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# Fully distributed containment control of high-order multi-agent systems with nonlinear dynamics ☆

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ABSTRACT

In this paper, distributed containment control problems for high-order multi-agent systems with nonlinear dynamics are investigated under directed communication topology. The states of the leaders are only available to a subset of the followers and the inputs of the leaders are possibly nonzero and time varying. Distributed adaptive nonlinear protocol is proposed based only on the relative state information, under which the states of the followers converge to the dynamic convex hull spanned by those of the leaders. As the special case with only one dynamic leader, leader–follower consensus problem is also solved with the proposed protocol. The adaptive protocol here is independent of the eigenvalues of the Laplacian matrix, which means the protocol can be implemented by each agent in a fully distributed fashion. A simulation example is provided to illustrate the theoretical results.



## 1. Introduction

In the past decade, cooperative control of multi-agent systems has received compelling attention due to its broad applications in several areas such as consensus, rendezvous, flocking, formation control and sensor networks [1,2]. An important feature in distributed cooperative control of multiple agents is that each agent updates its own state based on the information from itself and its local (time-varying) neighbors. Existing consensus problems can be roughly categorized into three classes, namely, leaderless consensus problem [3–6], leader–follower consensus problem with one leader (also called cooperative tracking problem) [7–13] and containment control problem with multiple leaders.

Recently, containment control problems with multiple leaders have been investigated a lot. The main objective of containment control is to drive the states of the followers into the convex hull spanned by the leaders. The study of containment control is motivated by numerous natural phenomena and potential applications in practice. For instance, a group of autonomous agents moves from one target to another when only a portion of the agents (designated as leaders) is equipped with necessary sensors to detect the hazardous obstacles such that the agents (designated as followers) who are not equipped will stay in a safety area formed by the leaders. In [14], some necessary and sufficient conditions were established to guarantee the achievement of containment control for both continuous-time and sampled-data based protocols. Distributed containment control problems for first-order and second-order integrator dynamics in the presence of both stationary and dynamic leaders under fixed and switching directed communication topologies were considered in [15,16]. In [17], containment control problem was considered for a second-order multi-agent system with time-varying delays. In [18], distributed containment control problem was investigated for continuous time heterogeneous multi-agent system which is composed of first-order and second-order integrator agents under directed topologies. The containment control problems for multi-agent systems with general linear dynamics were considered in [19,20].

Note that the agent dynamics in the above-mentioned literature are focused on linear systems, even simple integrators in some cases. However, many physical systems are inherently nonlinear in practice. There are many commonly observed phenomena that cannot be described by linear equations, such as multiple equilibria, limit cycles, bifurcations, and complex dynamical behavior, to name a few [21]. Therefore, it is important and meaningful to investigate distributed consensus problem for multi-agent systems with nonlinear dynamics. Consensus problems with one leader for nonlinear multi-agent systems were studied in references [8,13]. In [8], leader–follower consensus problem was studied for a group of non-identical second-order nonlinear systems. Note that, the above results required the information of the graph Laplacian matrix, which is global information and unknown to any of the agent in distributed consensus problems. In [13], adaptive protocols without using any Laplacian matrix information were proposed for

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consensus problem of nonlinear multi-agent systems via relative output feedback. Adaptive control techniques are used to tackle both the unknown connectivity and the nonlinear term in the systems. Recent work on containment control of nonlinear systems often focuses on some special nonlinear models, such as Lagrangian systems [22], attitude dynamics of rigid bodies [23], and integrator systems with nonlinear dynamics [24–27]. Specifically, in [25], the authors studied the distributed containment control problem for first-order multi-agent systems with inherent nonlinear dynamics. The work of [25] was extended to second-order nonlinear systems in [26], where the inputs of the leaders are assumed to be zero. In [27], adaptive containment control problem for second-order multi-agent systems with inherent nonlinear dynamics was investigated. In particular, the leaders' control inputs are nonzero, bounded, and not available to any follower.

Motivated by the limitation of existing results, we consider the distributed containment control problem for high-order multi-agent systems with nonlinear dynamics under directed communication topology. The inputs of the leaders are possibly nonzero and time varying. A distributed adaptive nonlinear protocol is proposed to guarantee that the states of the followers asymptotically converge to the convex hull spanned by those of the leaders. When there is only one dynamic leader, leader–follower consensus problem is also solved with the proposed protocol. Compared with the existing works in the literature, the main contribution of this paper is threefold. First, in contrast to the case with one leader for second-order multi-agent systems with unknown nonlinearities in [28], we deal with the containment control problem with multiple leaders. As the special case with only one dynamic leader, leader–follower consensus problem considered in [28] is also solved with the proposed protocol. Second, in [25–27], the authors designed adaptive protocols for first-order and second-order nonlinear systems when the interaction topology among the followers is undirected graph. Compared with these results, our adaptive protocol is applicable for high-order nonlinear multi-agent systems under directed communication topologies. This extension is nontrivial and the main difficulty lies in that the Laplacian matrices of directed graphs are asymmetric, which makes the construction of adaptive protocol and the selection of Lyapunov function difficult. Due to the complexity caused by the directed graph, the adaptive protocol designed in this paper is nonlinear rather than the conventional linear protocol [25–27]. Finally, the adaptive containment protocols here is in fully distributed fashion, because the design of the protocol is independent of the eigenvalues of the Laplacian matrix associated with the entire communication topology, which are the global information, actually.

The rest of this paper is organized as follows. Section 2 provides some preliminaries and problem formulation. In Section 3, containment problem for nonlinear systems with directed communication topology is studied. The effectiveness of the proposed distributed protocols is illustrated by examples in Section 4. Some concluding remarks are given in Section 5.

Notations: $\mathbb{R}^{n\times m}$ denotes a set of $n \times m$ real matrices and $I_n$ represents the identity matrix of dimension $n$. $A \otimes B$ denotes the Kronecker product of matrices $A$ and $B$. For a vector $x \in \mathbb{R}^n$, $\| x \|$ denotes its Euclidean norm. Denote by $\mathrm{dist}(x, \mathcal{C})$ the distance from $x \in \mathbb{R}^n$ to the set $\mathcal{C} \subset \mathbb{R}^n$ in the sense of Euclidean norm, that is

$$\mathrm{dist}(x, \mathcal{C}) = \inf_{y\in\mathcal{C}} \|x - y\|.$$

## 2. Preliminaries and problem setup

### 2.1. Graph theory

A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is used to represent the communication topology in a networked multi-agent system, where $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ represent, respectively, a nonempty finite set of vertices and a set of directed edges. Each agent is represented by a vertex in $\mathcal{V}$ and a directed edge is an ordered pair $(v_i, v_j)$ which represents the information flow from agent $i$ to agent $j$. The neighborhood of the $i$th agent is denoted by $\mathcal{N}_i = \{v_j \in \mathcal{V} | (v_j, v_i) \in \mathcal{E}\}$. A directed path $\mathcal{P}$ in $\mathcal{G}$ is a sequence $\{v_{i_0}, v_{i_1}, \ldots, v_{i_k}\}$ where $(v_{i_{j-1}}, v_{i_j}) \in \mathcal{E}$ for $j = 1, 2, \ldots, k$ and the vertices are distinct. The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ associated with the directed graph $\mathcal{G}$ is defined by $a_{ii} = 0$, $a_{ij} > 0$ if $(v_j, v_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Note that $a_{ij}$ denotes the weight for the edge $(v_j, v_i) \in \mathcal{E}$. The Laplacian matrix of the directed graph $\mathcal{G}$ is defined as $\mathcal{L} = [l_{ij}]_{N\times N} = \mathcal{D} - \mathcal{A}$, where $\mathcal{D} = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$ is called the degree matrix of $\mathcal{G}$ with $d_i = \sum_{j\in\mathcal{N}_i} a_{ij}$, $i = 1, 2, \ldots, N$.

### 2.2. Problem formulation

Suppose that the multi-agent system consists of $N$ followers and $M$ leaders. Denote the set of followers as $\mathcal{F} = \{1, 2, \ldots, N\}$ and the set of leaders as $\mathcal{R} = \{N + 1, N + 2 \ldots, N + M\}$.

**Definition 1** (*[14]*). For an $N+M$ agent system, an agent is called a leader if the agent has no neighbor, and an agent is called a follower if the agent has at least one neighbor.

In this paper, each agent with nonlinear dynamics is represented as

$$\begin{aligned} &\dot{x}_{i,j} = x_{i,j+1}, \quad j = 1, 2, \ldots, n-1 \\ &\dot{x}_{i,n} = u_i + f(x_i, t), \quad i = 1, 2, \ldots, N + M \end{aligned} \tag{1}$$

where $x_{i,j} \in \mathbb{R}^p$, $x_i = [x_{i,1}^T, x_{i,2}^T, \ldots, x_{i,n}^T]^T \in \mathbb{R}^{np}$ represents the state and $u_i \in \mathbb{R}^p$ represents the control input of agent $i$. $f : \mathbb{R}^{np} \times \mathbb{R} \to \mathbb{R}^p$ is the intrinsic nonlinear dynamics satisfies the following assumption.

**Assumption 1.** Given $\eta_1, \eta_2, \ldots, \eta_M$ with $\sum_{i=1}^{M}\eta_i = 1$, and $\eta_i \geq 0$, $i = 1, 2, \ldots, M$. There exists a nonnegative constant $l$, such that for $x, y_i \in \mathbb{R}^{np}$, $i = 1, 2, \ldots, M$,

$$\left\| f(x, t) - \sum_{i=1}^{M} \eta_i f(y_i, t) \right\| \leq l \left\| x - \sum_{i=1}^{M} \eta_i y_i \right\|. \tag{2}$$

**Remark 1.** For the single leader case, that is, $M = 1$, the condition (2) reduces to the following Lipschitz condition

$$\|f(x, t) - f(y, t)\| \leq l \|x - y\|. \tag{3}$$

Since we considered containment control problem for high-order multi-agent systems with nonlinear dynamics under directed communication topology and the followers may be affected by multiple dynamic leaders which have possibly nonzero and time varying inputs, a stronger condition (2) on the nonlinear function $f$ is needed. Similar assumptions also used in [25–27]. This assumption facilitates the following analysis of the derivative of the Lyapunov function. The condition (2) can be applicable to all linear functions and some nonlinear functions, such as $x\sin(t)$, $x\cos(t^2)$ and $xe^{(-t)}$.

**Definition 2.** Let $X$ be a set in a real vector space $V \subset \mathbb{R}^m$, where $m$ is a positive integer. The convex hull $\mathrm{Co}(X)$ of the set $X$ is defined as $\mathrm{Co}(X) = \left\{ \sum_{i=1}^{k} a_i x_i | x_i \in X, a_i \in R, a_i \geq 0, \sum_{i=1}^{k} a_i = 1, k = 1, 2, \ldots \right\}$.

**Definition 3** (*Containment Problem*). The multi-agent system (1) achieves containment if the control law for each follower can be