Systems & Control Letters 98 (2016) 79–85

Contents lists available at ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Performance output exponential tracking for a wave equation with a general boundary disturbance[☆]

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ARTICLE INFO

Article history:
Received 12 May 2016
Received in revised form 31 August 2016
Accepted 19 October 2016
Available online 16 November 2016

Keywords:
Disturbance
Hidden regularity
Performance output tracking
Wave equation

ABSTRACT

In this paper, we consider performance output tracking for a wave equation with a general boundary disturbance. The control and the disturbance are unmatched. Different from the existing results, the hidden regularity, instead of the high gain or variable structure, is utilized in the adaptive servomechanism design. As a result, we are able to cope with more complicated and general disturbances. Moreover, the performance output can track the reference signal exponentially, and at the same time, all the states of the subsystems involved are uniformly bounded. More specially, the overall closed-loop system is exponentially stable when the disturbance and reference are disconnected to the system. The numerical simulations are presented to illustrate the effect of the proposed scheme.

1. Introduction

Adaptive servomechanism design is an important and interesting problem in control theory [1–6], not only for finite-dimensional systems but also for infinite-dimensional systems. In this paper, we are concerned with the performance output tracking for a one-dimensional wave equation with a general boundary disturbance at one end and a boundary control at the other end. This problem can described by the following equation:

$$\begin{cases} u_{tt}(x,t) = u_{xx}(x,t), \ x \in (0,1), \ t > 0, \\ u_x(0,t) = qu_t(0,t) + d(t), \ t \geq 0, \\ u_x(1,t) = U(t), \ t \geq 0, \\ u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x), \ x \in [0,1], \\ y_{out}(t) = \{u_t(1,t), u(0,t)\}, \end{cases} \tag{1.1}$$

where $u_x(x,t)$ and $u_t(x,t)$ denote the derivative of $u(x,t)$ with respect to x and t, respectively, $y_{out}(t)$ is the output (measurement), $U(t)$ is the input (control), q is a positive constant, $(u_0(x), u_1(x))$ is the initial value, and $d \in L^\infty(0,\infty)$ is the external disturbance. The main objective of this paper is to design an output feedback controller such that, for any given reference signal $u_{ref}(t)$,

$$u(1,t) \to u_{ref}(t) \ \text{as} \ t \to \infty, \tag{1.2}$$

where $u(1,t)$ is the performance output. Since system (1.1) decays exponentially provided $d = 0$ and $U = 0$, it is obvious that $u(1,t)$ is measurable.

The problem of performance output tracking for wave equation has been considered in [7] where the Lyapunov functional method is used in the controller design. Since the disturbance is matched to the control in [7], the method of active disturbance rejection control (ADRC) can be used to cope with the disturbance. However, the main technique used in disturbance estimation is the variable structure which gives rise to difficulties for not only the controller design but also the well-posedness of closed-loop system. Another drawback of this method is that the derivative of disturbance is required to be bounded. In [8], the problem of performance output tracking for system (1.1) with harmonic disturbance is considered. The structure of the disturbance is supposed to be

$$d(t) = \sum_{j=1}^{m} (\bar{\theta}_j \sin \alpha_j t + \bar{\vartheta}_j \cos \alpha_j t), \tag{1.3}$$

where the amplitudes $\bar{\theta}_j$ and $\bar{\vartheta}_j$ are unknown parameters while all the frequencies α_j are supposed to be known. By exploiting the harmonic structure (1.3), an adaptive controller was proposed to achieve both the unknown parameters estimation and performance output reference tracking. However, this method in [8] is invalid to more general disturbances. For example, when we come across an aperiodic disturbance $d(t) = \sin((t+1)^{-1})$, such an approach does not apply anymore. Furthermore, since they deal with the disturbance by finite dimensional estimator in both [7] and [8], the exponential convergence of the performance output tracking cannot be obtained.

☆ This work was supported by the National Natural Science Foundation of China (11671240, 61403239, 61503230), the STIP 2014101, and the Science Council of Shanxi Province (2015021010).

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http://dx.doi.org/10.1016/j.sysconle.2016.10.007
0167-6911/

Different from [7,8], we use the infinite dimensional property like the hidden regularity to cope with the general disturbance $d \in L^\infty(0,\infty)$ which contains the harmonic disturbance as a special case. Thanks to the infinite dimensional structure, the exponential convergence of the performance output tracking is proved. Meanwhile, all the states of the subsystems involved are uniformly bounded. More specially, the overall closed-loop system is exponentially stable when the disturbance and reference are disconnected to the system. In other words, the disturbance and reference free closed-loop system is internally exponentially stable.

We proceed as follows. In Section 2, we propose an infinite-dimensional disturbance estimator in terms of the output by using the hidden regularity of the wave equation. In Section 3, an estimator based output feedback law is designed. Hence, the closed-loop system is then obtained. Section 4 is devoted to the well-posedness and exponential stability of transformed system. The well-posedness and stability of the closed-loop system are presented in Section 5. Numerical simulations are presented in Section 6 to validate the theoretical results, followed by the concluding remarks in Section 7.

2. Infinite-dimensional disturbance estimator design

In this section, we will design an infinite-dimensional disturbance estimator for system (1.1). Instead of the high gain, the hidden regularity of the wave equation is utilized in the estimator design. The infinite dimensional disturbance estimator is designed as the following system:

$$\begin{cases}\hat{u}_{tt}(x,t)=\hat{u}_{xx}(x,t),\ x\in(0,1),\ t>0,\\ \hat{u}(0,t)=u(0,t),\ t\ge 0,\\ \hat{u}_x(1,t)=U(t)+c_0(u_t(1,t)-\hat{u}_t(1,t)),\ t\ge 0,\end{cases}\tag{2.1}$$

where c_0 is a positive tuning parameter. The estimator design is mainly inspired by [9] where an estimator based controller is proposed to stabilize an anti-stable wave equation. The motivation behind (2.1) is trivial. Indeed, if we let

$$\varepsilon(x,t)=u(x,t)-\hat{u}(x,t),\tag{2.2}$$

the error $\varepsilon(x,t)$ is governed by

$$\begin{cases}\varepsilon_{tt}(x,t)=\varepsilon_{xx}(x,t),\ x\in(0,1),\ t>0,\\ \varepsilon(0,t)=0,\ t\ge 0,\\ \varepsilon_x(1,t)=-c_0\varepsilon_t(1,t),\ t\ge 0,\end{cases}\tag{2.3}$$

which is a well-known wave equation with a "passive damper" boundary condition at $x=1$, and a Dirichlet boundary condition at $x=0$. On the other hand, it follows from (1.1), (2.1) and (2.3) that

$$\begin{aligned}\varepsilon_x(0,t)&=qu_t(0,t)-\hat{u}_x(0,t)+d(t)\\ &=q\hat{u}_t(0,t)-\hat{u}_x(0,t)+d(t).\end{aligned}\tag{2.4}$$

Therefore, $\hat{u}_x(0,t)-q\hat{u}_t(0,t)$ can be considered as an approximation of disturbance $d(t)$, provided $\varepsilon_x(0,t)\to 0$ as $t\to\infty$.

We consider system (2.3) in Hilbert space $\mathcal{H}=H_L^1(0,1)\times L^2(0,1)$, where $H_L^1(0,1)=\{f\in H^1(0,1)|f(0)=0\}$. The inner product in $\mathcal{H}$ is given by

$$\langle (f_1,g_1),(f_2,g_2)\rangle_{\mathcal{H}}=\int_0^1\left[f_1'(x)\overline{f_2'(x)}+g_1(x)\overline{g_2(x)}\right]\mathrm{d}x,\quad \forall (f_i,g_i)\in\mathcal{H},\ i=1,2.\tag{2.5}$$

System (2.3) can be rewritten as the following abstract form

$$\frac{\mathrm{d}}{\mathrm{d}t}\Big(\varepsilon(\cdot,t),\varepsilon_t(\cdot,t)\Big)=\mathcal{A}_0\Big(\varepsilon(\cdot,t),\varepsilon_t(\cdot,t)\Big),\tag{2.6}$$

where the operator $\mathcal{A}_0$ is defined by

$$\begin{cases}\mathcal{A}_0(f,g)=(g,f''),\quad \forall (f,g)\in D(\mathcal{A}_0),\\ D(\mathcal{A}_0)=\Big\{(f,g)\in H^2(0,1)\times H^1(0,1)\,|\,f(0)=g(0)=0,\\ \quad f'(1)=-c_0g(1)\Big\}.\end{cases}\tag{2.7}$$

It is well-known that the operator $\mathcal{A}_0$ generates an exponentially stable C_0-semigroup in $\mathcal{H}$: there exist two positive constants L and ω such that

$$\left\|e^{\mathcal{A}_0t}\right\|_{\mathcal{H}}\le Le^{-\omega t},\quad \forall\, t\ge 0.\tag{2.8}$$

Moreover, the following hidden regularity holds:

Lemma 2.1. *For any initial value $(\varepsilon(\cdot,0),\varepsilon_t(\cdot,0))\in\mathcal{H}$, there exist two positive constants L and ω such that the solution $(\varepsilon(\cdot,t),\varepsilon_t(\cdot,t))\in C(0,\infty;\mathcal{H})$ of system (2.3) satisfies*

$$\|(\varepsilon(\cdot,t),\varepsilon_t(\cdot,t))\|_{\mathcal{H}}\le Le^{-\omega t}\|(\varepsilon(\cdot,0),\varepsilon_t(\cdot,0))\|_{\mathcal{H}},\quad t\ge 0.\tag{2.9}$$

Moreover, the following hidden regularity holds

$$\varepsilon_x(0,t)\in L^2(0,\infty).\tag{2.10}$$

Proof. Since the well-posedness of (2.3) and (2.9) is well-known, we only need to prove (2.10). Since $D(\mathcal{A}_0)$ is dense in $\mathcal{H}$, it is sufficient to prove (2.10) for any $(\varepsilon(\cdot,0),\varepsilon_t(\cdot,0))\in D(\mathcal{A}_0)$. We let

$$\rho(t)=2\int_0^1(x-1)\varepsilon_x(x,t)\varepsilon_t(x,t)\mathrm{d}x.\tag{2.11}$$

Then

$$|\rho(t)|\le\|(\varepsilon(\cdot,t),\varepsilon_t(\cdot,t))\|_{\mathcal{H}}^2\le L^2\|(\varepsilon(\cdot,0),\varepsilon_t(\cdot,0))\|_{\mathcal{H}}^2.\tag{2.12}$$

We find the derivative of $\rho(t)$ along the solution of system (2.3) to obtain

$$\dot{\rho}(t)=\varepsilon_x^2(0,t)-\int_0^1\left[\varepsilon_t^2(x,t)+\varepsilon_x^2(x,t)\right]\mathrm{d}x,\tag{2.13}$$

which implies that

$$\begin{aligned}&\int_0^t\varepsilon_x^2(0,\tau)\mathrm{d}\tau\\ &=\rho(t)-\rho(0)+\int_0^t\int_0^1\left[\varepsilon_t^2(x,\tau)+\varepsilon_x^2(x,\tau)\right]\mathrm{d}x\mathrm{d}\tau\\ &\le 2L^2\|(\varepsilon(\cdot,0),\varepsilon_t(\cdot,0))\|_{\mathcal{H}}^2\\ &\quad+L^2\|(\varepsilon(\cdot,0),\varepsilon_t(\cdot,0))\|_{\mathcal{H}}^2\int_0^te^{-2\omega\tau}\mathrm{d}\tau\\ &\le\left(\frac{1}{2\omega}+2\right)L^2\|(\varepsilon(\cdot,0),\varepsilon_t(\cdot,0))\|_{\mathcal{H}}^2.\end{aligned}\tag{2.14}$$

Therefore, (2.10) holds due to the arbitrariness of time t. □

Corollary 2.1. *Suppose that $(\varepsilon(\cdot,0),\varepsilon_t(\cdot,0))\in D(\mathcal{A}_0)$. Then system (2.3) admits a unique classical solution $(\varepsilon(\cdot,t),\varepsilon_t(\cdot,t))\in C(0,\infty;D(\mathcal{A}_0))$ such that*

$$|\varepsilon_x(0,t)|\le V_0Le^{-\omega t},\quad \forall t\ge 0,\tag{2.15}$$

where V_0 is a positive constant independent of t.

Proof. By semigroup theory and Lemma 2.1, system (2.3) admits a unique classical solution $(\varepsilon(\cdot,t),\varepsilon_t(\cdot,t))\in C(0,\infty;D(\mathcal{A}_0))\bigcap C^1(0,\infty;\mathcal{H})$ that satisfies the hidden regularity $\varepsilon_x(0,t)\in H_{loc}^1(0,\infty)$. If we let $\bar{\varepsilon}(x,t)=\varepsilon_t(x,t)$, then we differentiate system (2.3) with respect to t to obtain that $(\bar{\varepsilon}(\cdot,t),\bar{\varepsilon}_t(\cdot,t))\in C(0,\infty;\mathcal{H})$ is a weak solution of the following system:

$$\begin{cases}\bar{\varepsilon}_{tt}(x,t)=\bar{\varepsilon}_{xx}(x,t),\ x\in(0,1),\ t>0,\\ \bar{\varepsilon}(0,t)=0,\ t\ge 0,\\ \bar{\varepsilon}_x(1,t)=-c_0\bar{\varepsilon}_t(1,t),\ t\ge 0.\end{cases}\tag{2.16}$$

By Lemma 2.1, system (2.16) admits a unique solution $(\bar{\varepsilon}(\cdot,t),\bar{\varepsilon}_t(\cdot,t))\in C(0,\infty;\mathcal{H})$ such that

$$\|(\bar{\varepsilon}(\cdot,t),\bar{\varepsilon}_t(\cdot,t))\|_{\mathcal{H}}\le Le^{-\omega t}\|(\varepsilon_t(\cdot,0),\varepsilon_{xx}(\cdot,0))\|_{\mathcal{H}}.\tag{2.17}$$