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Fault detection and isolation of two-dimensional (2D) systems[☆]

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ABSTRACT

In this work, we develop a novel fault detection and isolation (FDI) scheme for discrete-time two-dimensional (2D) systems that are represented by the Fornasini–Marchesini model II (FMII). This is accomplished by generalizing the basic invariant subspaces including unobservable, conditioned invariant and unobservability subspaces of 1D systems to 2D models. These extensions have been achieved and facilitated by representing a 2D model as an infinite dimensional (Inf-D) system on a Banach vector space, and by particularly constructing algorithms that compute these subspaces in a *finite and known* number of steps. By utilizing the introduced subspaces, the FDI problem is formulated and necessary and sufficient conditions for its solvability are provided. Sufficient conditions for solvability of the FDI problem for 2D systems using both deadbeat and LMI-based filters have also been developed. Moreover, the capabilities and advantages of our proposed approach are demonstrated by performing an analytical comparison with the currently available 2D geometric methods in the literature.

1. Introduction

Two-dimensional (2D) systems have received considerable interest and attention from various researchers, particularly from the system control theory community. For example, by discretizing spatial and time coordinates, one can approximate *PDE* systems by 2D systems [1]. Moreover, linear repetitive control systems can be modeled as 2D systems [2]. However, there are only a few results on fault detection and isolation (FDI) of 2D systems in the literature, as in dead-beat filters [3] and parity equations [4,5].

The geometric control theory [6,7] has provided a valuable tool for addressing the FDI problem of a large class of dynamical systems such as parabolic PDE systems [8,9], Markovian jump systems [10] and linear impulsive systems [11]. In this paper, we investigate the FDI problem of 2D systems by using a geometric methodology.

The geometric theory of 2D systems has recently attracted much interest, where basic concepts such as invariant subspaces are studied in detail for the Fornasini and Marchesini model I (FMI) [12,13]. The geometric FDI approach for 2D systems, for the first time, was addressed in [1], where invariant subspaces of the Roesser model are defined and the FDI problem is formulated based on these subspaces. In this paper, we investigate observability of 2D systems from a *new* geometric point of view which has its roots in system theory of infinite dimensional (Inf-D) systems defined on a Banach vector space.

Compared to results reported in [1,14], we have specific generalization and novel contributions in this work. We *first* investigate the Fornasini and Marchesini model II (FMII) as an Inf-D system that allows us to deal with Inf-D subspaces. In addition, in [1] by utilizing the existence of an LMI-based observer only sufficient conditions for solvability of the Roesser model FDI problem were provided. In other words, the procedure to design the observer gains is not provided in [1,14]. However, in our paper, we derive *both* necessary and sufficient conditions, where sufficient conditions are based on (a) an ordinary, (b) a delayed deadbeat, and (c) an LMI-based 2D Luenberger filters. Moreover, we develop a procedure to design LMI-based filter gains.

It should be pointed out that a related work has appeared in [15], where solvability of the FDI problem for three-dimensional (3D) FMII models is considered. The geometric FDI methodology that is developed in [15] is analytically investigated in Section 4.1, where it is shown that our proposed method is more general than the approach in [15].

Another approach that was developed in the literature [3,5] has its roots in 2D deadbeat filters [16]. In [3], by using polynomial matrices and unknown input deadbeat observers, it was shown that the solvability of the FDI problem (and estimating the fault severities) is equivalent to the right zero primeness of the 2D Popov–Belevitch–Hautus (PBH) matrix. In [5], this condition was

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relaxed and necessary and sufficient conditions that are based on an extended parity equation approach were obtained. We use these results to provide a novel geometric FDI scheme for 2D systems.

To summarize, the main contributions of this paper can be stated as follows:

(1) Investigate and introduce the Inf-D conditioned invariant and unobservability subspaces by utilizing a Fin-D representation.
(2) Develop an LMI-based methodology to design a 2D Luenberger observer for FDI of 2D systems.
(3) Derive the necessary and sufficient conditions for solvability of the FDI problem by using (a) the delayed, (b) the ordinary deadbeat, and (c) the 2D Luenberger filters.
(4) Analytically compare our proposed methodology with the available geometric FDI approach of 2D systems in the literature [15].

It should be pointed out that due to space limitations simulation results are not included here. However, these simulations as well as additional explanations can be found in the full electronic version of our paper in [17].

Notation: We use $\mathscr{A}, \mathscr{B}, \ldots$ to denote subspaces. For a given vector L, the subspace $\text{span}\{L\}$ is denoted by $\mathscr{L}$. The inverse image of the subspace $\mathscr{V}$ with respect to the operator A is denoted by $A^{-1}\mathscr{V}$. The block diagonal matrix is denoted by $\text{diag}(A, B)$. The real, complex, integer and positive integer numbers are denoted by $\mathbb{R}, \mathbb{C}, \mathbb{Z}$ and $\mathbb{N}$, respectively. $\underline{\mathbb{N}}$ denotes the set $\mathbb{N} \cup \{0\}$. In this paper, we deal with Inf-D subspaces and vectors. An Inf-D vector is designated by the bold letters $\mathbf{x}, \mathbf{y}, \ldots$. The Inf-D subspace $\cdots \oplus \mathscr{V} \oplus \mathscr{V} \oplus \cdots$ is denoted by $\oplus\mathscr{V}$, where $\mathscr{V} \subseteq \mathbb{R}^n$. Let $\mathbf{x} = (x_k)_{k\in\mathbb{Z}} = (\ldots, x_{-1}^{\mathrm{T}}, x_0^{\mathrm{T}}, x_1^{\mathrm{T}}, \ldots)^{\mathrm{T}} \subseteq \oplus\mathscr{V}$ and $|\mathbf{x}|_\infty = \sup_{i\in\mathbb{Z}} |x_i|$, where $x_i \in \mathscr{V}$. The vector space $\mathscr{V}_\infty = \sum \mathscr{V}$ is defined as $\{\mathbf{x} | \mathbf{x} \in \oplus\mathscr{V} \text{ and } |\mathbf{x}|_\infty < \infty\}$. It can be shown that $\mathscr{V}_\infty$ is a Banach (but not necessarily Hilbert) space. Other notations are provided within the text as appropriate.

2. Preliminary results

In this section, we first review the FMII formulation of a 2D system subjected to faults. Subsequently, a 2D system is expressed as an Inf-D system defined on the corresponding Banach vector space. This representation allows one to geometrically analyze the unobservable subspace and one of its subspaces (this is to be defined and specified in the next section). The FDI problem is also formulated in this section. Moreover, we review the 2D PBH matrix and 2D deadbeat observers. Finally, an LMI-based approach is introduced to design a 2D Luenberger observer (also known as a detection filter) for 2D systems.

2.1. Discrete-time 2D systems

In this work, we consider and concentrate on the FMII that includes Roesser model and FMI model as special cases [18]. Consider the following FMII model [18],

$$\begin{aligned} x(i+1, j+1) &= A_1 x(i, j+1) + A_2 x(i+1, j) \\ &\quad + B_1 u(i, j+1) + B_2 u(i+1, j) + \sum_{k=1}^{p} L_k^1 f_k(i, j+1) \\ &\quad + \sum_{k=1}^{p} L_k^2 f_k(i+1, j), \\ y(i, j) &= C x(i, j), \quad i, j \in \mathbb{Z}, \end{aligned} \tag{1}$$

where $x \in \mathbb{R}^n$, $u \in \mathbb{R}^m$, and $y \in \mathbb{R}^q$ denote the state, input and output vectors, respectively. The fault signals and the corresponding fault signatures are designated byf_k, L_k^1 and L_k^2, respectively. Also, p denotes the number of faults in the system. Since in this work all the introduced invariant subspaces are based on the operators A_1, A_2 and C, we designate the system (1) by the triple (C, A_1, A_2).

It is assumed that A_1 and A_2 in model (1) are not necessarily commutative (i.e. $A_1A_2 \neq A_2A_1$), and hence, the results that are subsequently developed can also be applied to the Roesser model.

2.2. Infinite dimensional (Inf-D) representation

Consider the fault free system (1), that is with $f_k \equiv 0$, and with zero input. By considering $\mathbf{x}(k) = (\ldots, x(-1+k, 1)^{\mathrm{T}}, x(k, 0)^{\mathrm{T}}, x(1+k, -1)^{\mathrm{T}}, \ldots)^{\mathrm{T}} \in \sum \mathbb{R}^n$, the system (1) can be represented as,

$$\begin{aligned} \mathbf{x}(k+1) &= \mathcal{A}\mathbf{x}(k), \quad k \in \underline{\mathbb{N}} \\ \mathbf{y}(k) &= \mathcal{C}\mathbf{x}(k), \end{aligned} \tag{2}$$

where the global state and outputs are denoted by $\mathbf{x}(k) \in \mathcal{X} = \sum \mathbb{R}^n$, $\mathbf{y}(k) = (\ldots, y(-1+k, 1)^{\mathrm{T}}, y(k, 0)^{\mathrm{T}}, y(1+k, -1)^{\mathrm{T}}, \ldots)^{\mathrm{T}} \in \sum \mathbb{R}^q$, respectively. Also, $\mathcal{A}$ is an Inf-D matrix with A_1 and A_2 as diagonal and upper diagonal blocks, respectively, with the remaining elements set to zero, and $\mathcal{C} = \text{diag}(\ldots, C, C, \ldots)$. Note that since we invoke an Inf-D representation to investigate an unobservable subspace, and where this subspace is defined by only $\mathcal{A}$ and $\mathcal{C}$, therefore to represent the Inf-D system (2), we consider the 2D system (1) that is subjected to no faults.

There are various formulations for the initial conditions of the FMII model (1) based on the separation set that is introduced in [19]. There are two separation sets that are commonly used in the literature. In the first formulation the initial conditions are denoted by $\mathbf{x}(0) = (\ldots, x(-1, 1)^{\mathrm{T}}, x(0, 0)^{\mathrm{T}}, x(1, -1)^{\mathrm{T}}, \ldots)^{\mathrm{T}} \in \sum \mathbb{R}^n$ [18] (this is compatible with the model (2)). The second formulation is expressed as $x(i, 0) = h_1(i)$ and $x(0, j) = h_2(j)$, where $h_1(i), h_2(j) \in \mathbb{R}^n$ and $i, j \in \mathbb{N}$ [18]. Given that we derive the solvability conditions of the FDI problem based on the finite invariant unobservable subspace (this is defined explicitly in Section 3.1), our proposed methodology is applicable to *both* initial condition formulations. In other words, we use the Inf-D representation to only show the results and evaluate the developed algorithms. However, to apply our results there is no need to deal with Inf-D systems and subspaces, and therefore, one can apply our proposed methods to 2D systems corresponding to both initial condition formulations. For more detail, refer to Remark 4.

The system theory corresponding to Inf-D systems is significantly more challenging than Fin-D systems (1D systems) (refer to [20]). However, as shown subsequently, the operator $\mathcal{A}$ is bounded and consequently, one can readily extend the result of 1D systems to the Inf-D system (2) [20]. Let us first define the notion of bounded operators.

Definition 1 (*[20]*)**.** Consider the operator $\mathcal{A} : \mathcal{X}_1 \to \mathcal{X}_2$, where $\mathcal{X}_1$ and $\mathcal{X}_2$ are Banach vector spaces with the norms $|\cdot|_1$ and $|\cdot|_2$, respectively. The operator $\mathcal{A}$ is bounded if there exists a real number G such that $|\mathcal{A}\mathbf{x}|_2 \le G|\mathbf{x}|_1$ for all $\mathbf{x} \in \mathcal{X}_1$.

Lemma 1. *The operator $\mathcal{A}$ as defined in the Inf-D system (2) is bounded.*

Proof. Let $G = 2\max(|A_1|, |A_2|)$, where $|A_i|$ denotes the norm of A_i and $\mathbf{x} = (x_k)_{k\in\mathbb{Z}} \in \mathcal{X}$. It follows readily that $|\mathcal{A}\mathbf{x}|_\infty = \sup_{k\in\mathbb{Z}} |A_1x_k + A_2x_{k+1}| \le \sup_{k\in\mathbb{Z}} [G\max(|x_k|, |x_{k+1}|)] = G\sup_{k\in\mathbb{Z}} |x_k|$. Therefore, $|\mathcal{A}\mathbf{x}|_\infty \le G|\mathbf{x}|_\infty$. This completes the proof of the lemma.

Remark 1. Although, in [21] all the results such as the controlled invariant subspaces are presented on $\mathbb{R}^n$, the developed approach in [21] has its roots in the theory of systems over rings. In this paper, we propose an alternative approach that is based on Inf-D systems that are defined on a Banach vector space. Similar to