatched disturbances is considered in [15]. In [16], SMC is designed to guarantee minimization of unmatched disturbance effects on system motions in a sliding mode. However, the problem of feasible controller design for coupled ODE–PDE systems as well as for coupled PDE–PDE systems is far from being complete, and this problem is rather challenging.

In the present paper, to the best of our knowledge, the backstepping-based sliding mode controller is designed for PDEs in the presence of both, unmatched and matched disturbances. Moreover, boundary backstepping-based SMC is extended to a new class of PDEs: cascade of ODE-Schrödinger equation. We consider

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the following cascade with disturbances:

$$\begin{cases}\dot{X}(t)=AX(t)+Bu(0,t)+B_1d_1(t),\ t>0,\\ u_t(x,t)=-\mathrm{i}u_{xx}(x,t),\ 0<x<1,\ t>0,\\ u_x(0,t)=CX(t),\ t>0,\\ u_x(1,t)=U(t)+d_2(t),\ t>0,\end{cases} \tag{1.1}$$

where $A\in\mathbb{C}^{n\times n}$, $B\in\mathbb{C}^{n\times 1}$, $C\in\mathbb{C}^{1\times n}$, $X(t)\in\mathbb{C}^{n\times 1}$ is the state of ordinary differential equation, $u(x,t)\in\mathbb{C}$ is the displacement of Schrödinger equation, and $U(t)\in\mathbb{C}$ is the control actuation. The unmatched disturbance d_1 and the matched one d_2 are assumed to be measurable and bounded functions: $|d_1(t)|\le\varDelta$ and $|d_2(t)|\le M$, where $\varDelta>0$ and $M>0$ are known upper bounds.

Our main objective is state-feedback practical stabilization of the coupled system in the presence of small unmatched disturbance $d_1(t)$ and matched bounded disturbance $d_2(t)$. We design a SMC to reject the matched disturbance. We further establish the reachability of the sliding manifold in finite time and the existence and uniqueness of the solution. Finally, input-to-state stability (ISS) of the target closed-loop system is analyzed.

The structure of the paper is as follows. In the next section, we transform system (1.1) into the equivalent target system by the backstepping method. Section 3 is devoted to the matched disturbance rejection by SMC approach. We design a sliding mode control and prove the existence and uniqueness of solution of the closed-loop system. The reachability of the sliding manifold in finite time is presented. In Section 4, the Lyapunov method is used to show that the closed-loop system on the sliding mode surface is input-to-state stable. An example with numerical simulation is presented in Section 5 for illustration of the effectiveness of the method. Concluding remarks are presented in Section 6.

Notation. The Sobolev space $W^{k,p}(\varOmega)$ is defined as $W^{k,p}(\varOmega)=\{u\ :\ D^\alpha u\in L^p(\varOmega),\ \textit{for all}\ 0\le|\alpha|\le k\}$ with norm $\|u\|_{W^{k,p}}=\left\{\sum_{0\le|\alpha|\le k}\|D^\alpha u\|^p_{L^p}\right\}^{\frac{1}{p}}$. $W^{k,2}(\varOmega)=H^k(\varOmega)$ is the Sobolev space of absolutely continuous scalar functions on $\varOmega$ with square integrable derivatives of the order $k\ge 1$.

2. Backstepping transformation

First, following (M. Krstic, A. Smyshlyaev [4]), we introduce a transformation for $[X,u]\to[X,w]$ in the form

$$\begin{cases}X(t)=X(t),\\ w(x,t)=u(x,t)-\displaystyle\int_0^x q(x,y)u(y,t)dy-\gamma(x)X(t)\end{cases} \tag{2.1}$$

where

$$q(x,y)=\int_0^{x-y}\mathrm{i}\gamma(\sigma)Bd\sigma, \tag{2.2}$$

$$\gamma(x)=\begin{bmatrix}K & C & \mathrm{i}KA\end{bmatrix}e^{\begin{bmatrix}0 & 0 & -\mathrm{i}BC\\ I & 0 & \mathrm{i}A\\ 0 & I & 0\end{bmatrix}x}\begin{bmatrix}I\\0\\0\end{bmatrix}. \tag{2.3}$$

The transformations (2.1) transform the system (1.1) into the intermediate system of ODE-Schrödinger cascades of the following form:

$$\begin{cases}\dot{X}(t)=(A+BK)X(t)+Bw(0,t)+B_1d_1(t),\\ w_t(x,t)=-\mathrm{i}w_{xx}(x,t),\\ w_x(0,t)=0,\\ w_x(1,t)=W(t)+d_2(t),\end{cases} \tag{2.4}$$

where $w(x,t)\in\mathbb{C}$. Assume that (A,B) is stabilizable and $K\in\mathbb{C}^{1\times n}$ is chosen such that $A+BK$ is Hurwitz. Here $W(t)$ is intermediate system controller of the form:

$$W(t)=U(t)-q(1,1)u(1,t)-\int_0^1 q_x(1,y)u(y,t)dy-\gamma'(1)X(t). \tag{2.5}$$

The transformation (2.1) is invertible,

$$\begin{cases}X(t)=X(t),\\ u(x,t)=w(x,t)+\displaystyle\int_0^x l(x,y)w(y,t)dy+\psi(x)X(t),\end{cases} \tag{2.6}$$

where

$$l(x,y)=\int_0^{x-y}\mathrm{i}\psi(\sigma)Bd\sigma, \tag{2.7}$$

$$\psi(x)=\begin{bmatrix}K & C\end{bmatrix}e^{\begin{bmatrix}0 & \mathrm{i}(A+BK)\\ I & 0\end{bmatrix}x}\begin{bmatrix}I\\0\end{bmatrix}. \tag{2.8}$$

Next, a further transformation from $[X,w]\to[X,z]$ is given by

$$\begin{cases}X(t)=X(t),\\ z(x,t)=w(x,t)-\displaystyle\int_0^x k(x,y)w(y,t)dy,\end{cases} \tag{2.9}$$

where

$$k(x,y)=-cix\frac{I_1\left(\sqrt{ci(x^2-y^2)}\right)}{\sqrt{ci(x^2-y^2)}}, \tag{2.10}$$

and I_1 is the modified Bessel function,

$$I_1(x)=\sum_{n=0}^{\infty}\frac{\left(\frac{x}{2}\right)^{2n+1}}{n!(n+1)!}. \tag{2.11}$$

Hence, we obtain the target system:

$$\begin{cases}\dot{X}(t)=(A+BK)X(t)+Bz(0,t)+B_1d_1(t),\\ z_t(x,t)=-\mathrm{i}z_{xx}(x,t)-cz(x,t),\\ z_x(0,t)=0,\\ z_x(1,t)=Z(t)+d_2(t),\end{cases} \tag{2.12}$$

where $c>0$ and

$$Z(t)=W(t)-k(1,1)w(1,t)-\int_0^1 k_x(1,y)w(y,t)dy. \tag{2.13}$$

Then,

$$\begin{aligned}Z(t)=&\,U(t)-q(1,1)u(1,t)-\int_0^1 q_x(1,y)u(y,t)dy-\gamma'(1)X(t)\\ &-k(1,1)\left[u(1,t)-\int_0^1 q(1,y)u(y,t)dy-\gamma(1)X(t)\right]\\ &-\int_0^1 k_x(1,y)\left[u(y,t)-\int_0^y q(y,\tau)u(\tau,t)d\tau-\gamma(y)X(t)\right]dy.\end{aligned} \tag{2.14}$$

The inverse of the transformation (2.9) can be found as follows

$$\begin{cases}X(t)=X(t),\\ w(x,t)=z(x,t)+\displaystyle\int_0^x p(x,y)z(y,t)dy,\end{cases} \tag{2.15}$$

where

$$p(x,y)=-cix\frac{J_1(\sqrt{ci(x^2-y^2)})}{\sqrt{ci(x^2-y^2)}}, \tag{2.16}$$

and J_1 is the Bessel function of first kind.